Universidad de Burgos
Mathematical Physics Group

Q-CAYLE
COMUNICACIONES CUÁNTICAS

Relative Locality in curved spacetimes

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- In most of them a minimal length appears $\Longrightarrow$ Planck length (l_P)??
- This is closely related to an energy scale $\Longrightarrow$ Planck energy (Λ)??
- Problem: there are no experimental evidences of a fundamental QGT

Spacetime: the last frontier

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- New effects $\rightarrow$ Micro black holes creation?
- Spacetime can be regarded as a "foam"

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- There is a privileged observer $\rightarrow$ physical laws depending on the observer

Doubly Special Relativity (DSR)

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- There is a relativity principle
- Two invariants in every inertial frame: speed of light c and Planck length l_P

Kinematics

Ingoing particles $\Rightarrow$ Interaction $\Rightarrow$ Outgoing particles

past ∞ future ∞

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- In the interaction, the conservation of total momentum holds

Kinematics in DSR

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- Dispersion relation and conservation law compatible with relativity principle $\rightarrow$ deformed Lorentz transformations

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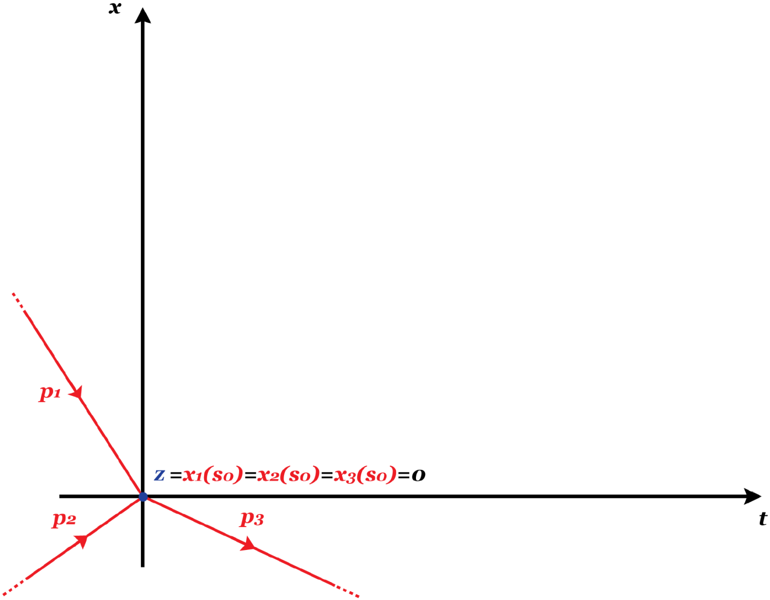
- Snyder kinematics [Battisti and Meljanac, 2010]

$$(p \oplus q)_\mu \;=\; p_\mu \left(1 + \frac{q_\mu q_\nu \eta^{\mu\nu}}{2\Lambda^2} + \frac{p_\mu q_\nu \eta^{\mu\nu}}{2\Lambda^2}\right) + q_\mu$$

Relative locality



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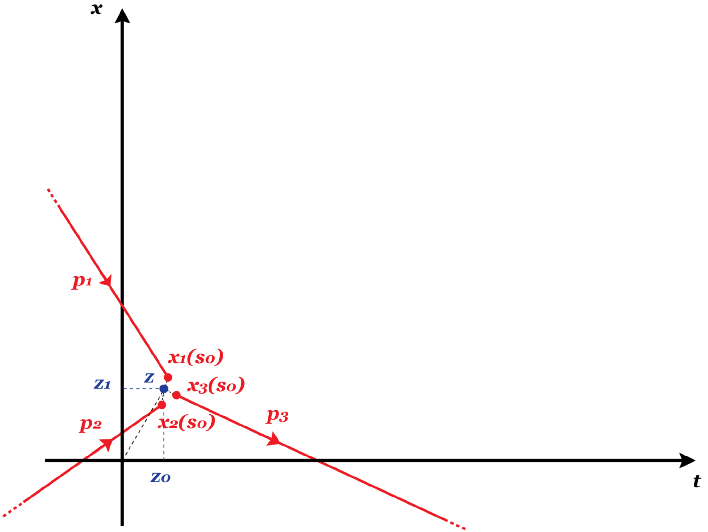

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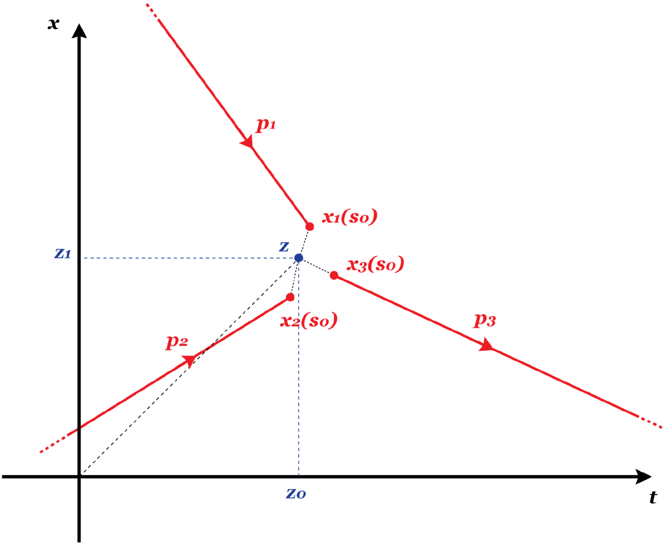

Implementation of locality

- From an action

$$S = \int_{-\infty}^{0} d\tau \sum_{i=1,2} \left[x^{\mu}_{-(i)}(\tau) \dot{p}^{-(i)}_{\mu}(\tau) + N_{-(i)}(\tau) \left[C(p^{-(i)}(\tau)) - m^2_{-(i)} \right] \right]$$

$$+ \int_{0}^{\infty} d\tau \sum_{j=1,2} \left[x^{\mu}_{+(j)}(\tau) \dot{p}^{+(j)}_{\mu}(\tau) + N_{+(j)}(\tau) \left[C(p^{+(j)}(\tau)) - m^2_{+(j)} \right] \right]$$

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- Phenomenological consequences: time delay of flight of high-energy particles

Our geometrical perspective [Carmona et al., 2019]

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- Translations, deformed "Lorentz" generators $\rightarrow$ 10 isometries of the metric!
- Only a maximally symmetric momentum space satisfies this! $\rightarrow$ Minkowski, de Sitter or anti de Sitter

Relative locality from geometry [Relancio, 2021]

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- In spacetime the composition law is not an isometry

$$g_{\mu\nu}\left(p\oplus k\right)dx^\mu dx^\nu + g^{\mu\nu}\left(p\oplus k\right)d\left(p\oplus k\right)_\mu d\left(p\oplus k\right)_\nu \neq$$
$$g_{\mu\nu}(k)dx^\mu dx^\nu + g^{\mu\nu}(k)dk_\mu dk_\nu$$

Relative locality from geometry [Relancio, 2021]

- We need to consider a different space-time point

$$g_{\mu\nu}\left(p\oplus q\right)d\xi^{\mu}d\xi^{\nu}+g^{\mu\nu}\left(p\oplus q\right)d\left(p\oplus q\right)_{\mu}d\left(p\oplus k\right)_{\nu}=$$
$$g_{\mu\nu}(p)dx^{\mu}dx^{\nu}+g^{\mu\nu}(p)dp_{\mu}dp_{\nu}$$

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- Solution: consider a two-particle line element with an 8-dimensional (momentum dependent) metric
- This metric can be obtained by imposing that the composition law and Lorentz transformations are isometries

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- SR $\to$ GR $\implies$ flat spacetime $\to$ curved spacetime

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Momentum and space-time curvature

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- SR $\rightarrow$ DSR $\Longrightarrow$ flat momentum space $\rightarrow$ curved momentum space!
- GR $\rightarrow$ DGR?$\rightarrow$ curved phase space?
- This leads to a metric depending on all phase-space variables [Relancio and Liberati, 2020]
- This metric possesses the symmetries of the relativistic deformed kinematics for each fixed space-time point

Relative locality from geometry [Mercati and Relancio, 2023]

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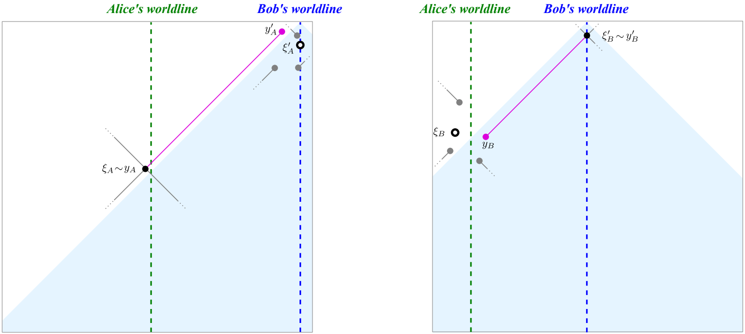
- But it is possible from our geometrical setup
- The new (differential) equations of relative locality are

$$\frac{\partial y^\mu}{\partial \xi^\nu} e^\sigma_\mu(y) \;=\; e^\rho_\nu(\xi)\frac{\partial \left(\bar{p}\oplus\bar{q}\right)_\rho}{\partial \bar{p}_\sigma}\,, \quad \frac{\partial z^\mu}{\partial \xi^\nu} e^\sigma_\mu(z) \;=\; e^\rho_\nu(\xi)\frac{\partial \left(\bar{p}\oplus\bar{q}\right)_\rho}{\partial \bar{q}_\sigma}$$

with $(\bar{p}_\mu, \bar{q}_\mu) = (\bar{e}^\nu_\mu(y) p_\nu, \bar{e}^\nu_\mu(z) q_\nu)$, e^ρ_ν the space-time tetrad and $\bar{e}^\nu_\mu$ its inverse

Relative locality in De Sitter spacetime



Alice ***Bob***

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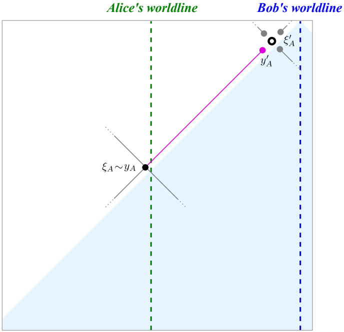

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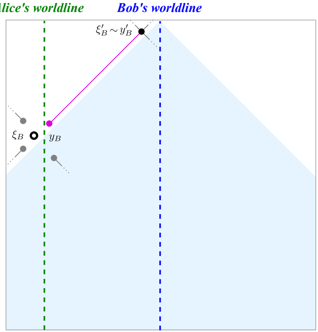

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Results

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- This means that our construction of DGR is compatible with horizons

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- This interpretation allowed us to extend relative locality to curved spacetimes
- We find that relative locality is compatible with the usual GR notion of horizon
- This was checked for two different kinematics, but only with the dominant deformation term
- Future work: extend the work at all order in Λ and for different horizons

Thanks for your attention!

Amelino-Camelia, G., Freidel, L., Kowalski-Glikman, J., and Smolin, L. (2011).
The principle of relative locality.
Phys. Rev., D84:084010.

Battisti, M. V. and Meljanac, S. (2010).
Scalar Field Theory on Non-commutative Snyder Space-Time.
Phys. Rev., D82:024028.

Borowiec, A. and Pachol, A. (2010).
Classical basis for kappa-Poincare algebra and doubly special relativity theories.
J. Phys., A43:045203.

Carmona, J. M., Cortés, J. L., and Relancio, J. J. (2019).
Relativistic deformed kinematics from momentum space geometry.
Phys. Rev., D100(10):104031.

Majid, S. and Ruegg, H. (1994).

Bicrossproduct structure of kappa Poincare group and noncommutative geometry.
Phys. Lett., B334:348–354.

Mercati, F. and Relancio, J. J. (2023).
Relative Locality in curved spacetimes and event horizons.

Relancio, J. J. (2021).
Geometry of multiparticle systems with a relativistic deformed kinematics and the relative locality principle.
Phys. Rev. D, 104(2):024017.

Relancio, J. J. and Liberati, S. (2020).
Phenomenological consequences of a geometry in the cotangent bundle.
Phys. Rev., D101:064062.