AstroHEP-PPCC24 Zaragoza

Using Large-scale structure correlations in Gravitational-wave Cosmology

A Bayesian Approach

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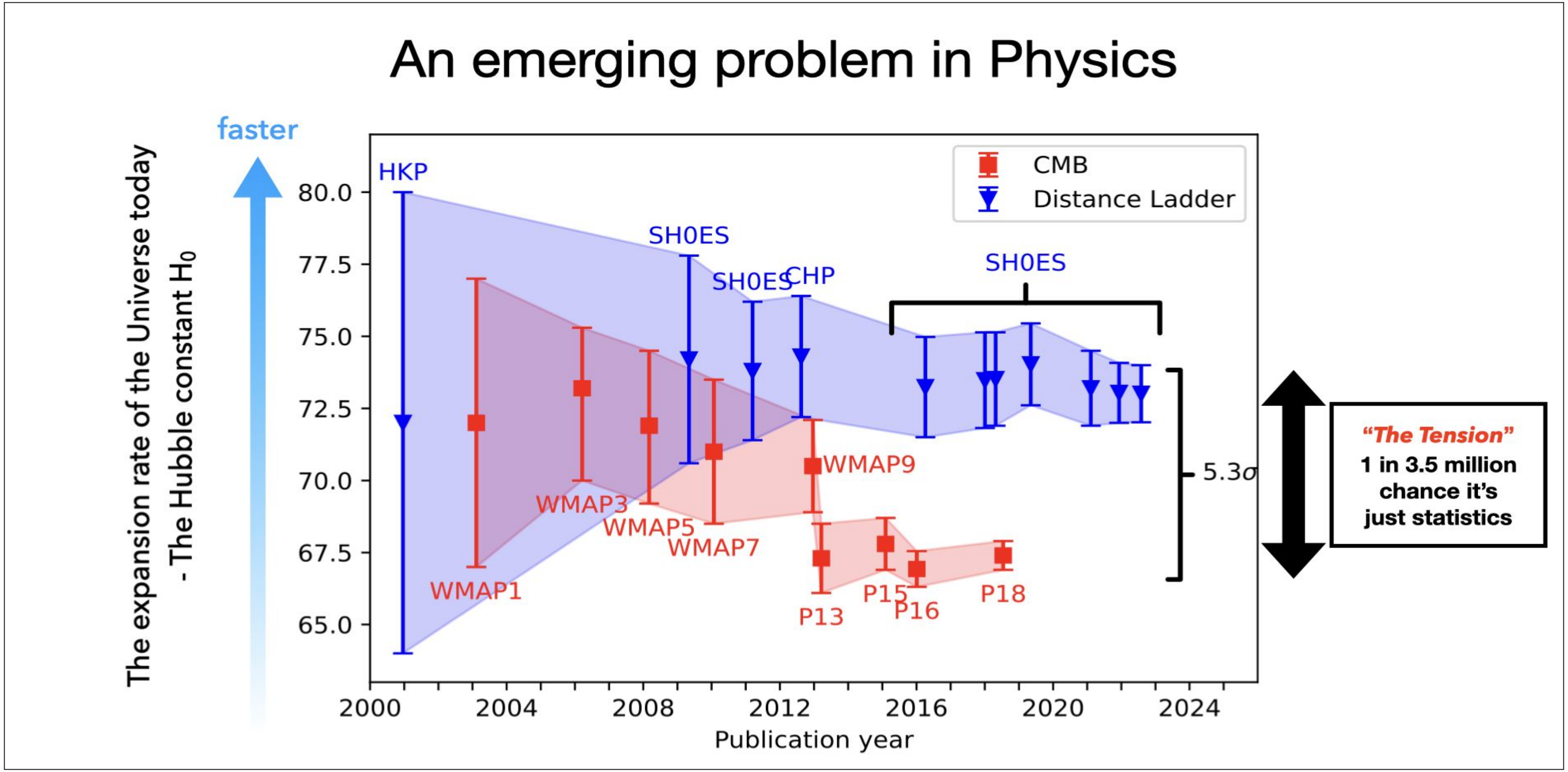
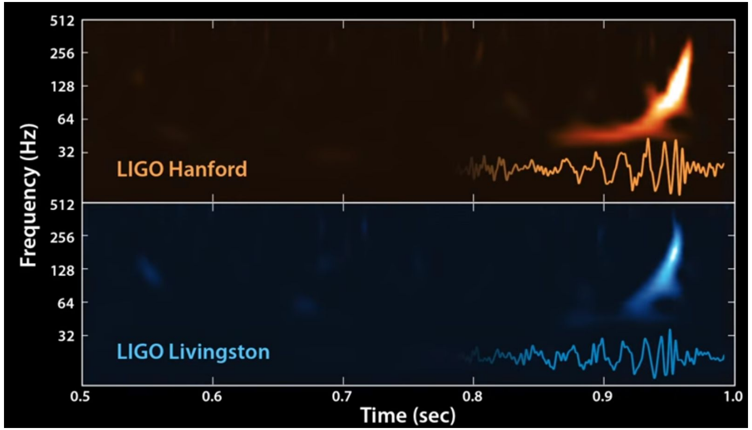
Based on: Astrophys.J. 902 (2020)
ArXiv: 2312.16305 (under review)

The Hubble Tension

Image: *D'arcy Kenworthy*



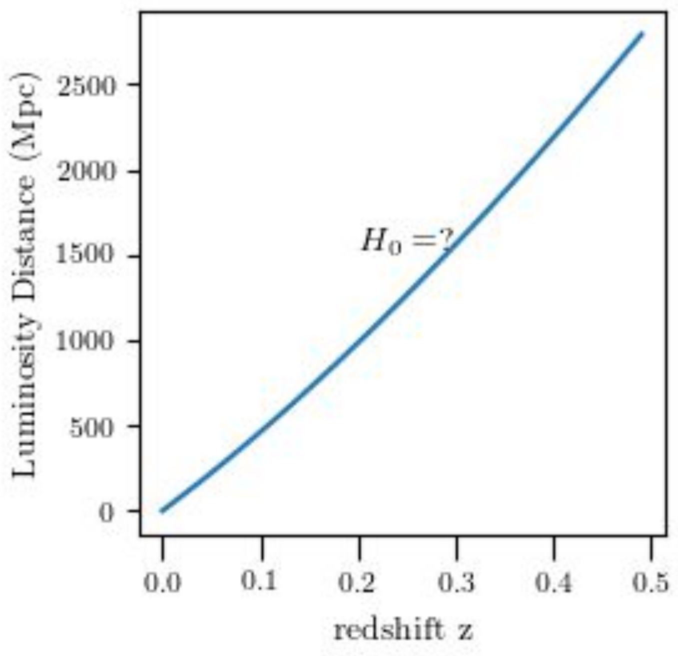
Distance measurement from Gravitational Waves



$$h(t) = \frac{M_z^{5/3} f(t)^{2/3}}{d_L} F(\iota, \theta) \cos(\Phi(t))$$

M_z: Redshifted chirp mass
ι: inclination angle
$\Phi(t)$: Accumulated phase

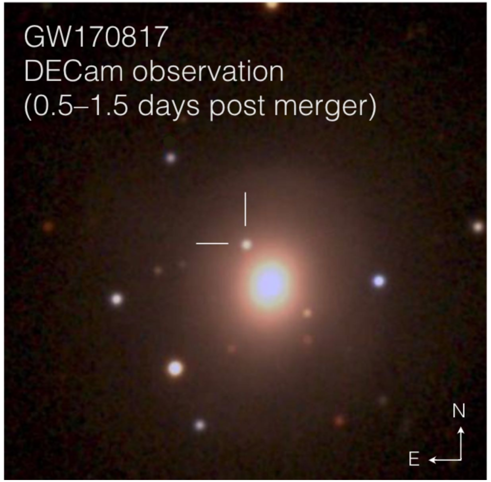
Measuring H_0 with “standard sirens”



- ❑ Luminosity distance - redshift curve depends on the value of the Hubble parameter H_0
- ❑ Luminosity distance - GW observation
- ❑ Redshift - from an electromagnetic counterpart

Thus an independent estimate of H_0 is possible

GW170817



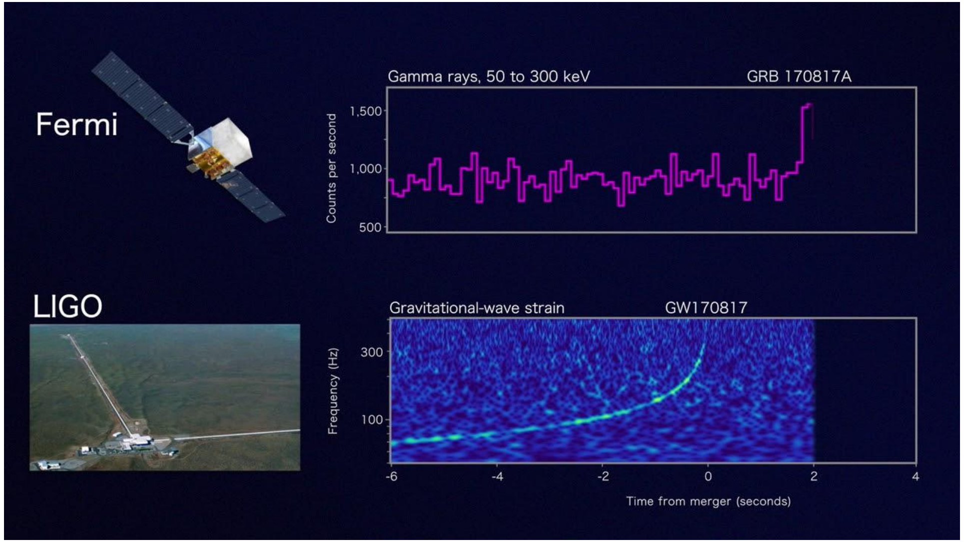
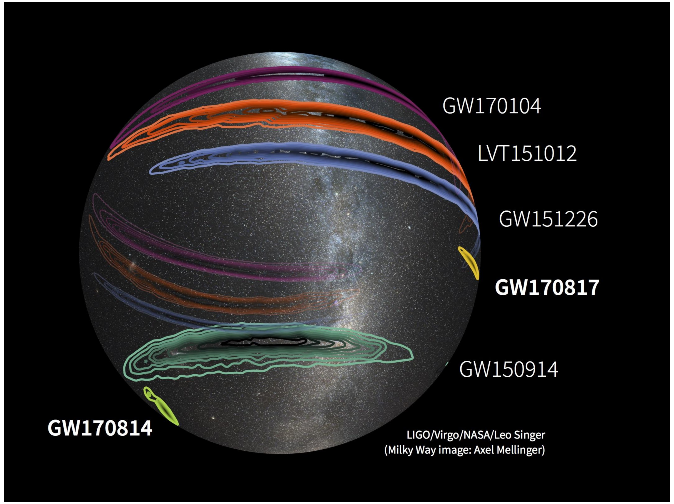


Image: https://www.ligo.org

- The only GW event detected along with a GRB: GRB 170817A
- Luminosity distance ~ 40 Mpc
- Host identification : NGC 4993

For most of the detected events, the host identification is not possible

Inferring H_0 using population statistics



Credit : Leo Singer

- Consider galaxies (with known redshifts) in the localization region as potential hosts.
- Compute H_0 distribution for each potential host

Schutz(1986)

An alternative approach: *The Large Scale Structures*

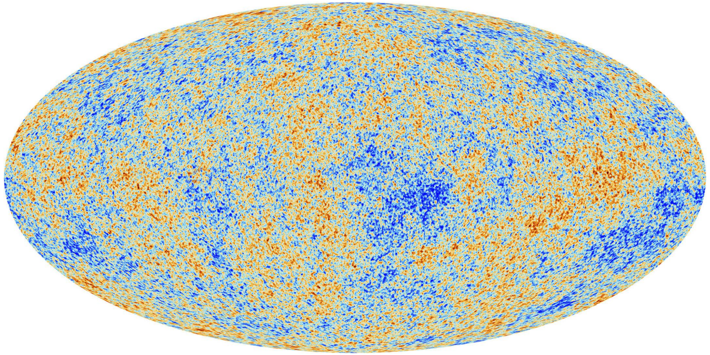


Image: ESA

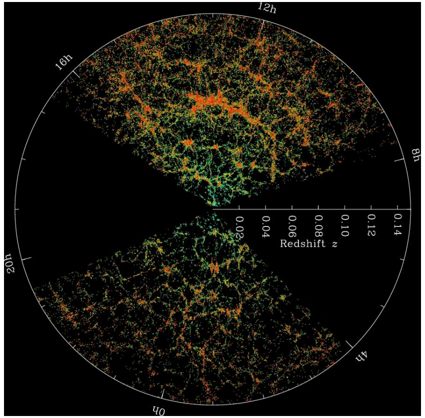


Image: SDSS

The Millennium simulation (z=0)

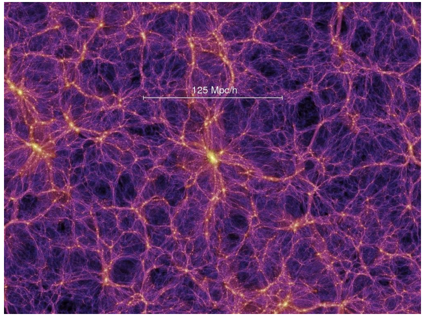


Image: https://wwwmpa.mpa-garching.mpg.de

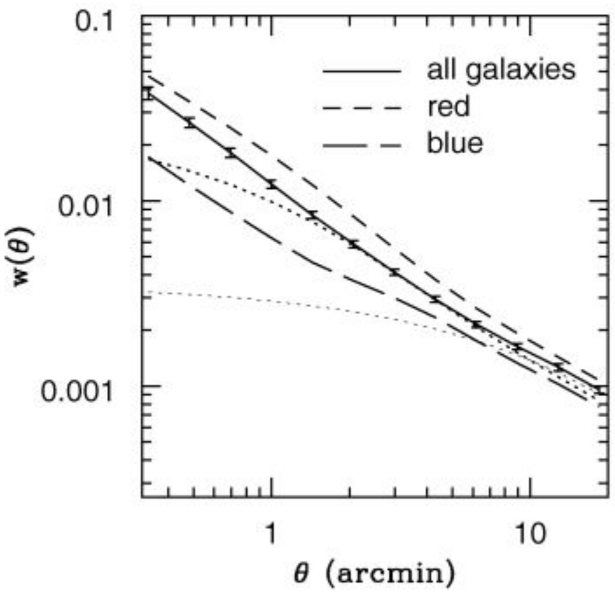
Measures of clustering: Density Contrast and cross-correlation

$$\delta(\mathbf{x}) \sim \frac{\rho(\mathbf{x})}{\bar{\rho}} - 1$$

$$\xi(\mathbf{x}, \mathbf{x}') \sim \langle \delta(\mathbf{x}) \delta(\mathbf{x}') \rangle$$

Angular cross-correlation

$$w(\theta, \theta') \sim \langle \delta(\theta) \delta(\theta') \rangle$$



Jain, Scranton, Sheth (2003)

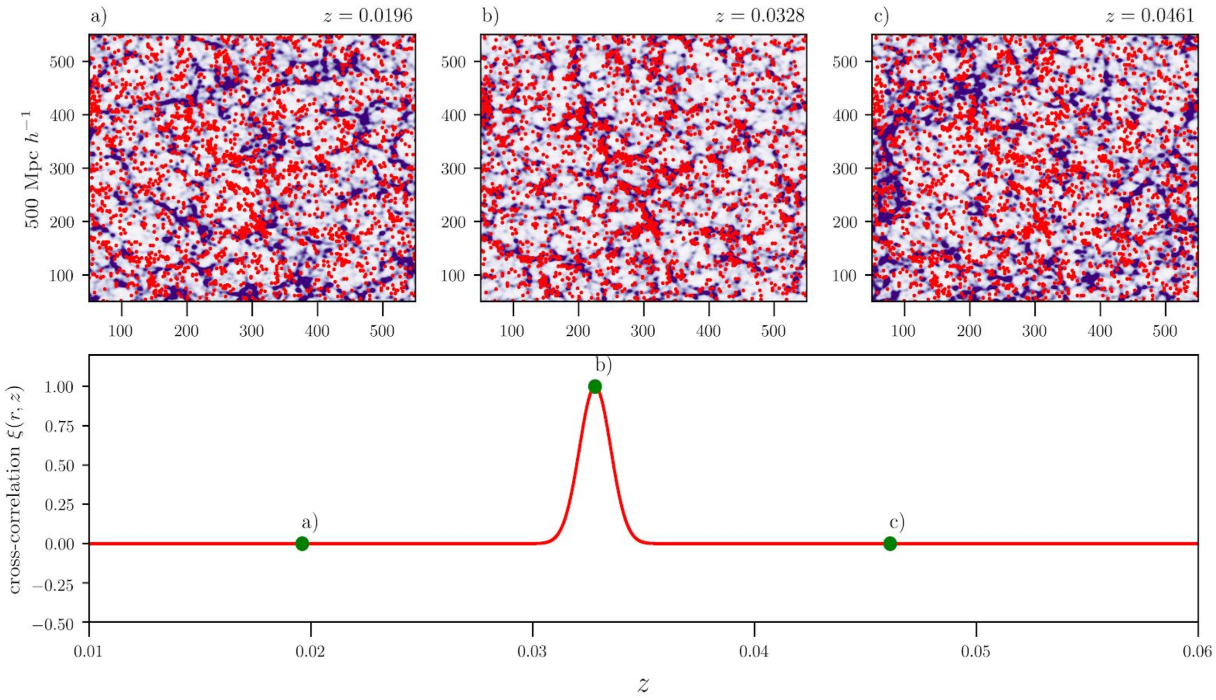
Inferring redshift from cross-correlations

Red : BBH sources at a fixed unknown redshift

Blue: Galaxy distribution at different redshift slices

The BBH distribution is a part of the same large scale structure as the galaxies.

Cross-correlation of the two distributions provide a redshift estimate for the unknown BBH population



Newman (2008), Oguri (2016)

A realistic Simulation of the catalogs

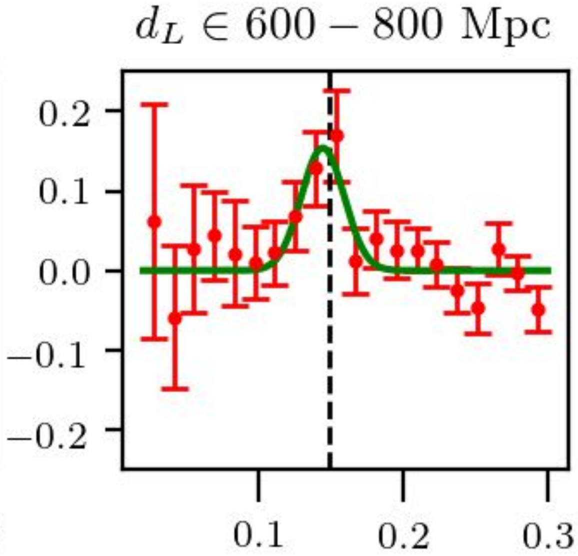
- ❑ The true locations of the GW events are sampled from the dark matter distribution of a cosmological N-body simulation (Big-MultiDark Planck)

- ❑ Massive dark matter halos act as galaxy markers in our simulation.

- ❑ Realistic simulation of the GW events and parameter estimations run using BILBY: A free Bayesian Inference library for GW (Ashton et al. 2019)

- ❑ 3 detector network (Advanced Ligo L +H + Advanced Virgo): combined SNR threshold of 8

Modelling the cross-correlation

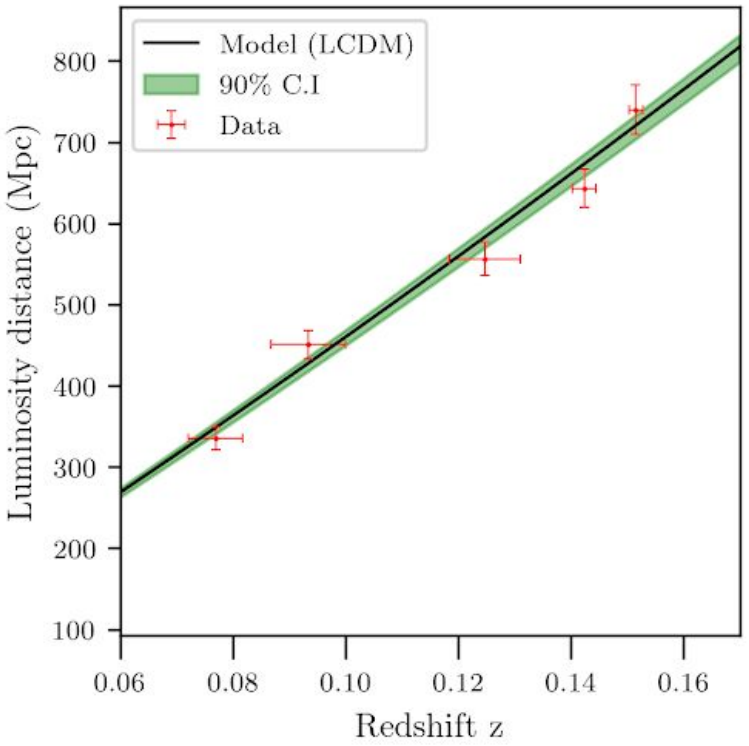


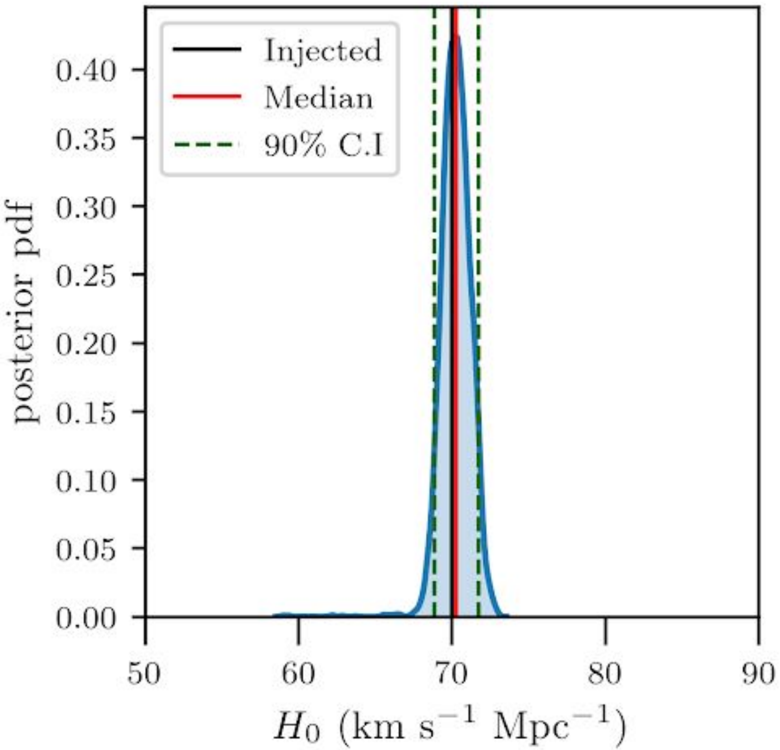
Assume power law three-dimensional cross-correlation function:

$$\xi_{\rm gw,g}(r) = \left[\frac{r}{r_0}\right]^{-\gamma}$$

$$w(\leq \theta_{\rm max}, z, z') \propto \exp\left[-\frac{(z-z')^2}{2\sigma_z^2}\right]$$

Hubble-Lemaitre diagram : 500 events





SB, Rana, More, Bose (2020)

An event-by-event analysis

$$p(H_0 \mid \boldsymbol{d}_{\text{strain}}, \boldsymbol{d}_g^{\text{obs}}) = \int p(H_0, \boldsymbol{d}_{\text{gw}} \mid \boldsymbol{d}_{\text{strain}}, \boldsymbol{d}_g^{\text{obs}}) d\boldsymbol{d}_{\text{gw}}$$

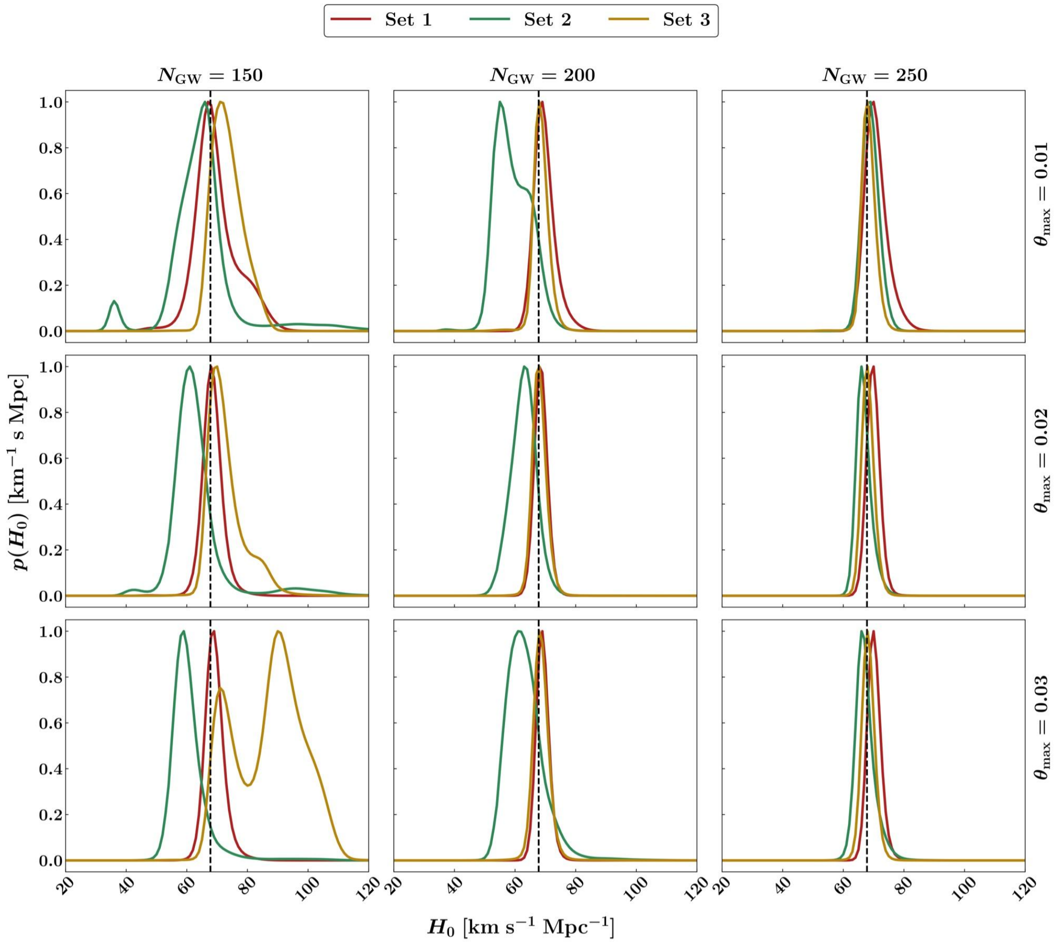
$$\propto \int \mathcal{L}(\boldsymbol{d}_{\text{strain}}, \boldsymbol{d}_{\text{g}}^{\text{obs}} | H_0, \boldsymbol{d}_{\text{gw}}) P(H_0, \boldsymbol{d}_{\text{gw}}) d\boldsymbol{d}_{\text{gw}}$$

→ For each GW event, the posterior is obtained by marginalizing over localization uncertainties $\boldsymbol{d}_{\text{gw}}$

Assuming independent probability distributions,
the single-event posteriors can be combined as :

$$P(H_0 \mid \{\boldsymbol{d}_{\text{strain}}\}, \{\boldsymbol{d}_g^{\text{obs}}\}) \propto P(H_0) \prod_i \mathcal{L}(\boldsymbol{d}_{\text{strain}_i}, \boldsymbol{d}_{g_i}^{\text{obs}} | H_0)$$

$$\propto P(H_0) \prod_i \int \mathcal{L}(\boldsymbol{d}_{\text{strain}_i}, \boldsymbol{d}_{g_i}^{\text{obs}} | H_0, \boldsymbol{d}_{\text{gw}}) P(\boldsymbol{d}_{\text{gw}}) d\boldsymbol{d}_{\text{gw}}$$



Dependence on **sample size** and **correlation scale**

Injected value of H_0 = 70 km/s/Mpc

Ghosh, More, **SB**, Bose (arXiv: 2312.16305)

Final Takeaway

Taking into account the large-scale structure correlations is crucial to a more robust inference of the background cosmology

Caveats:

- Need ~**200 or more** well-localised GW sources for a meaningful estimate. Expected to be achieved in the 3G era of GW detectors!

- Effects due to weak lensing (SB *et al.*, in preparation).

ACKNOWLEDGMENTS

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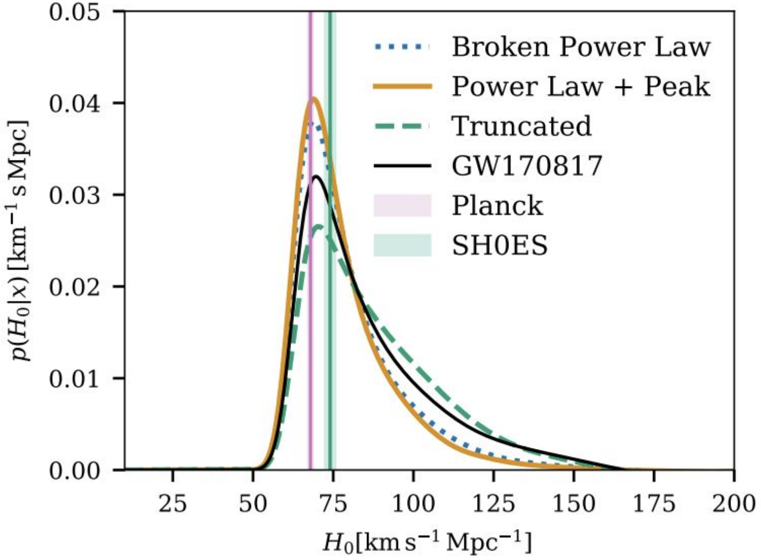
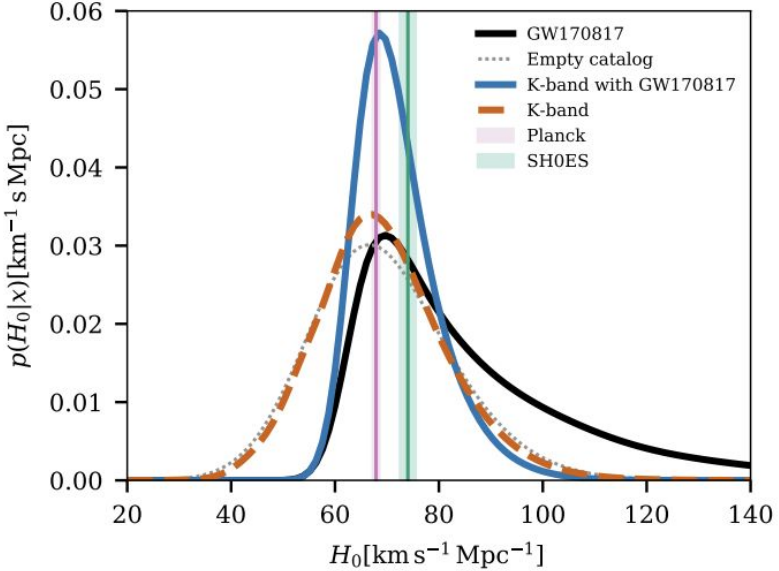
Thank You!

Constraints from GWTC-3 catalog

Figures: LVK collaboration(2021), arXiv:2111.03604



Method 1: mass distribution



method 2: galaxy catalog

Waveform simulation: inputs

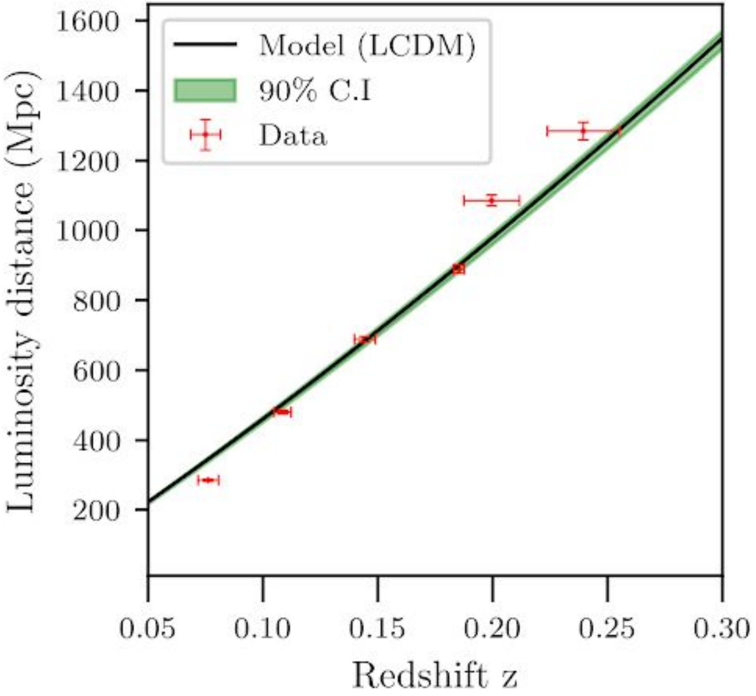
Detectors	Sensitivity
Livingston	Advanced LIGO
Hanford	Advanced LIGO
Virgo	Advanced Virgo

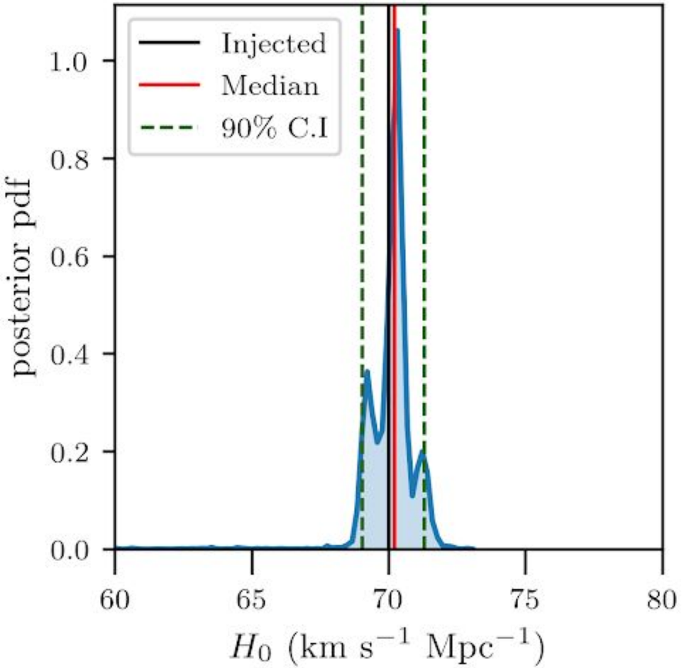
Injection Parameters		
Parameters	Distribution	Limits
$m_{1,2}$	uniform	[10, 35] $M_\odot$
$\chi_{1,2}$	uniform	[0, 0.8]
ϕ_{12} , ϕ_{jl}	uniform	[0, 2π)
$\cos\theta_{1,2}$, $\cos\iota$	uniform	[-1, 1)
ψ , ϕ_c	Fixed	0

Detection criteria: At least two of the detectors SNR above a threshold value of 5 each, the third an SNR greater than 2.5, and network SNR of greater than 8.

Hubble-Lemaitre diagram : 5000 events

Red points: d_L inferred from BBH merger waveforms, redshift from cross-correlations

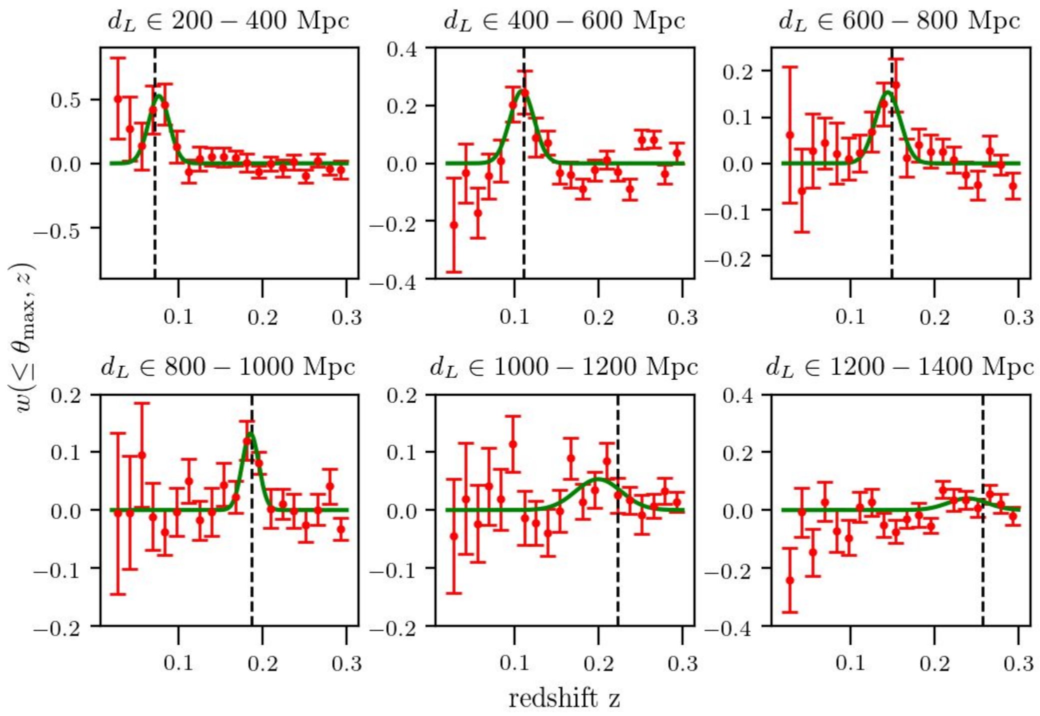




Black solid line: the true value of H_0 in the simulation
Dashed lines: 90 percent credible interval

SB, Rana, More, Bose (2020)

Redshift from angular cross-correlation

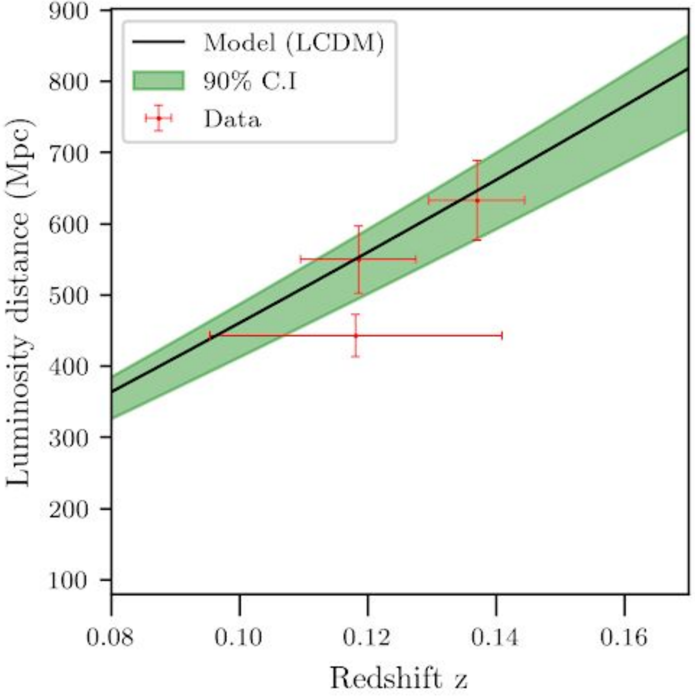


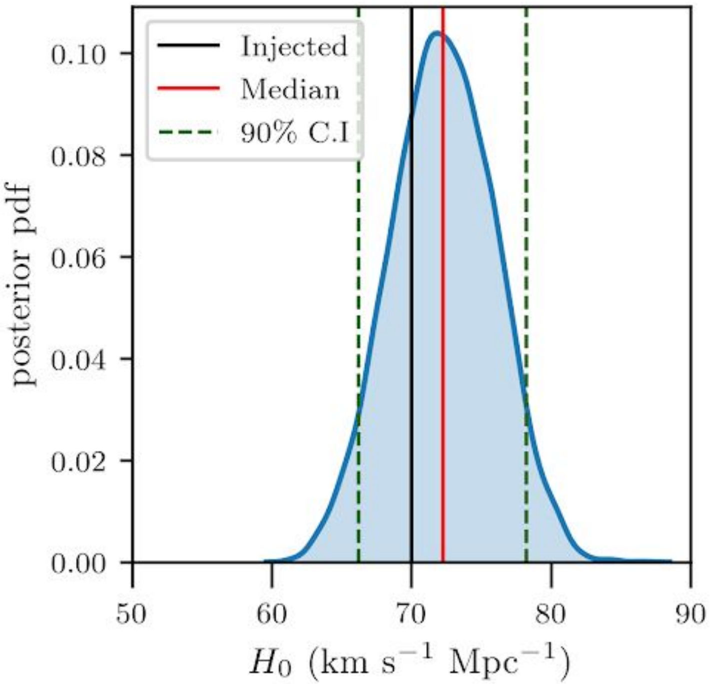
- ❏ 5000 BBH mergers divided into 6 bins in the inferred luminosity distances
- ❏ The mock galaxies are divided into 20 redshift bins

- ❏ Red points are the measured cross-correlations with error bars, peaking at the correct redshift
- ❏ The injected value of H_0 = 70 km/s/Mpc gives an average redshift of the GW sources in each bin (black vertical line)

SB, Rana, More, Bose (2020)

Hubble-Lemaitre diagram : 50 events





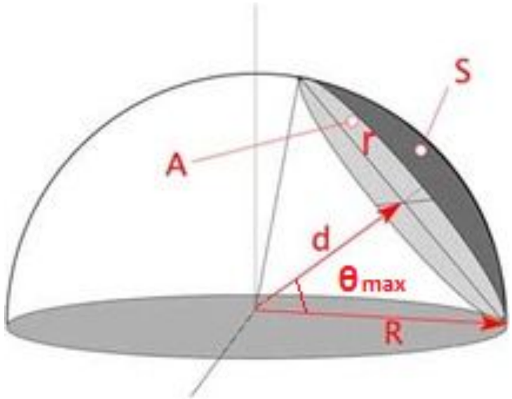
SB, Rana, More, Bose (2020)

Constraints from the three samples

Constraints on H_0			
No. of GW events	Max d_L (Mpc)	Injected H_0 (km s^{-1} Mpc^{-1})	Constraints on H_0 (km s^{-1} Mpc^{-1})
5100	1400	70	$70.22^{+1.09}_{-1.18}$
500	900	70	$70.26^{+1.47}_{-1.40}$
50	900	70	$72.24^{+5.98}_{-6.05}$

The error bars signify 90% credible interval around the the median of H0 posterior

Angular Cross-correlation Estimator



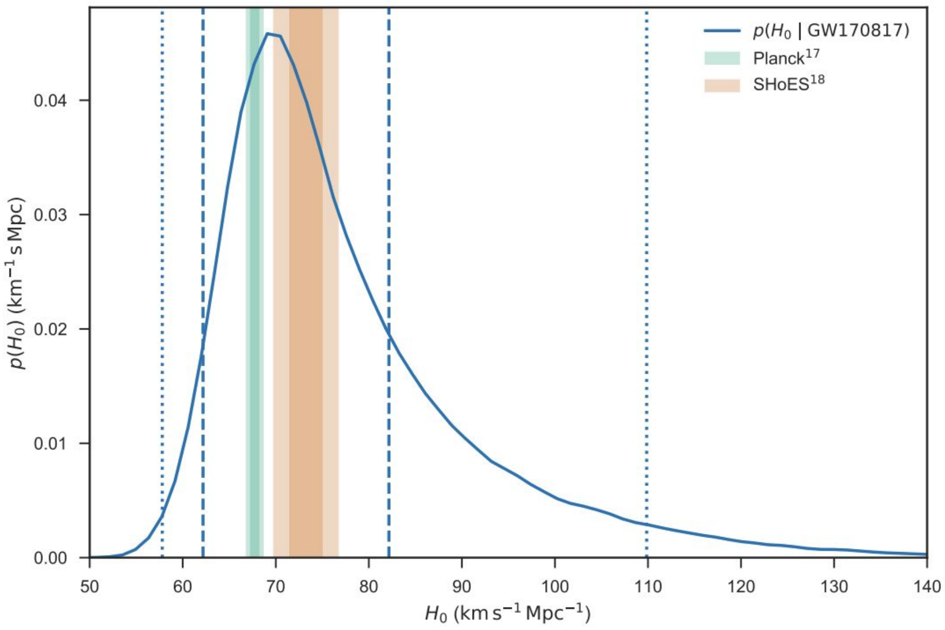
We count the number of galaxy-BBH pairs which have an angular separation θ_{max} or less in the actual catalog and in a randomly distributed catalog.

Angular cross-correlation estimator

$$w(\leq \theta_{\rm max}) = \frac{n_{\rm D_1 D_2}(\leq \theta_{\rm max})}{n_{\rm R_1 R_2}(\leq \theta_{\rm max})} - 1$$

D_1, D_2 : Data catalogs
R_1, R_2 : Random catalogs

GW170817



Abbott et al (2017)

Measurement of H_0 with ~ 15% accuracy at 68.3% confidence