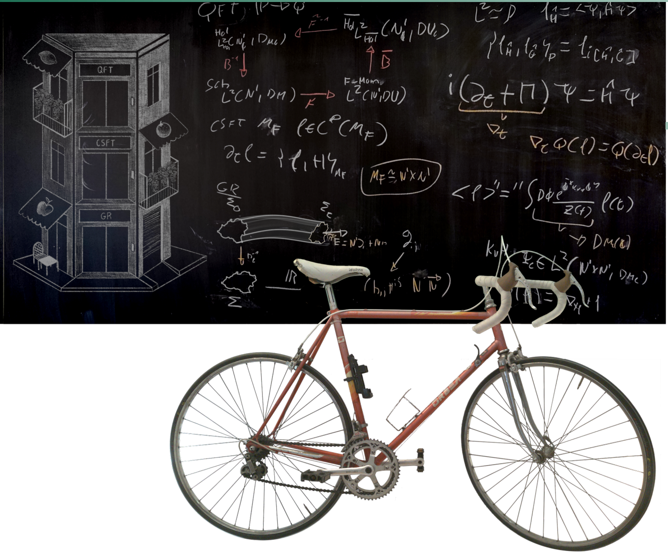
Geometric Flavours of Quantum Field Theory on a Cauchy hypersurface

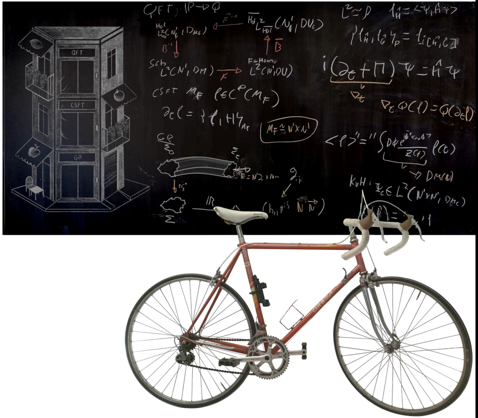


Modifications to the Schrödinger equation and Applications to Cosmology

David Martínez Crespo

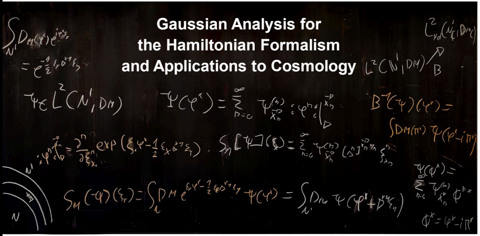
Universidad Zaragoza

CAPA
Centro de Astropartículas y Física de Altas Energías
Universidad Zaragoza



Ph.D. Thesis

Geometric Flavours of Quantum Field Theory on a Cauchy Hypersurface:



Gaussian Analysis for the Hamiltonian Formalism and Applications to Cosmology

David Martínez Crespo

Director:
Jesús Clemente Gallardo

September 2024

Universidad Zaragoza

David Martínez Crespo 2024 Geometric Flavours of Quantum Field Theory

Quantum Field Theory on a Curved Spacetimes

The Schördinger equation is

$$i\hbar\partial_t\Psi = \hat{H}\Psi$$

with $\hat{H}(t)$ selfadjoint at every step in time. This equation does not conserve probability.

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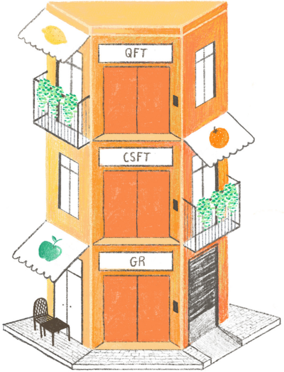
From a geometric point of view we will modify

$$i\hbar(\partial_t + \Gamma_t)\Psi = \hat{H}\Psi$$

Presentation Outline

1. **Hamiltonian Gravity**
2. **Classical Statistical Field Theory**
3. **Quantum Field Theory**
4. **Modification of the Schrödinger Equation**
5. **Particle Creation in FLRW spacetimes**



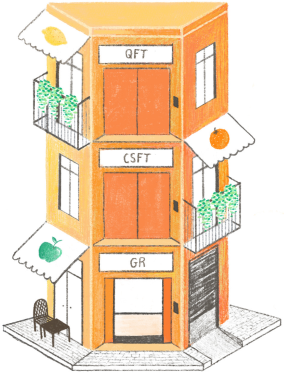
1 Hamiltonian Gravity

2 Classical Statistical Field Theory

3 Quantum Field Theory

4 Modification of the Schrödinger Equation

5 Particle Creation in FLRW spacetimes



Ground floor: space + time

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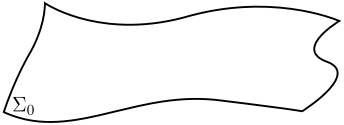
- In globally hyperbolic spacetimes the spacetime manifold is diffeomorphic to $\Sigma \times \mathbb{R}$



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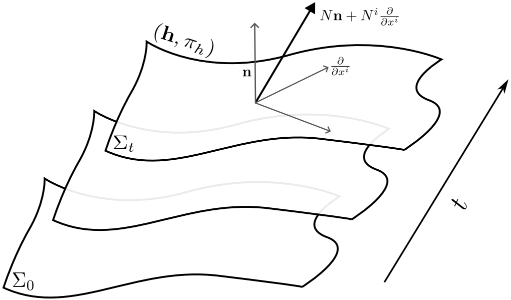
ADM formalism

The ADM Hamiltonian formalism of general relativity using the phase space $T^* \operatorname{Riem}(\Sigma)$ with coordinates $(\mathbf{h}, \pi_h)$ in components $(h_{ij}(x), \pi_h^{ij}(x))$. The diffeomorphism is fully implemented by Lapse function and Shift vector $N, \vec{N}$.



Elements for QFT in curved spacetimes

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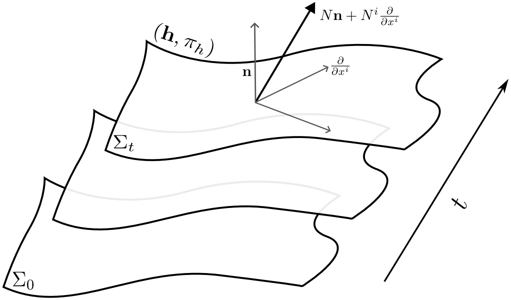

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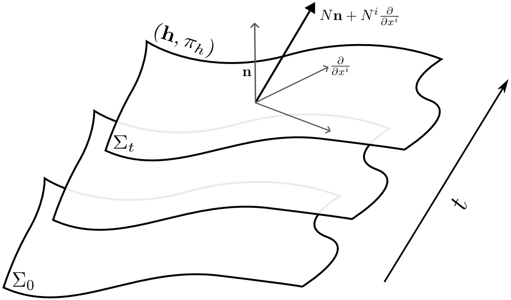

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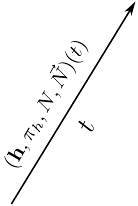
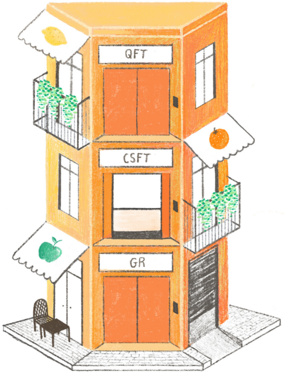
The geometric structures of the upper floors will depend on the time parameter t only through these parameters




QFT
CSFT
GR

Phase space in Classical Statistical Field Theory

The phase space of scalar fields is often modelled over the compact Cauchy hypersurface Σ

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This space does not admit Gaussian probability distributions because there is no Gaussian probabilty measure over it. Instead we must use

$$\mathcal{M}_F = D'(\Sigma) \times D'(\Sigma)$$

Phase space in Classical Statistical Field Theory

Notation

$\varphi_{\mathbf{x}} \in C^\infty(\Sigma)$ represents test functions $\varphi^{\mathbf{x}} \in D'(\Sigma)$ represents distributions. The Riemannian metric h induces a musical isomorphism relating both of them when we restrict $\mathrm{Den}(\Sigma) \subset D'(\Sigma)$.

$$\mathcal{M}_F = D'(\Sigma) \times D'(\Sigma) \text{ covered with a chart } (\varphi^{\mathbf{x}}, \pi^{\mathbf{x}})$$

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$$\mathcal{M}_F = D'(\Sigma) \times D'(\Sigma) \text{ covered with a chart } (\varphi^{\mathbf{x}}, \pi^{\mathbf{x}})$$

Admits a symplectic structure densely defined over $\mathrm{Den}(\Sigma) \times \mathrm{Den}(\Sigma)$

$$\omega_M = \int_{\Sigma} \frac{d^3x}{\sqrt{h}}\, d\pi(x) \wedge d\varphi(x)$$

Gaussian states in Classical Statistical Field Theory

The Gaussian state is provided by a measure μ. Bochner-Minlos theorem provides those states with the characteristic functional

$$\int_{D'(\Sigma)\times D'(\Sigma)} D\mu(\varphi^{\mathbf{x}},\pi^{\mathbf{x}}) e^{i(\chi_x\varphi^x+\eta_x\pi^x)} := \exp\left(\frac{1}{2}\mu^{-1}_{\mathcal{M}_F}[(\chi_{\mathbf{x}},\eta_{\mathbf{x}}),(\chi_{\mathbf{x}},\eta_{\mathbf{x}})]\right)$$

defined via a positive definite covariance $\mu^{-1}_{\mathcal{M}_F} : C^\infty(\Sigma)\times C^\infty(\Sigma)\to\mathbb{R}$.

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We introduce the covariance using a Kähler structure $(\mu, \omega, J)_{\mathcal{M}_F}$ on the space of fields compatible with the symplectic structure $\mu_{\mathcal{M}_F}(,) = \omega_{\mathcal{M}_F}(, -J_{\mathcal{M}_F})$:

$$J_{\mathcal{M}_F} = (\partial_{\varphi^y}, \partial_{\pi^y}) \begin{pmatrix} A^y_x & \Delta^y_x \\ D^y_x & -(A^t)^y_x \end{pmatrix} \begin{pmatrix} d\varphi^x \\ d\pi^x \end{pmatrix}$$

$$\mu_{\mathcal{M}_F} = (d\varphi^y, d\pi^y) \begin{pmatrix} \Delta_{yx} & -A_{yx} \\ -A^t_{yx} & -D_{yx} \end{pmatrix} \begin{pmatrix} d\varphi^x \\ d\pi^x \end{pmatrix}$$

with

$\Delta_{\mathbf{xy}} > 0 > D_{\mathbf{xy}}$

$J^2_{\mathcal{M}_F} = -\mathbb{1}$

Brief comments on dynamics

It is convenient to use a stationary Gaussian state μ_0 as a reference to describe other states.

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This implies choosing a complex structure and a Gaussian measure dependent on the Hamiltonian $H \in \mathcal{F}(\mathcal{M}_F)$ that generates the dynamics

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Annals of Physics 313 (2004) 446–478

ANNALS of PHYSICS

Schrödinger and Fock representation for a field theory on curved spacetime

Alejandro Corichi,[a,b,*] Jerónimo Cortez,[a] and Hernando Quevedo[a,c]

Elements for QFT in curved spacetimes

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The Kähler structure that is a geometric ingredient regarded as a kinematical ingredient is intertwined with the dynamics

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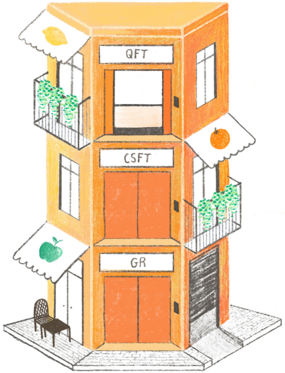

Quantum Field Theory: Hilbert space

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The Hilbert space of the **quantum theory** is obtained using the Gaussian measure of the classical theory. **This state is interpreted as part of the vacuum of the theory**

$$L^2(D'(\Sigma) \times D'(\Sigma), D\mu) \to \mathscr{H}$$

This Hilbert space has too many degrees of freedom and has to be reduced to a **Lagrangian submanifold** of $\mathcal{M}_F$. We obtain infinitely many representations from this procedure. The most representative ones are:

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- $\mathscr{H}_{Hol} = L^2_{Hol}(D'(\Sigma)_{\mathbb{C}}, D\mu_c)$ $\quad C_{\mu_c}(\rho_{\mathbf{x}}, \bar{\rho}_{\mathbf{x}}) = e^{-\bar{\rho}_x \Delta^{xy} \rho_y}$ Holomorphic
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Introducing the complex coordinate $\phi^{\mathbf{x}} = \varphi^{\mathbf{x}} - i\pi^{\mathbf{x}}$

Quantum Field Theory: Space of observables

The observables are obtained with a quantization mapping $\mathcal{Q} : C^\infty(\mathcal{M}_F) \to B(\mathscr{H})$

Q1) $\mathcal{Q}(F + G) = \mathcal{Q}(F) + \mathcal{Q}(G)$

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Q4) **(Irreducibility condition)** For a given set of classical observables $\{f_i\}_{\mathcal{I}}$ such that $\{f_i, g\} = 0 \ \forall i \in \mathcal{I}$ implies g is constant; then if an operator A commutes with every $\mathcal{Q}(f_i)$, A is a multiple of the identity.

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For linear operators we get

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for higher order polynomials we complete the definition using an ordering prescription.

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for higher order polynomials we complete the definition using an ordering prescription.

Implications to define the evolution

- The Gaussian measures of the Hilbert spaces μ, ν and the quantization mappings $\mathcal{Q}$ depend on $J_{\mathcal{M}_C} = |X_H|^{-1} X_H$. As a consequence, every aspect of the construction will acquire a dependence on time

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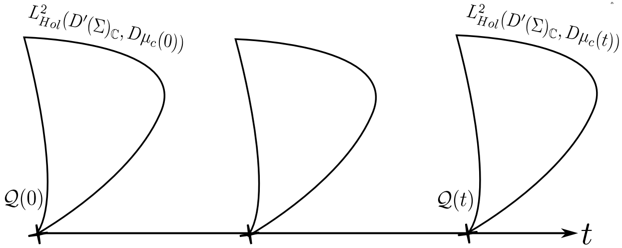
> The structure of the phase space of QFT is such that the Kähler structure and the quantization mapping are time dependent structures $(\omega(t), \mu(t), J(t))_{\mathcal{M}_F}$ and $\mathcal{Q}(t)$



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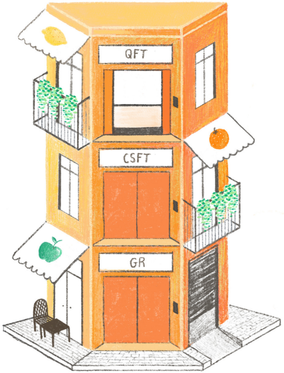
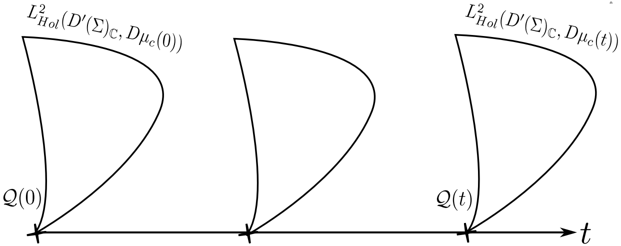


Second quantized Kähler structure

We must consider pure states Ψ a section of a bundle $B \to \mathbb{R}$ locally equivalent to

$$``L^2(D'(\Sigma), D\mu_c(t)) \times \mathbb{R}"$$

.



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$$i\partial_t \Psi = \hat{H}\Psi$$

with $\hat{H}(t)$ selfadjoint at every moment in time. This equation does not conserve probability because it does not respect the geometric structure of the bundle.

Connection term

In order to respect the Geometric structure we simply substitute the time derivative by a covariant time derivative

$$\nabla_t \Psi = \partial_t \Psi + \Gamma_t \Psi$$

such that $\nabla_t \langle \cdot, \cdot \rangle_{\mu_c} = 0$.

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such that $\nabla_t \left\langle \cdot, \cdot \right\rangle_{\mu_c} = 0$. This condition together with

$$\nabla_t \mathcal{Q}(F) = \mathcal{Q}(\partial_t F)$$

are enough to select a unique connection term.

Integral transforms

The relation among every representation can be provided in terms of integral transforms

$$\begin{array}{ccc}
\text{Holomorphic} & & \text{Antiholomorphic}\\
\left(L^2_{Hol}(\mathcal{N}'_{\mathbb{C}}, D\mu_c), \mathcal{Q}\right) & \underset{\tilde{\mathcal{F}}^{-1}}{\overset{\tilde{\mathcal{F}}}{\rightleftarrows}} & \left(L^2_{\overline{Hol}}(\mathcal{N}'_{\mathbb{C}}, D\nu_c), \overline{\mathcal{Q}}\right)\\
\tilde{\mathcal{B}}\uparrow \quad \downarrow \tilde{\mathcal{B}}^{-1} & & \tilde{\overline{\mathcal{B}}}\uparrow \quad \downarrow \tilde{\overline{\mathcal{B}}}^{-1}\\
\left(L^2(\mathcal{N}', D\mu), \mathcal{Q}_s\right) & \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} & \left(L^2(\mathcal{N}', D\nu), \overline{\mathcal{Q}}_m\right)\\
\text{Schrödinger} & & \text{Momentum-field}
\end{array}$$

We defined the Gaussian integral transforms $\mathcal{F}$ and $\tilde{\mathcal{B}}$ that we dubbed **Fourier** and **Segal-Bargmann** transforms.

The modified Schrödinger equation

For a selfadjoint Hamiltonian $\hat{H}$ and the unique connection Γ_t that defines a covariant time derivative ∇_t fulfilling $\nabla_t \langle \cdot, \cdot \rangle_{\mu_c} = 0$ and $\nabla_t \mathcal{Q}(F) = \mathcal{Q}(\partial_t F)$ the evolution is provided by the modified Schrödinger equation

$$i\left(\partial_t + \Gamma_t\right)\Psi = \hat{H}\Psi$$

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- The study of the Kinematical aspects of the theory, i.e. the geometrical structures involved in the Hamiltonian pictures of QFT induce an unambiguous correction to the dynamics

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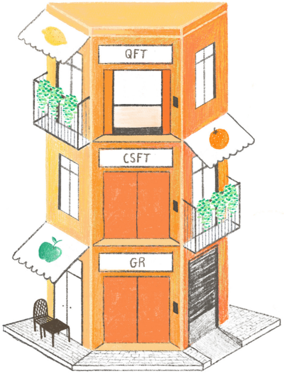
4 Modification of the Schrödinger Equation

5 Particle Creation in FLRW spacetimes



Frieman-Lemaître-Robertson-Walker spacetimes

The cosmological models are represented by FLRW spacetimes

$$\mathbf{g} = -dt \otimes dt + a^2(t)\left[\frac{dr \otimes dr}{1 - kr^2} + r^2\left(d\theta + \sin^2\theta d\phi\right)\right]$$

with $k = 0, \pm 1$, $(r, \theta, \phi) \in [0, \infty) \times [0, \pi] \times [0, 2\pi]$

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Choosing as Cauchy hypersurface $\mathbb{R}^3$ we obtain

$$\begin{aligned} N = 1, \vec{N} &= 0, \\ \mathbf{h} =& a^2(t)\left[\frac{dr \otimes dr}{1 - kr^2} + r^2\left(d\theta + \sin^2\theta d\phi\right)\right], \end{aligned}$$

Klein Gordon theory

We can illustrate the effects of the correction with the Klein Gordon theory with Hamiltonian

$$H = \int_\Sigma d^d x [N \mathscr{H} + N^i \mathscr{H}_i]$$

where

$$\mathscr{H}^x = \frac{\sqrt{h}(x)}{2}[\pi_x^2 + h^{ij} D_i \varphi_x D_j \varphi_x + m^2 \varphi_x^2]$$
$$\mathscr{H}_i^x = \sqrt{h} \pi_x D_i \varphi_x$$

With D representing the Levi-Civita connection of the Riemannian metric $\mathbf{h}$ over Σ.

Klein-Gordon theory in FLRW spacetimes

The Hamiltonian is

$$H_{KG} = \int_{\Sigma} \frac{a^3(t)}{2} \left(\pi^2(u) - \frac{\varphi(u)\nabla^2\varphi(u)}{a^2(t)} + m^2\varphi(u)^2 \right) \mathrm{Vol}_k(u) du^3$$

$$\mathrm{Vol}_k, \nabla^2 = \begin{cases} \sin^2\chi \sin\theta, & \frac{1}{\sin^2\chi}\left[\partial_\chi(\sin^2\chi\partial_\chi) + \frac{1}{\sin\theta}\left[\partial_\theta(\sin\theta\partial_\theta) + \partial_\phi^2\right]\right] & k = 1 \\ 1, & \partial_x^2 + \partial_y^2 + \partial_z^2 & k = 0 \\ \sinh^2\chi \sin\theta, & \frac{1}{\sinh^2\chi}\left[\partial_\chi(\sinh^2\chi\partial_\chi) + \frac{1}{\sin\theta}\left[\partial_\theta(\sin\theta\partial_\theta) + \partial_\phi^2\right]\right] & k = -1 \end{cases}$$

Klein-Gordon theory in FLRW spacetimes

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The complex structure of this model is

$$J_{\mathcal{M}_F} = \begin{pmatrix} 0 & a\left(\sqrt{-\nabla^2 + M^2}\right)^{-1} \\ -\frac{1}{a}\sqrt{-\nabla^2 + M^2} & 0 \end{pmatrix}$$

The modified Schrödinger equation

The corrections to the Schrödinger equation are

$$
\begin{aligned}
i\left[\partial_t + \frac{\dot a}{a}\phi^y\left(2-\frac{1}{2}\frac{M^2}{M^2-\nabla^2}\right)^x_{\ y}\partial_{\phi^x}\right]\Psi = \\
\dot a a^3\left[\mathrm{Vol}_k\left(\frac{1}{2}+\frac{1}{4}\frac{M^2}{M^2-\nabla^2}\right)\frac{1}{\sqrt{M^2-\nabla^2}}\right]^{xy}\partial_{\phi^x}\partial_{\phi^y}\Psi + \\
-\frac{\dot a}{a^3}\left[\frac{1}{\mathrm{Vol}_k}\left(\frac{1}{2}+\frac{1}{4}\frac{M^2}{M^2-\nabla^2}\right)\sqrt{M^2-\nabla^2}\right]_{\mathbf{xy}}\phi^x\phi^y\Psi + \\
\frac{1}{a}\phi^x\left(\sqrt{-\nabla^2+M^2}\right)^y_{\ x}\partial_{\phi^y}\Psi
\end{aligned}
$$

Particle production

We can express the equation in terms of creation $a^{\dagger,\mathbf{x}}$ and annihilation $a^{\mathbf{x}}$ operators

$$\partial_t \begin{pmatrix} a^{\mathbf{x}} \\ a^{\dagger,\mathbf{x}} \end{pmatrix} = \frac{\dot{a}}{a}\left[\frac{1}{2}\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\frac{M^2}{M^2-\nabla^2} + \begin{pmatrix} -1 & -2 \\ -2 & -1 \end{pmatrix}\right]\begin{pmatrix} a^{\mathbf{x}} \\ a^{\dagger,\mathbf{x}} \end{pmatrix}$$

The correction term mixes these operators, this can be interpreted as a dynamical generation of Bogoliubov transformation that is interpreted as particle production on expanding universes.

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The mixing is proportional to the Hubble parameter $\frac{\dot{a}}{a}$.

6 Conclusions and Bibliography

Conclusions

- From a systematic analysis of the geometry of Hamiltonian QFT over a Cauchy hyperspace we found corrections to the Schrödinger equation.

$$i\hbar\Big(\partial_t + \Gamma_t\Big)\Psi = \widehat{H}\Psi$$

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$$i\hbar\Big(\partial_t + \Gamma_t\Big)\Psi = \widehat{H}\Psi$$

- We can identify that the source of particle production in an expanding universe, an effect already studied in the literature using asymptotic Minkowsky states, is precisely this correction.

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Thank You For Your Attention!