Spectrum of Global Strings and the Axion Dark Matter Mass

Mathieu Kaltschmidt
mkaltschmidt@unizar.es

based on JCAP10(2024)043 with K. Saikawa, J. Redondo & A. Vaquero

Saturnalia 2024
Zaragoza, December 18th 2024

Background: G. Pierobon

Departamento de Física Teórica
Universidad Zaragoza

Centro de Astropartículas y Física de Altas Energías
Universidad Zaragoza

GOBIERNO DE ARAGON

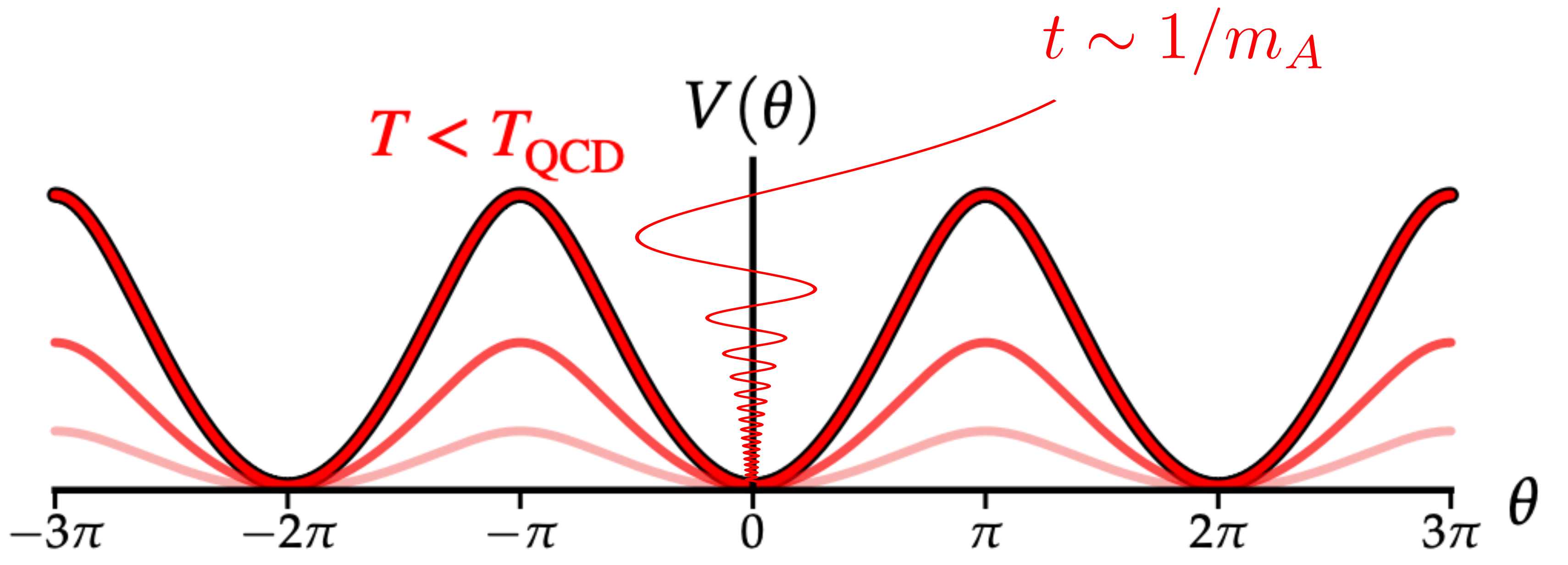
COST EUROPEAN COOPERATION IN SCIENCE & TECHNOLOGY

QCD Axion

- The QCD Axion
  - Pseudo Nambu-Goldstone boson associated with the spontaneous breaking of the global Peccei-Quinn (PQ) $U(1)$ symmetry at the high-energy scale f_a.
  - Dynamical solution to the strong-CP problem.
  - Suitable candidate for Cold Dark Matter.
- Acquires a mass below the QCD scale.

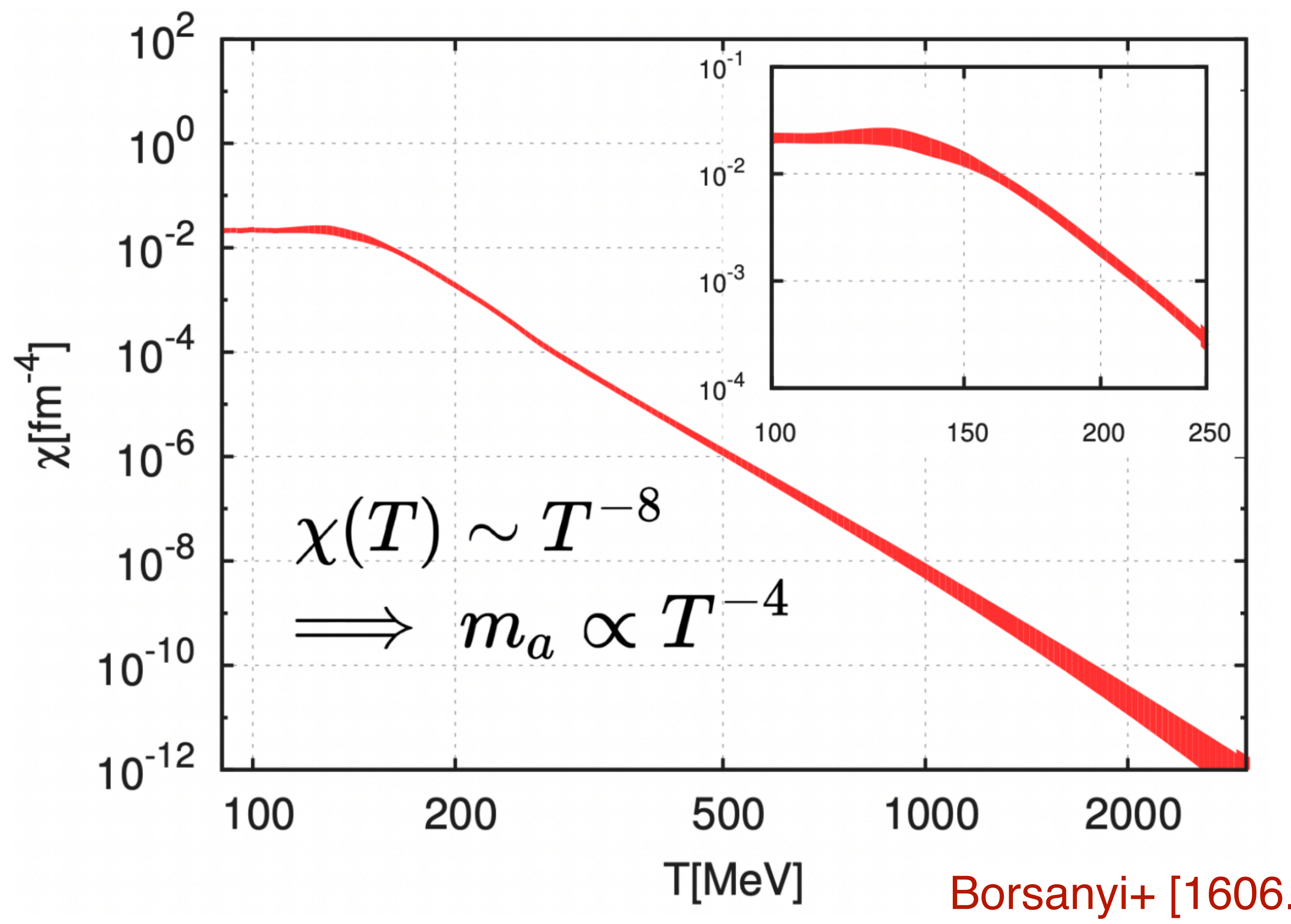
* Throughout this talk, when we refer to the axion, we implicitly mean the **QCD axion** (i.e. solves the strong CP problem)

$$V(\theta) \approx \chi(T)(1-\cos\theta) = m_a^2(T) f_a^2 (1-\cos\theta)$$



O'Hare [2403.17697]

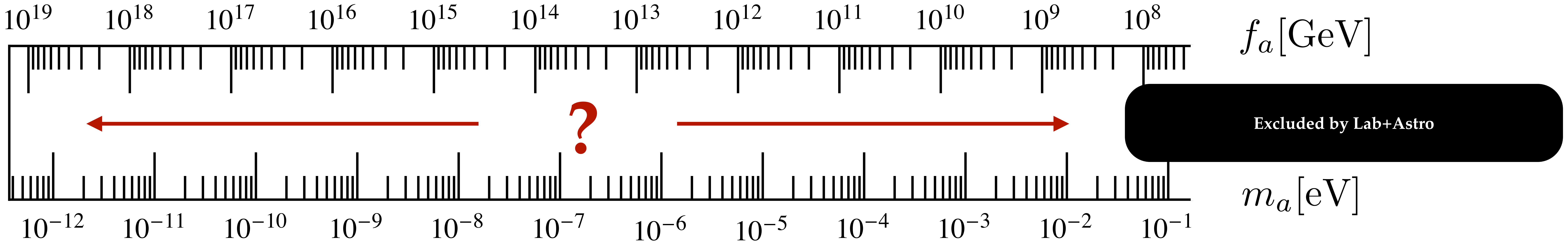
$$m_A|\theta|^2 R^3 = \frac{\rho_A}{m_A} R^3 \equiv N_A$$



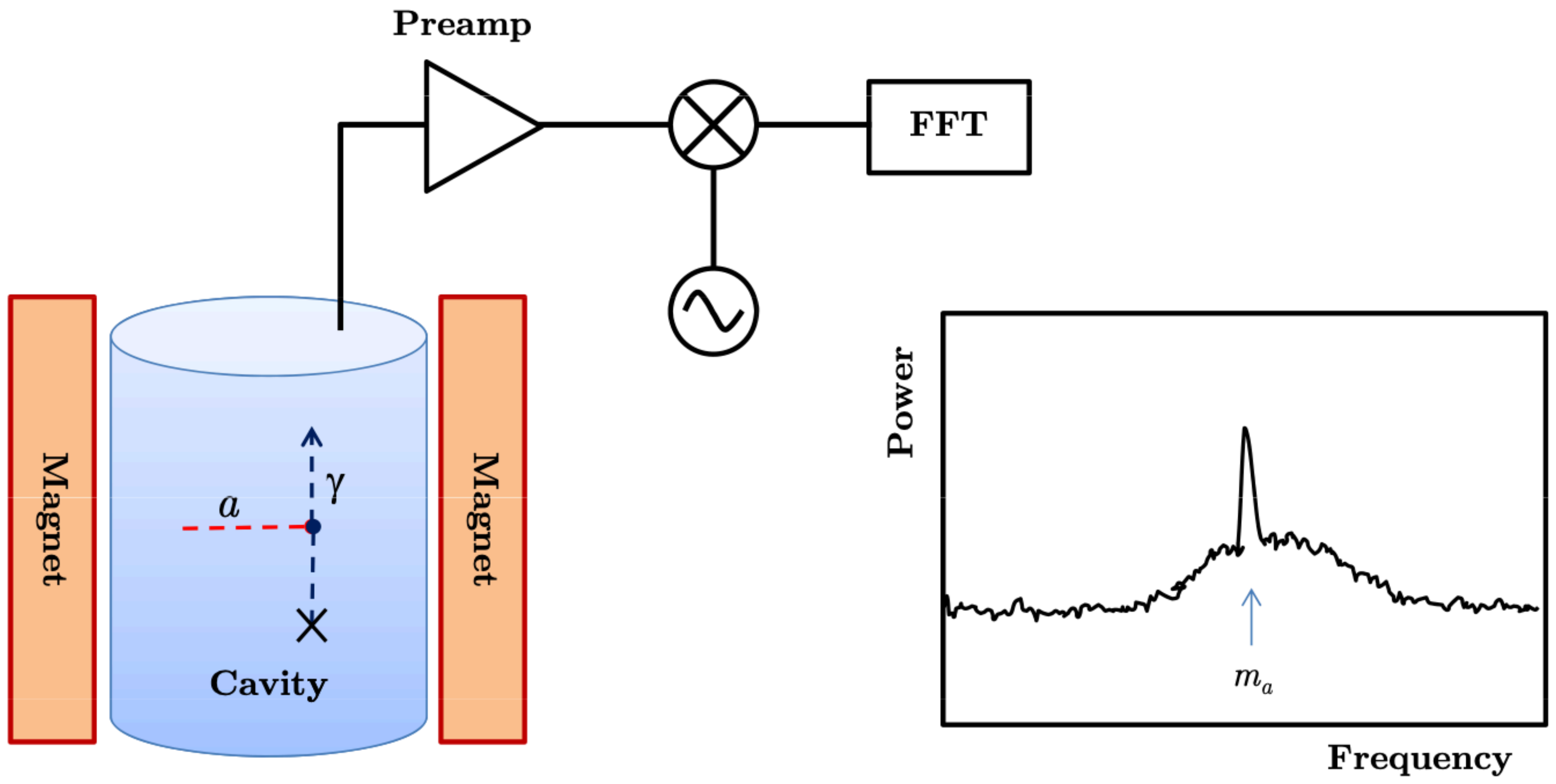
Borsanyi+ [1606.07494]

Axion Dark Matter Mass

- What is the "typical mass" of QCD axion dark matter?



- How can theory inform experimental searches?
- Information on theoretical predictions could also be used to interpret future experimental results.

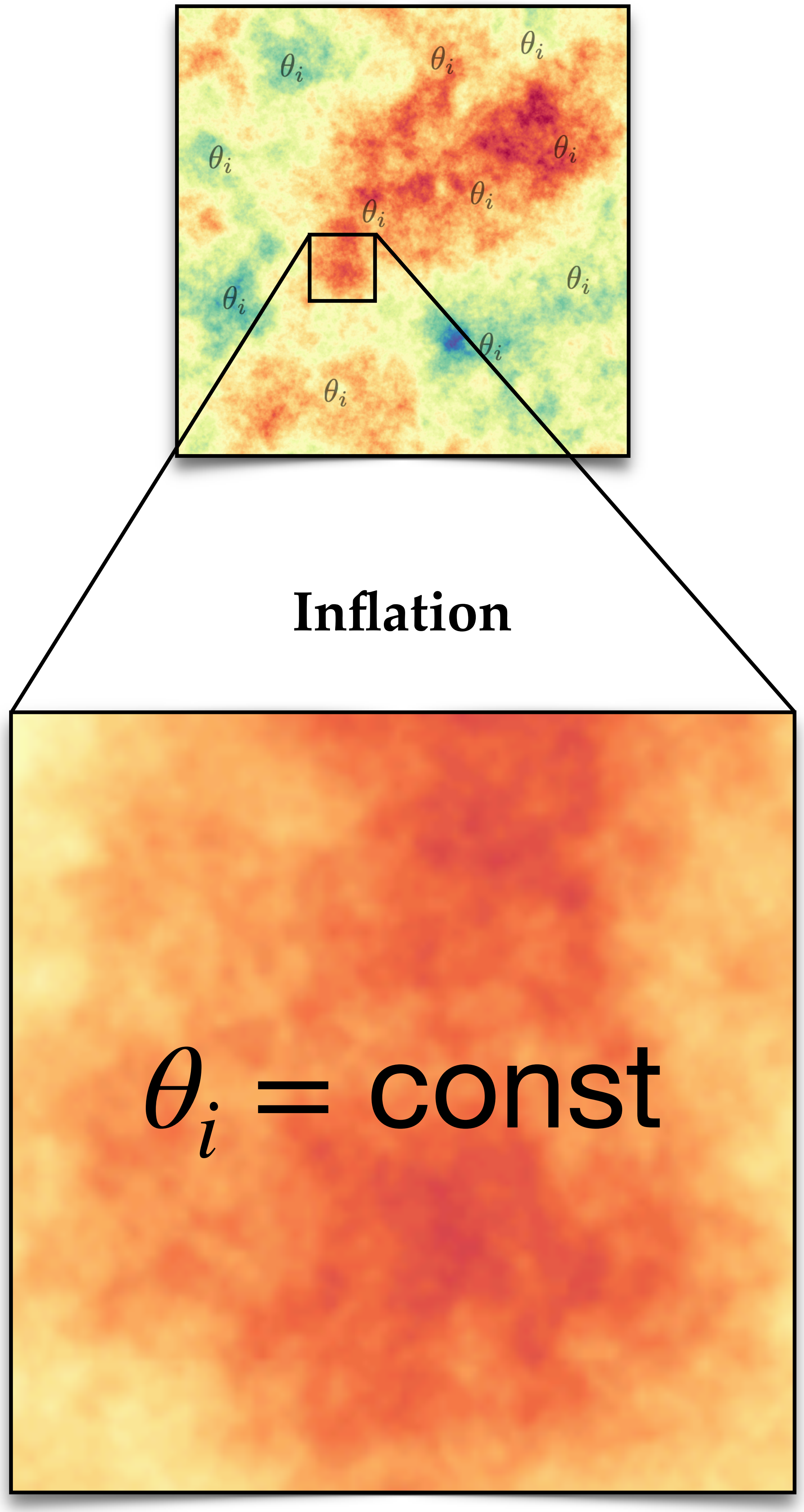


Redondo, Irastorza [1801.08127v2]

When did Inflation happen?

Pre-Inflationary Scenario

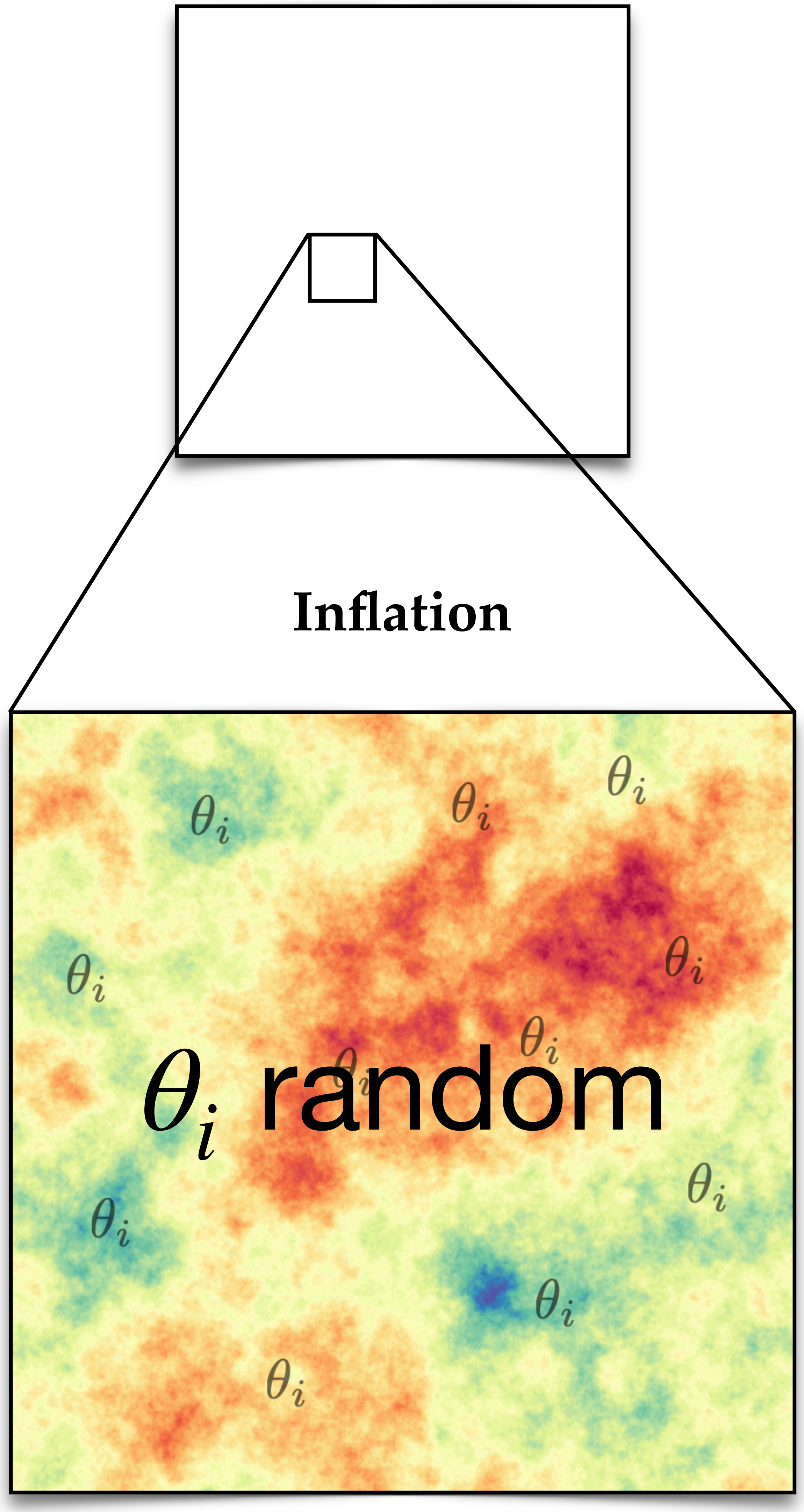
PQ broken **before** and **during** inflation



$$\sqrt{\langle \theta_i \rangle} = ???$$

Post-Inflationary Scenario

PQ broken **after** inflation

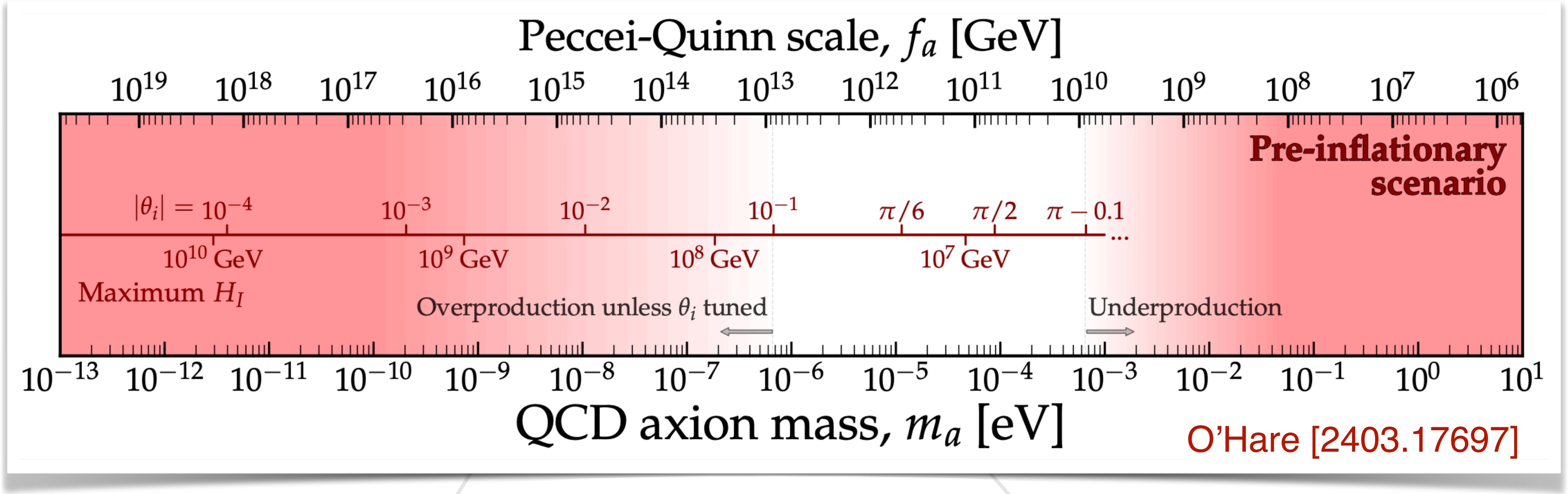


$$\sqrt{\langle \theta_i \rangle} \sim 2$$

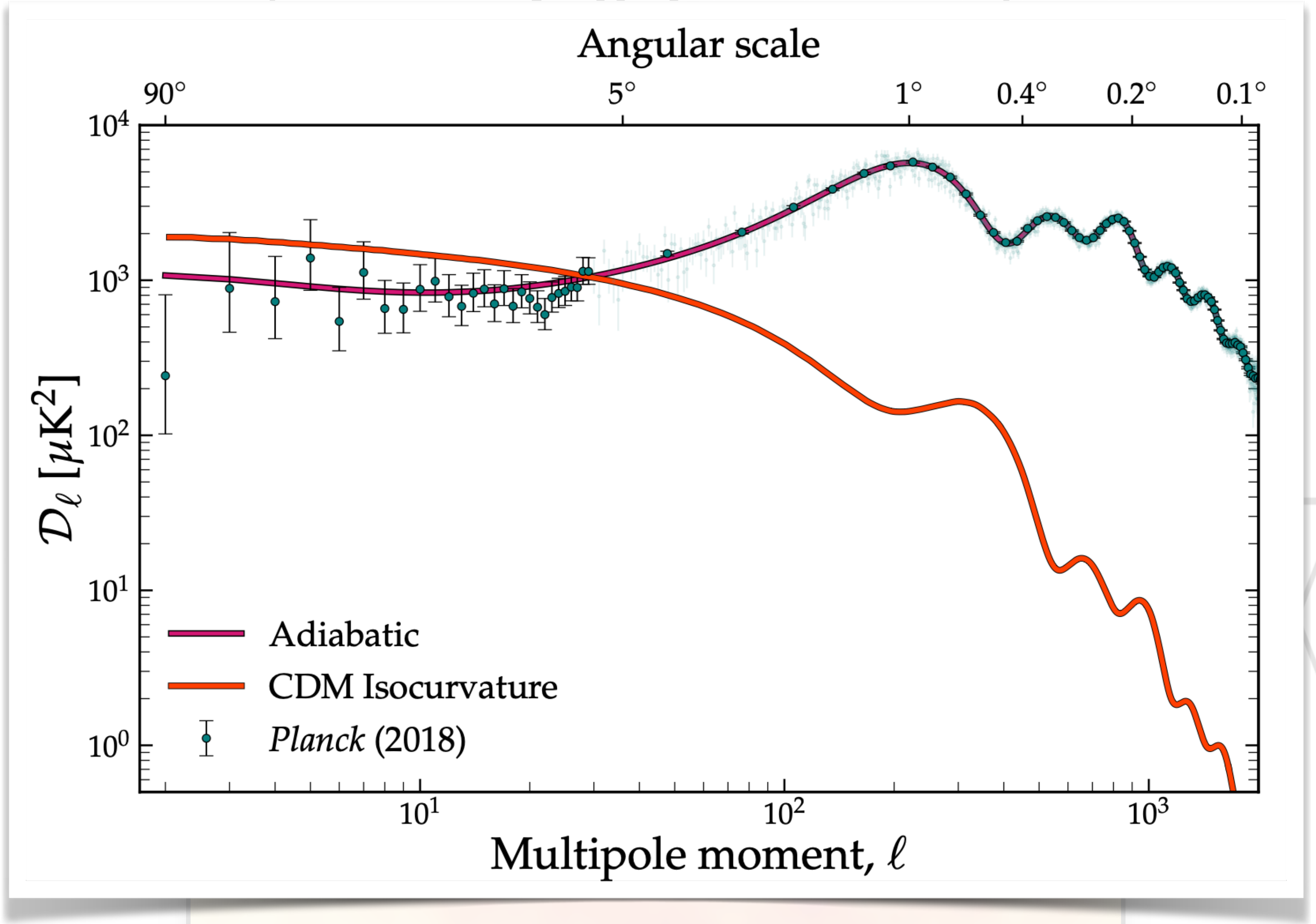
When did Inflation happen?

Pre-Inflationary Scenario

PQ broken **before** and **during** inflation



O'Hare [2403.17697]



PLANCK [1907.12875]

$\sqrt{\langle\theta_i\rangle} = ???$

Post-Inflationary Scenario

PQ broken **after** inflation

$\sqrt{\langle\theta_i\rangle} \sim 2$

When did Inflation happen?

Pre-Inflationary Scenario

PQ broken **before** and **during** inflation

$$\langle \theta_i \rangle = ???$$

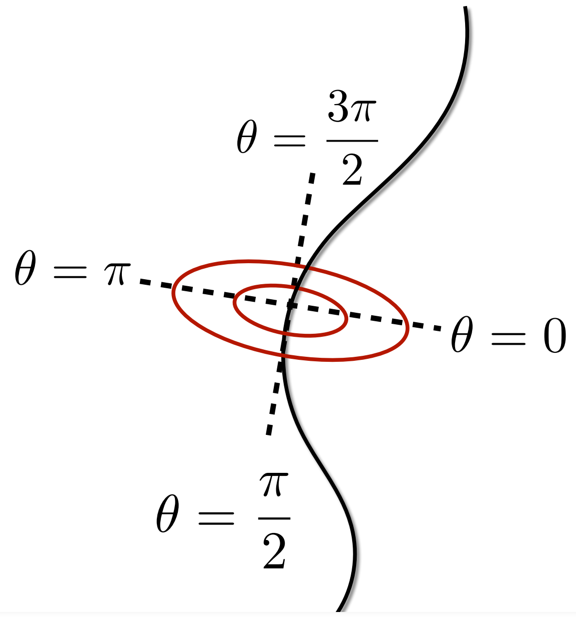
Post-Inflationary Scenario

PQ broken **after** inflation

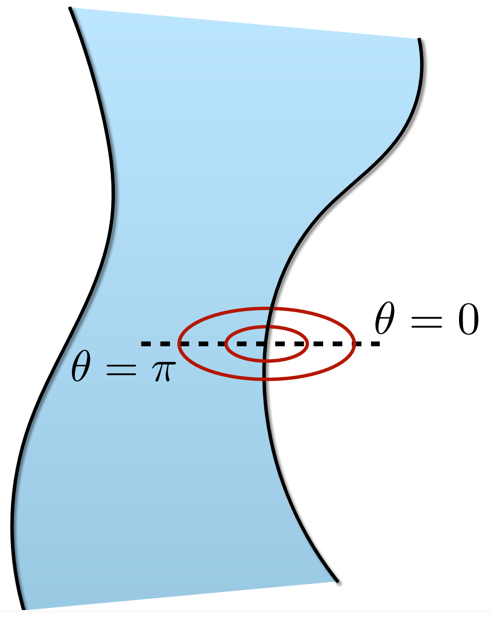
- Allows in principle for precise axion mass prediction

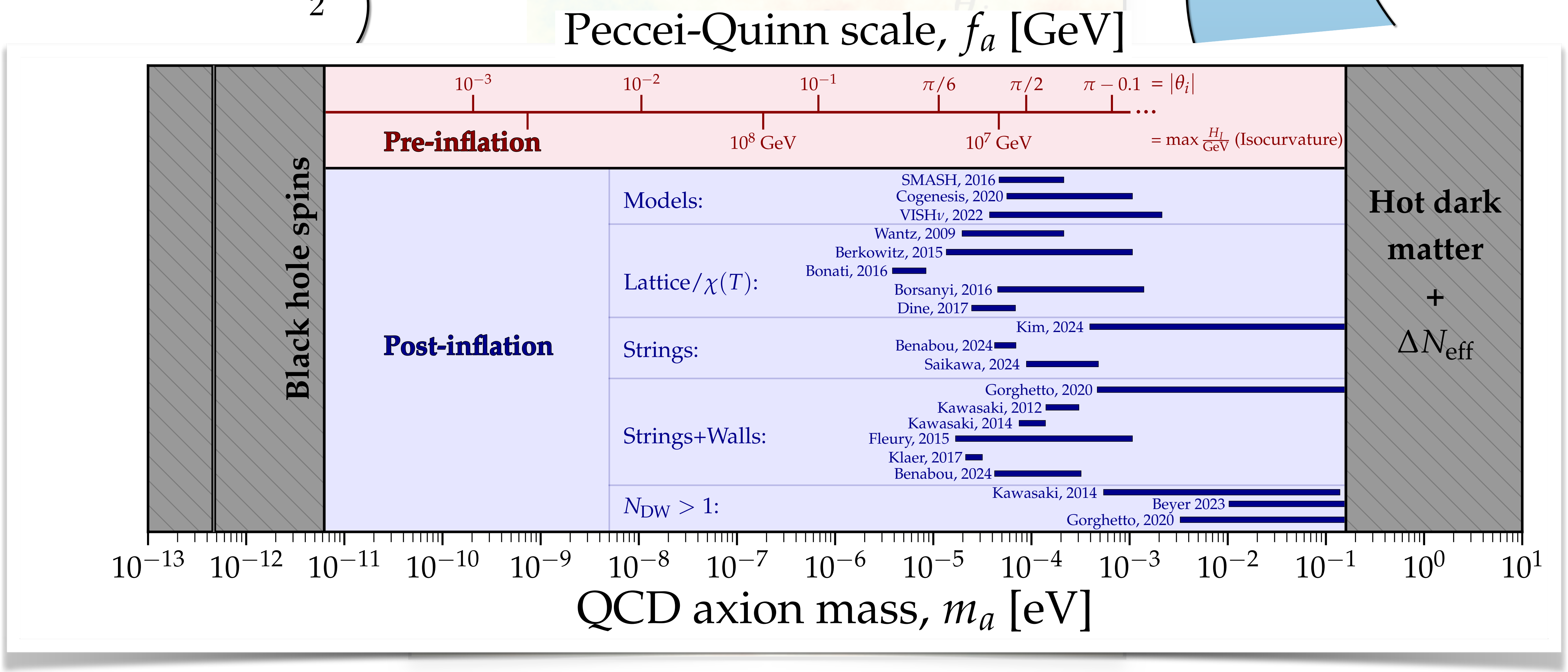
$$\Omega_a h^2 = 0.12 \quad \Rightarrow \quad m_a = ???\mu\text{eV}$$

- **Subtlety:** Formation of Strings (+ Domain Walls)



Buschmann, MK+ [to appear]





Formation of Topological Defects

- System undergoes a phase transition with order parameter θ
- Causally disconnected regions have **different** θ_i
- Those different patches meet and **spatial field gradients** lead to formation of **topological defects** via the **Kibble mechanism**

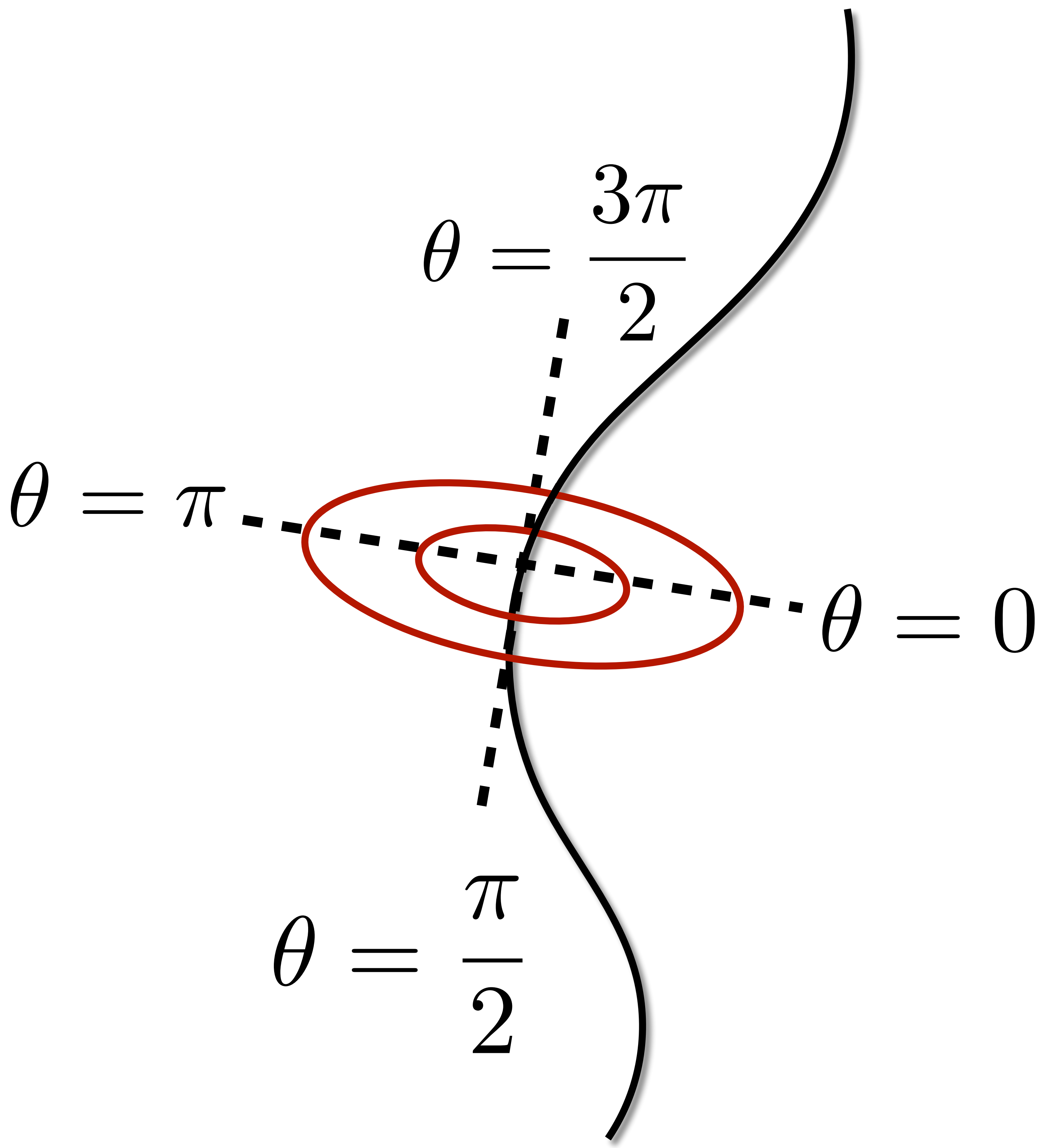
Kibble [J. Phys. A 9 (1976) 1387–1398]

$$\ddot{\theta} + 3H\dot{\theta} - \boxed{\frac{1}{a^2}\nabla^2\theta} + m_a^2\theta = 0$$

Formation of Topological Defects

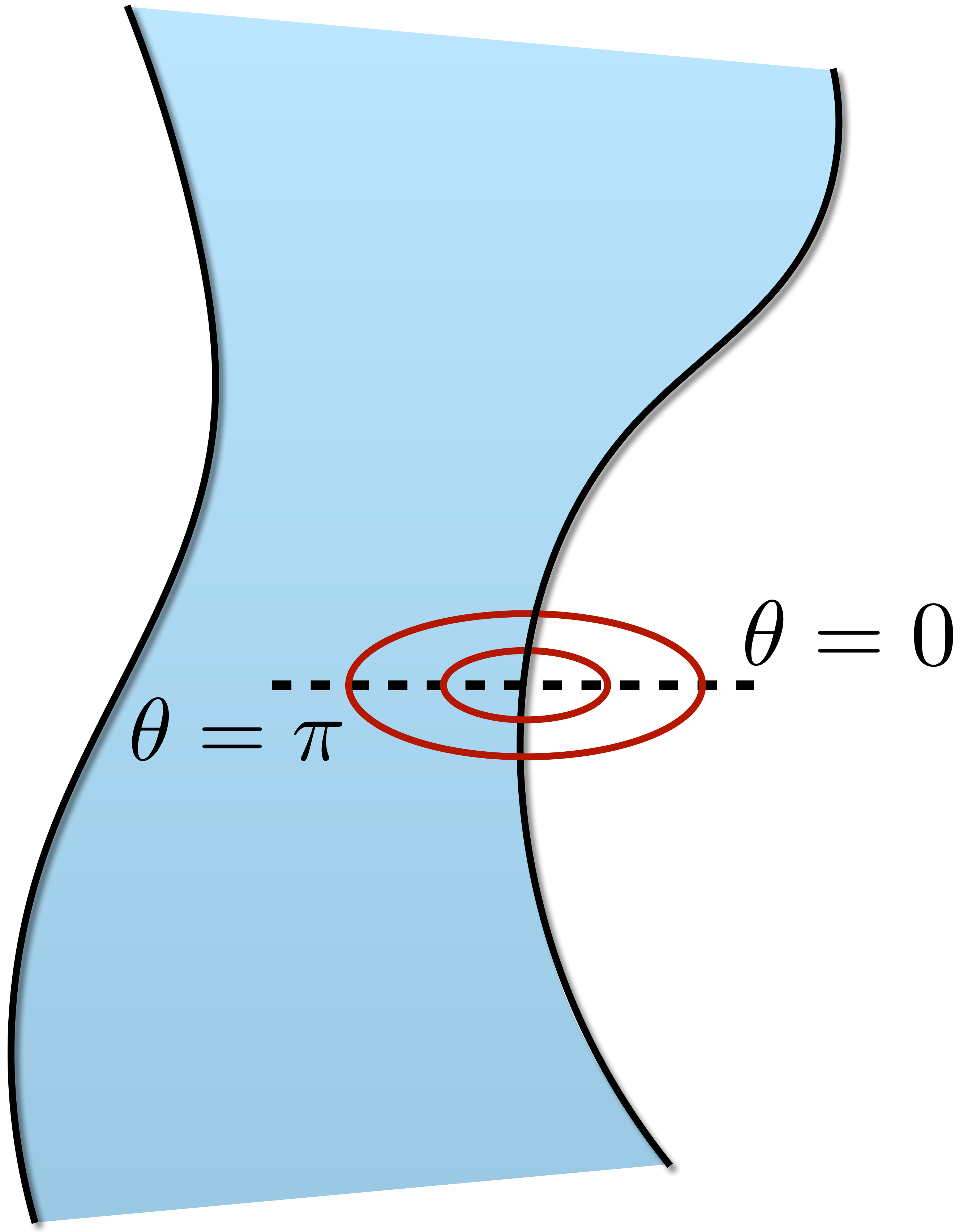
Strings

Axion field winds around 2π



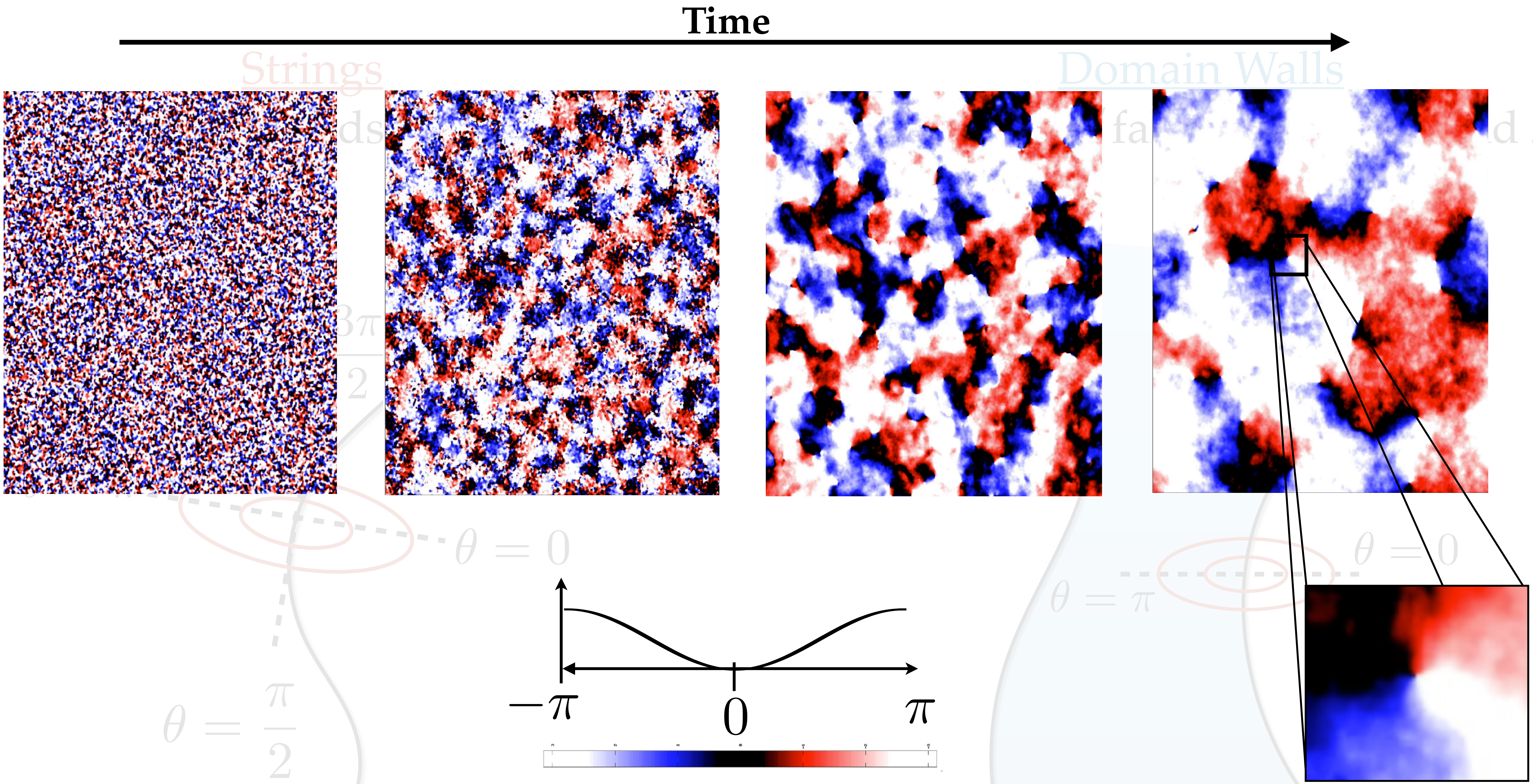
Domain Walls

between true/false vacuum (0 and π)



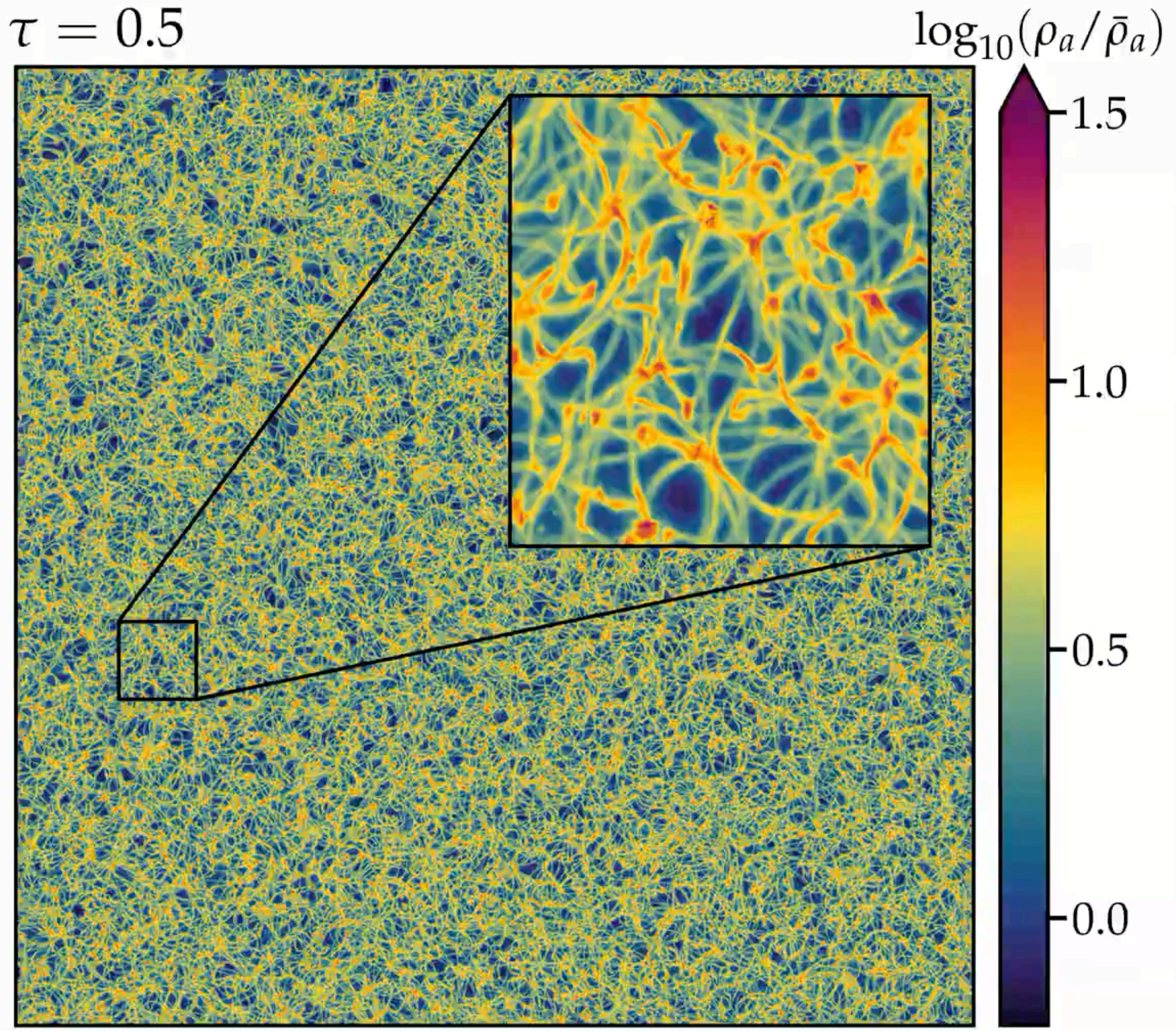
Buschmann, MK+ [to appear]

Formation of Topological Defects



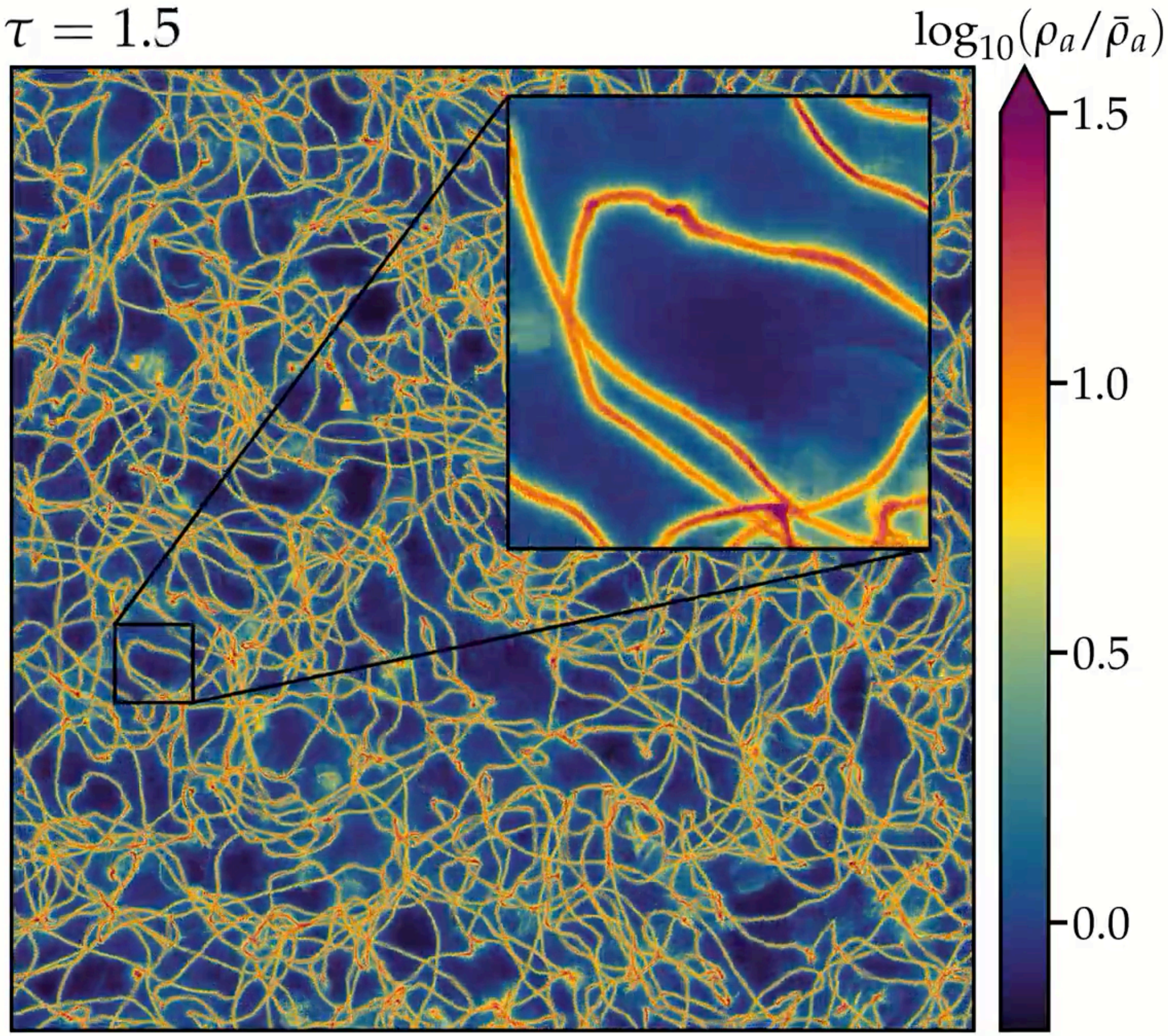
Courtesy of J. Redondo

Cosmological Evolution in the Post-Inflationary Scenario



O'Hare+ [2110.11014]

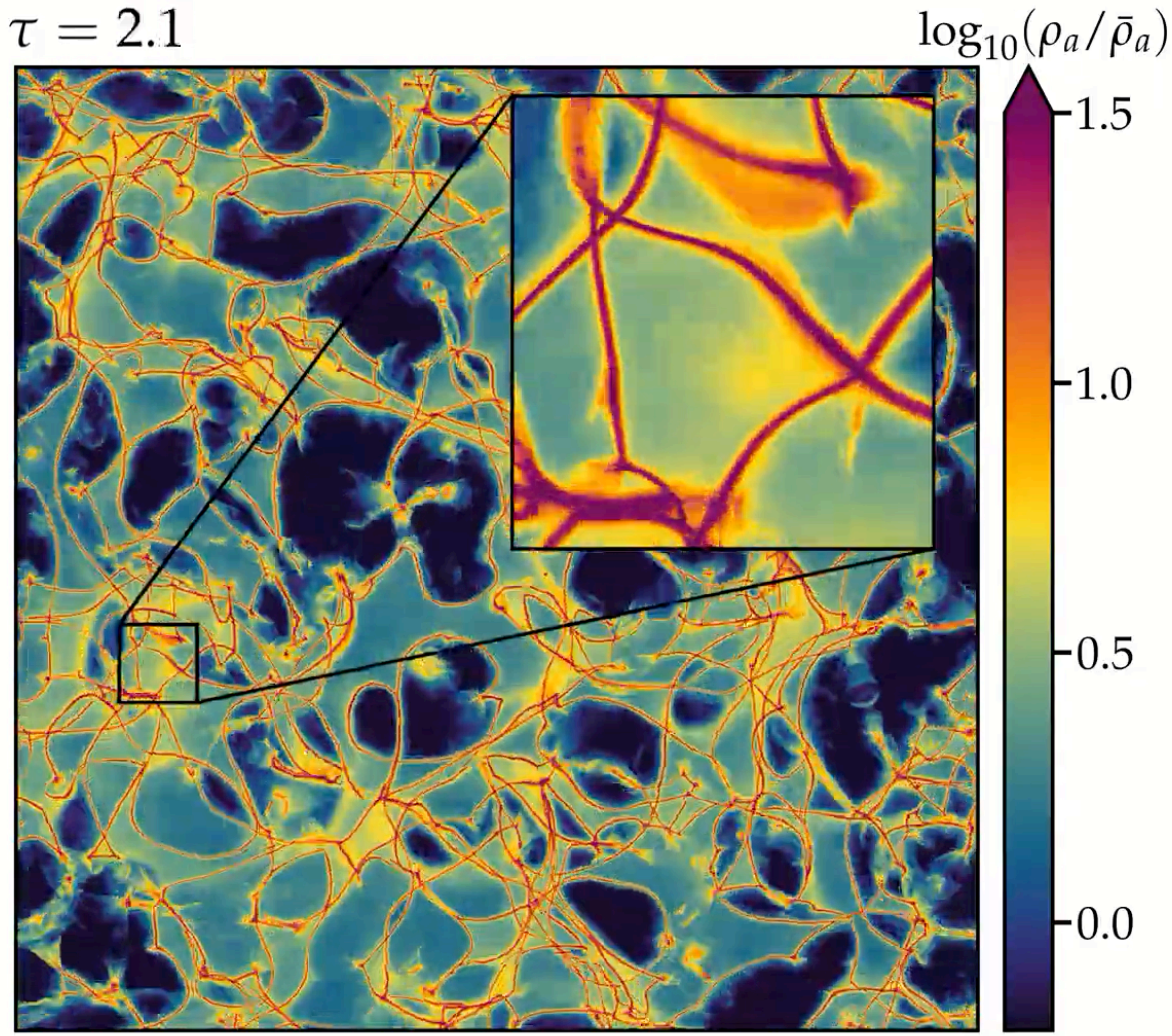
Cosmological Evolution in the Post-Inflationary Scenario



O'Hare+ [2110.11014]

- Network evolves to **scaling** solution
- Scaling maintained by radiating **relativistic, massless** axions

Cosmological Evolution in the Post-Inflationary Scenario



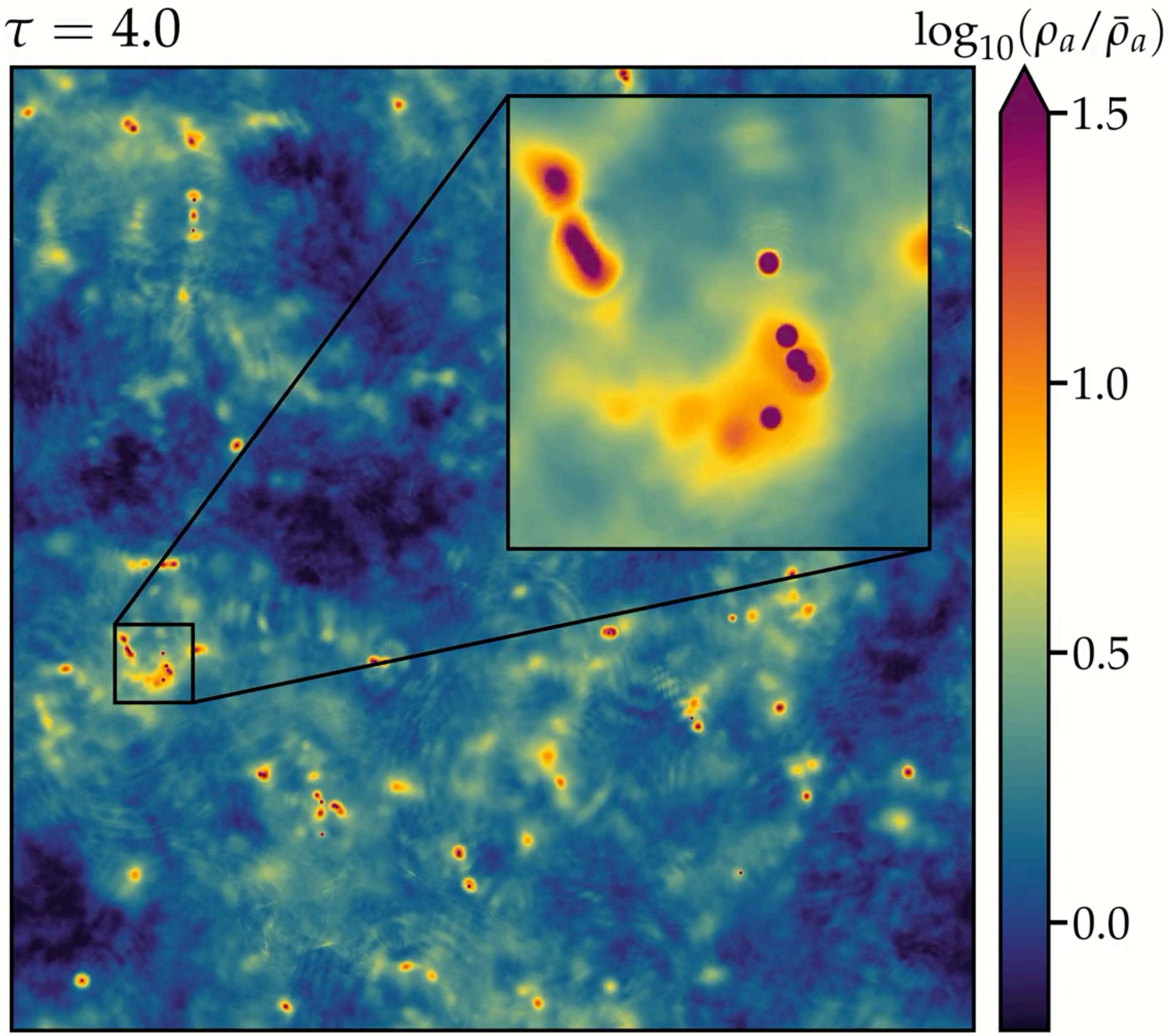
O'Hare+ [2110.11014]

- Network evolves to **scaling** solution
- Scaling maintained by radiating **relativistic, massless** axions
- QCD phase transition at $T \sim$ GeV
- **Domain Walls*** form and network collapses
- Rapidly increasing mass renders axions **nonrelativistic**

*In general more complex dynamics if $N_{\rm DW} > 1$

$$V(\theta) \sim \cos(\theta) \rightarrow \cos(N_{\rm DW}\theta)$$

Cosmological Evolution in the Post-Inflationary Scenario



O'Hare+ [2110.11014]

- Network evolves to **scaling** solution
- Scaling maintained by radiating **relativistic, massless** axions
- QCD phase transition at $T \sim \text{GeV}$
- **Domain Walls*** form and network collapses
- Rapidly increasing mass renders axions **nonrelativistic**
- **Axitons** form and serve as seeds for dark matter structure formation (**miniclusters/axion stars**)

see e.g. Vaquero+ [1809.09241]

How to simulate Axion Strings?

- Solve the classical EoM for a complex scalar field in comoving coordinates, discretised on a static lattice:

$$\partial_\tau^2\phi - \nabla^2\phi + \lambda\phi\left(|\phi|^2 - \tau^2\right) = 0$$

- **Tricky:** Simulations require proper resolution of two very different length scales

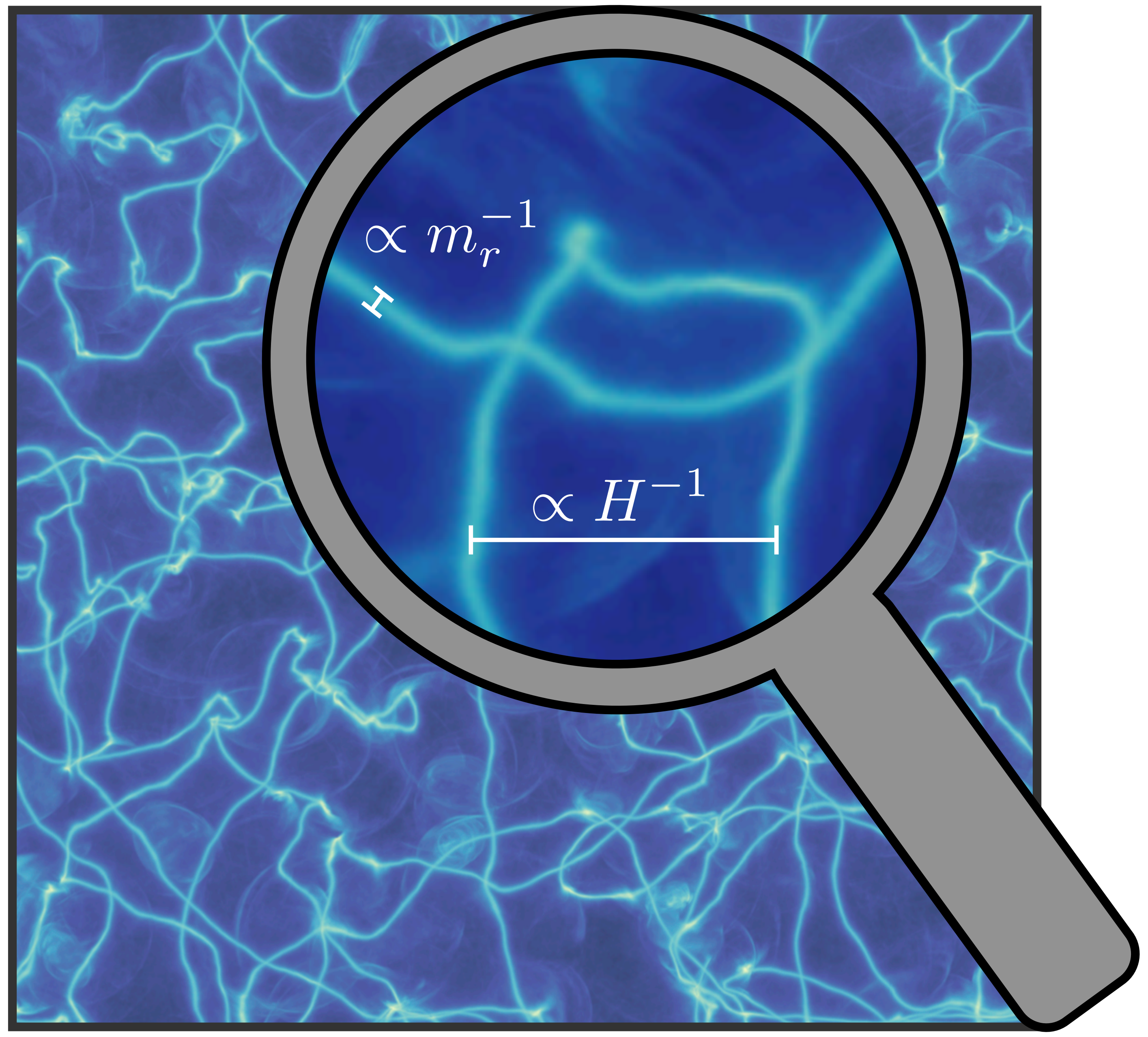
  - String core radius

    $$\propto \frac{1}{m_r} \propto \frac{1}{f_a}, \text{ where } m_r = \text{radial mass}$$

  - String separation given by Hubble radius

    $$\propto \frac{1}{H}$$

  - Realistic value: $\dfrac{f_a}{H_{\rm QCD}} \approx 10^{30} \implies \log\left(\dfrac{m_r}{H}\right) \approx 70$



Courtesy of K. Saikawa

Jaxions Code

- Highly parallelised `C++` code to simulate the evolution of the axion dark matter field in the early Universe

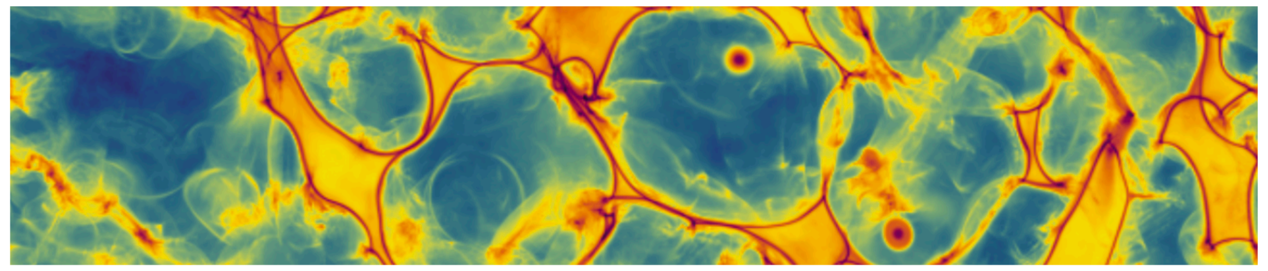
- Available on Github

Jaxions-docs

Search Jaxions-docs

View on GitHub

- Home
- Installation
- Running the code
- Physics
- Python tools



Jaxions

A grid-based massively parallel code to study the Axion field evolution before, around and after the QCD phase transition

View it on GitHub

Overview

- Axion string simulations to calculate emission spectra
- String-Wall network simulations with $N_{\rm DW} = 1, N_{\rm DW} = 2$
- Generalisation to axion-like-particles
- Interface with AxionNyx and gadget-4

Details on the physics of `jaxions` are found here.

Obtain the code

To download the source code from the public repository use:

```
git clone https://github.com/veintemillas/jaxions.git
```

`jaxions`: Simulating the Axion Dark Matter Field in the Post-Inflationary Scenario

Alejandro Vaquero,[a] Javier Redondo,[a,b] Ken'ichi Saikawa,[c] Mathieu Kaltschmidt,[a] Giovanni Pierobon[d]

[a]CAPA & Departamento de Física Teórica, Universidad de Zaragoza, C. Pedro Cerbuna 12, 50009 Zaragoza, Spain
[b]Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 München, Germany
[c]Institute for Theoretical Physics, Kanazawa University, Kakuma-machi, Kanazawa, Ishikawa 920-1192, Japan
[d]School of Physics, The University of New South Wales, NSW 2052 Kensington, Sydney, Australia

E-mail: alexv@unizar.es, jredondo@unizar.es, saikawa@hep.s.kanazawa-u.ac.jp, mkaltschmidt@unizar.es, g.pierobon@unsw.edu.au

Abstract. We present `jaxions`, a massively parallel code to simulate the evolution of the axion field on a uniform grid, specialised for the case of axion dark matter in the post-inflationary scenario.
The code tracks the evolution of the Peccei-Quinn complex scalar field ϕ, as long as topological defects are present, the subsequent evolution of the axion field θ, and the non-relativistic field Ψ, well after the QCD phase transition.
Additionally, we provide an option to create initial conditions suitable for running the simulations with `AMReX`-based adaptive mesh codes such as `axioNyx` and a utility function to map the final grid into a particle snapshot, to continue the simulation of the forming miniclusters with the N-body code `gadget4`. The code also features the extensive `python` library `pyaxions`, with a variety of tools and options to set up, run and analyse the simulations.

The Issue of large $\log(m_r/H)$

- Evolution of the string density suggests that the energy density of the system is of order

$$\rho \sim 8\pi\xi \log(m_r/H) H^2 f_a^2$$

This leads to an enhancement by a factor of $\sim \xi \log(m_r/H)$ in comparison to the typical density $H^2 f_a^2$ at QCD temperatures.

- Does this imply an enhancement of the axion abundance (and therefore of the dark matter mass)?

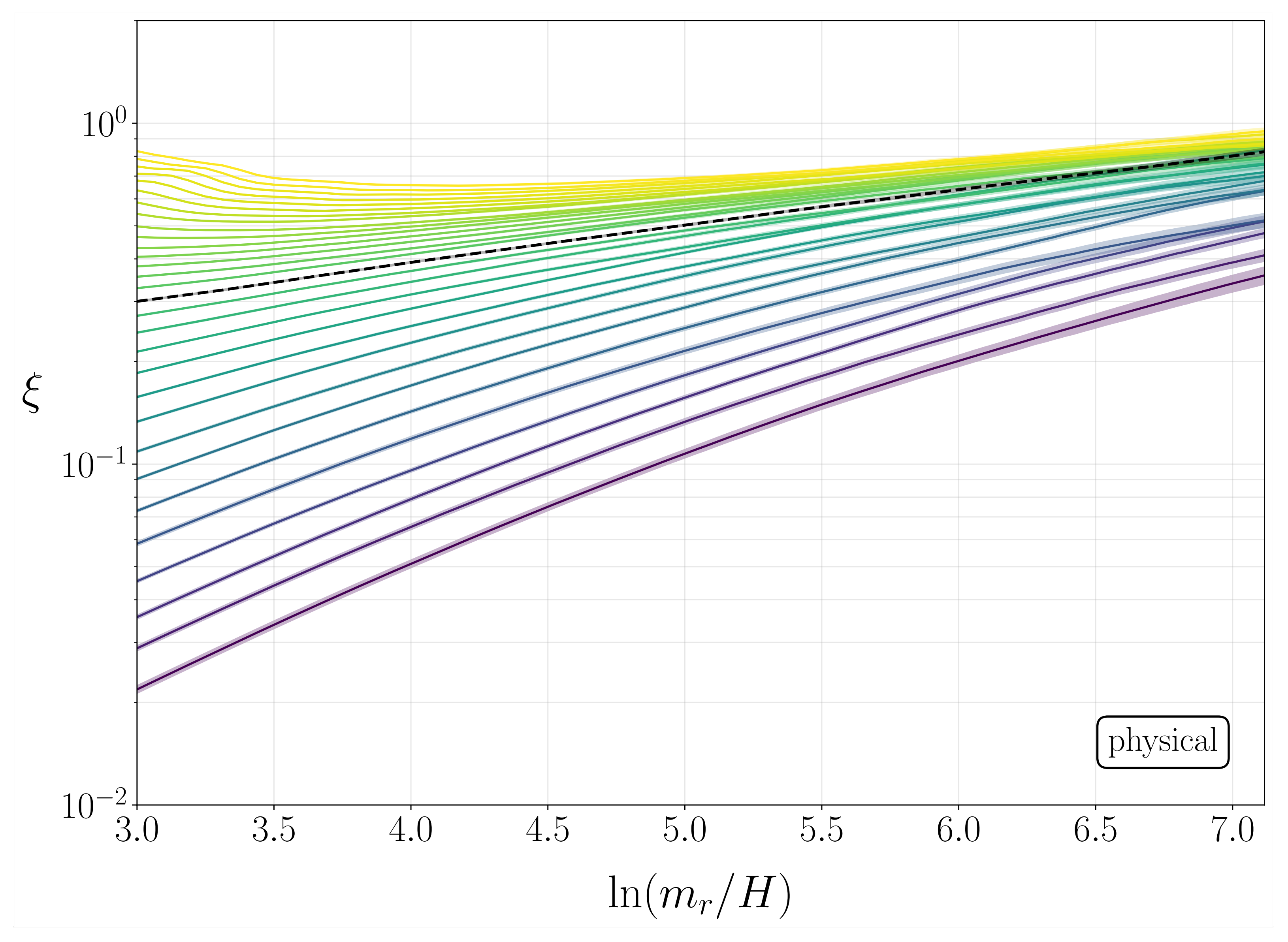
- We need to know how this energy is partitioned into radiated axions (i.e. the axion spectrum).

Logarithmic Growth of String Density

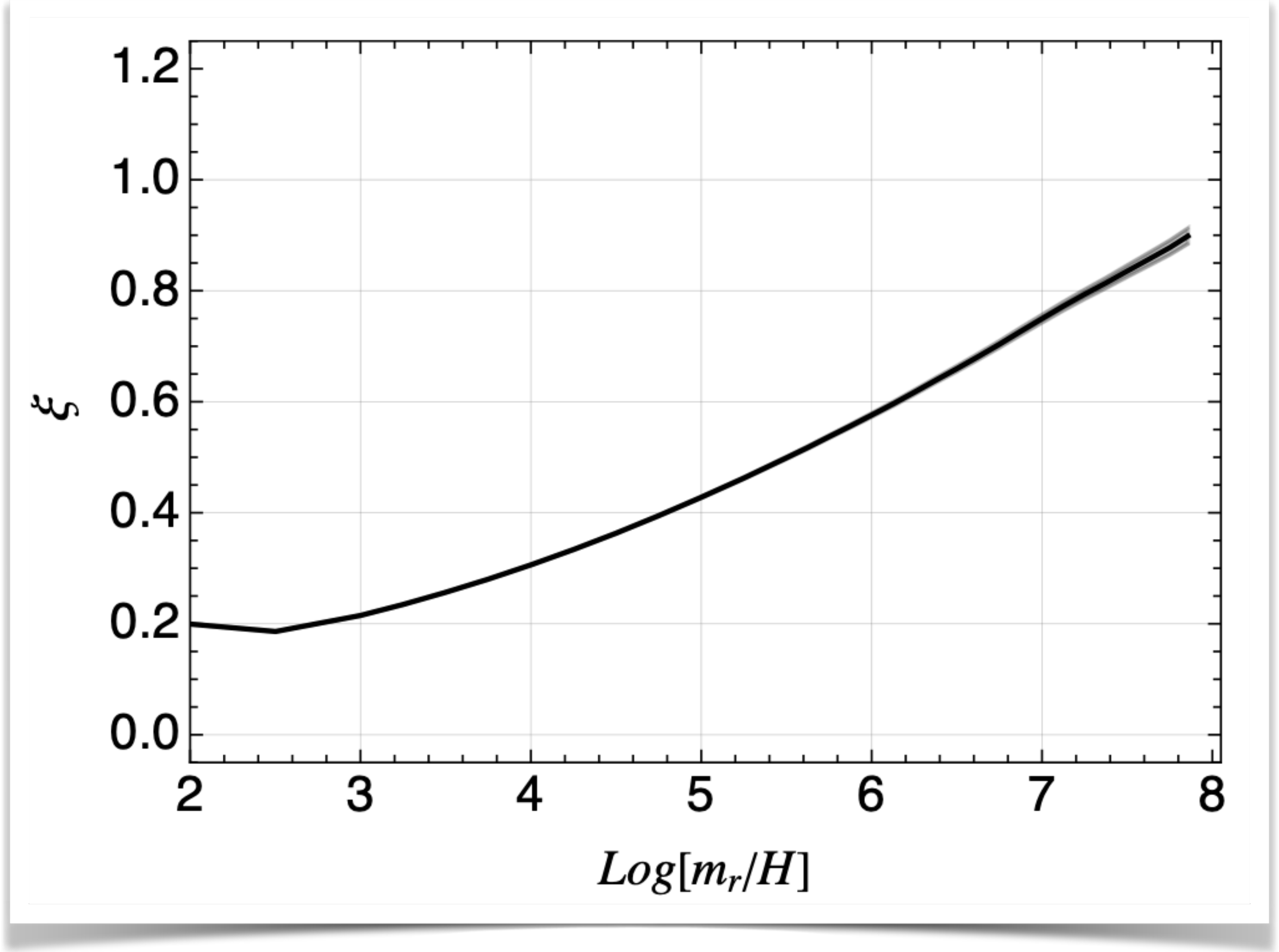
- String density parametrised as

$$\xi = \frac{l_{\text{string}} t^2}{\mathcal{V}} = \frac{\rho_{\text{string}} t^2}{\mu}$$

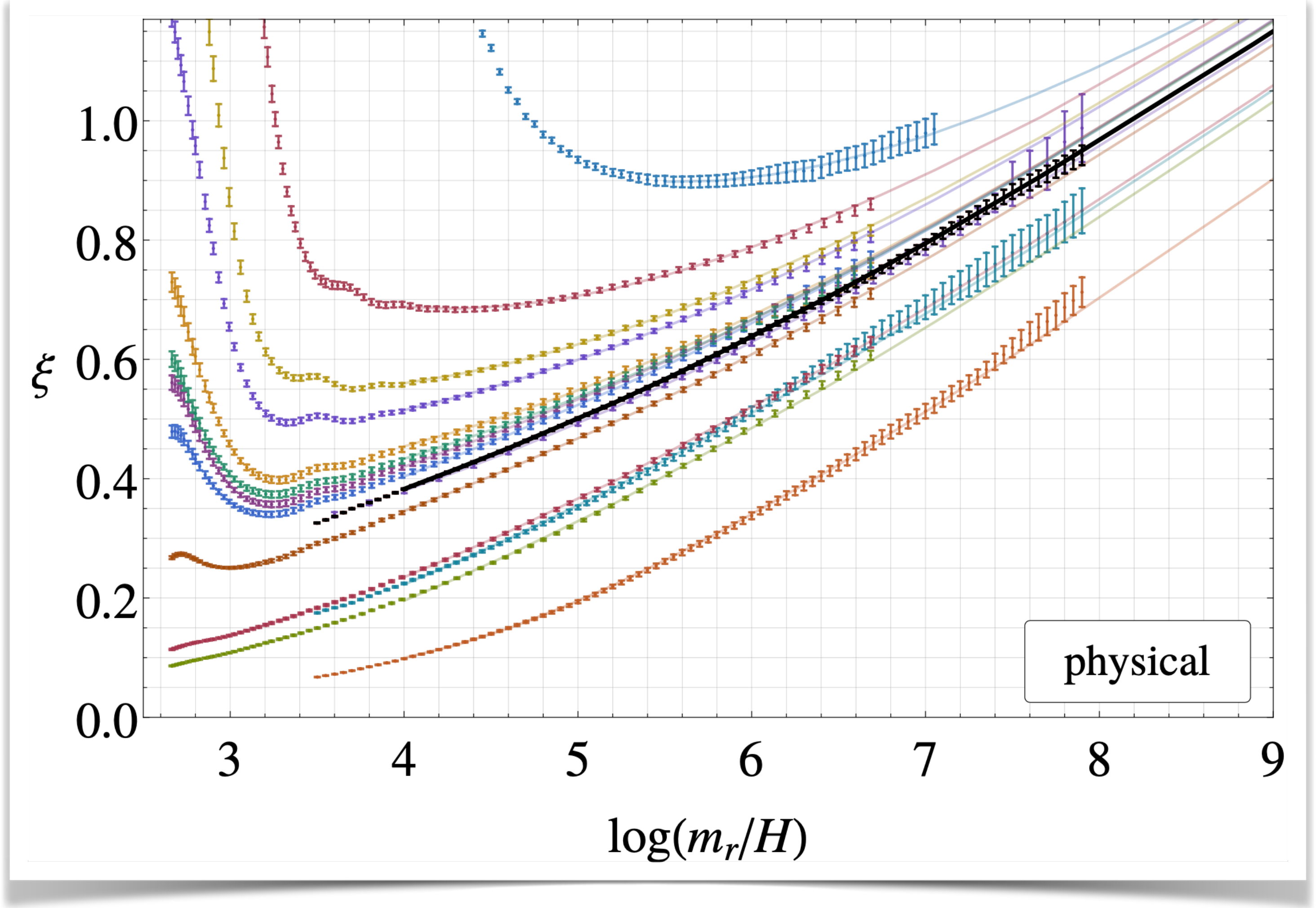
- Most recent simulations observe increase in ξ.



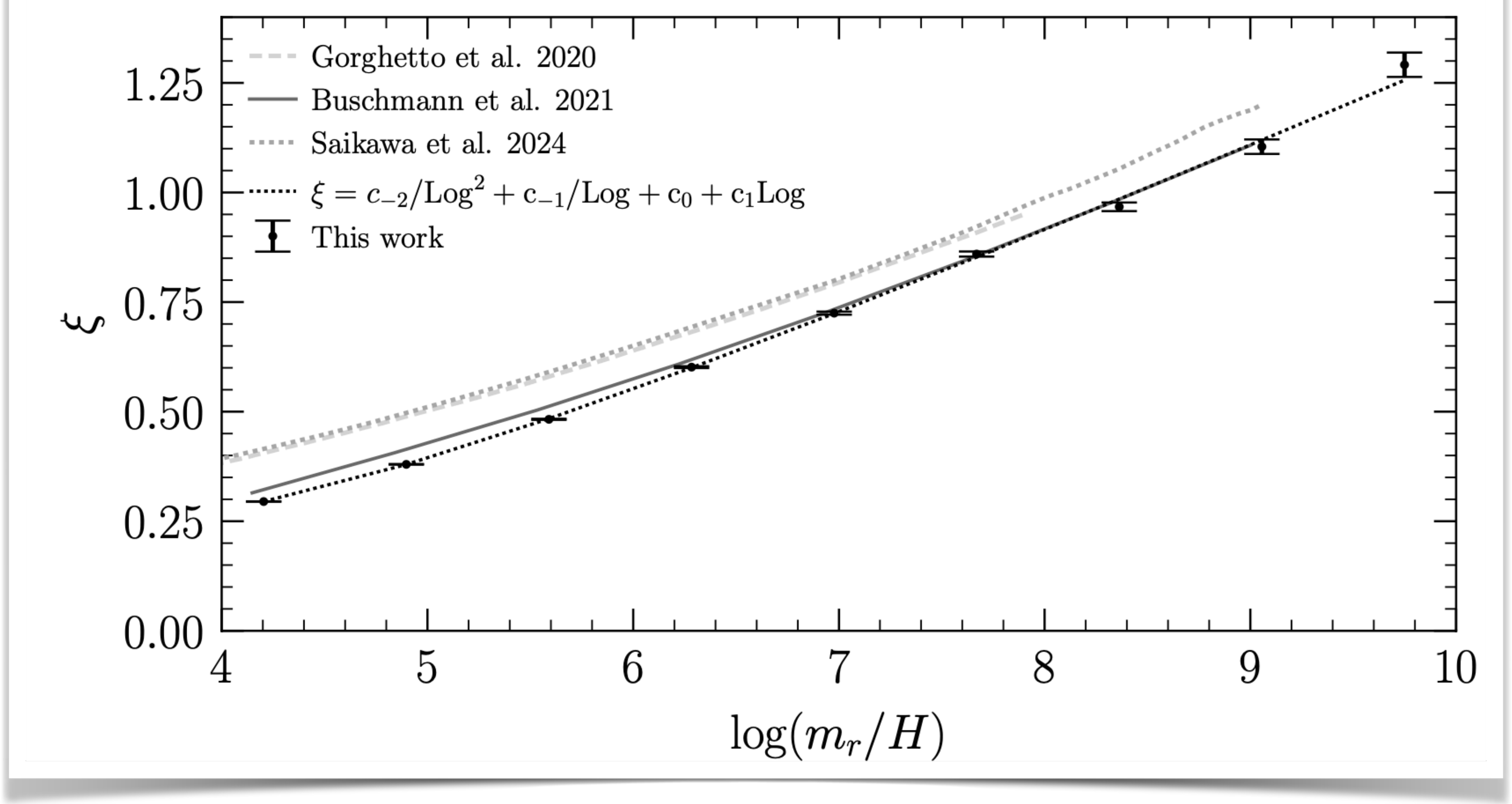
Saikawa, MK+ [2401.17253]



Kim+ [2402.00741v3]



Gorghetto+ [2007.04990]



Buschmann+ [2412.08699]

Evolution of String Density

- Model evolution of string network density with semi-analytic model:

Characteristic time scale of network restoration

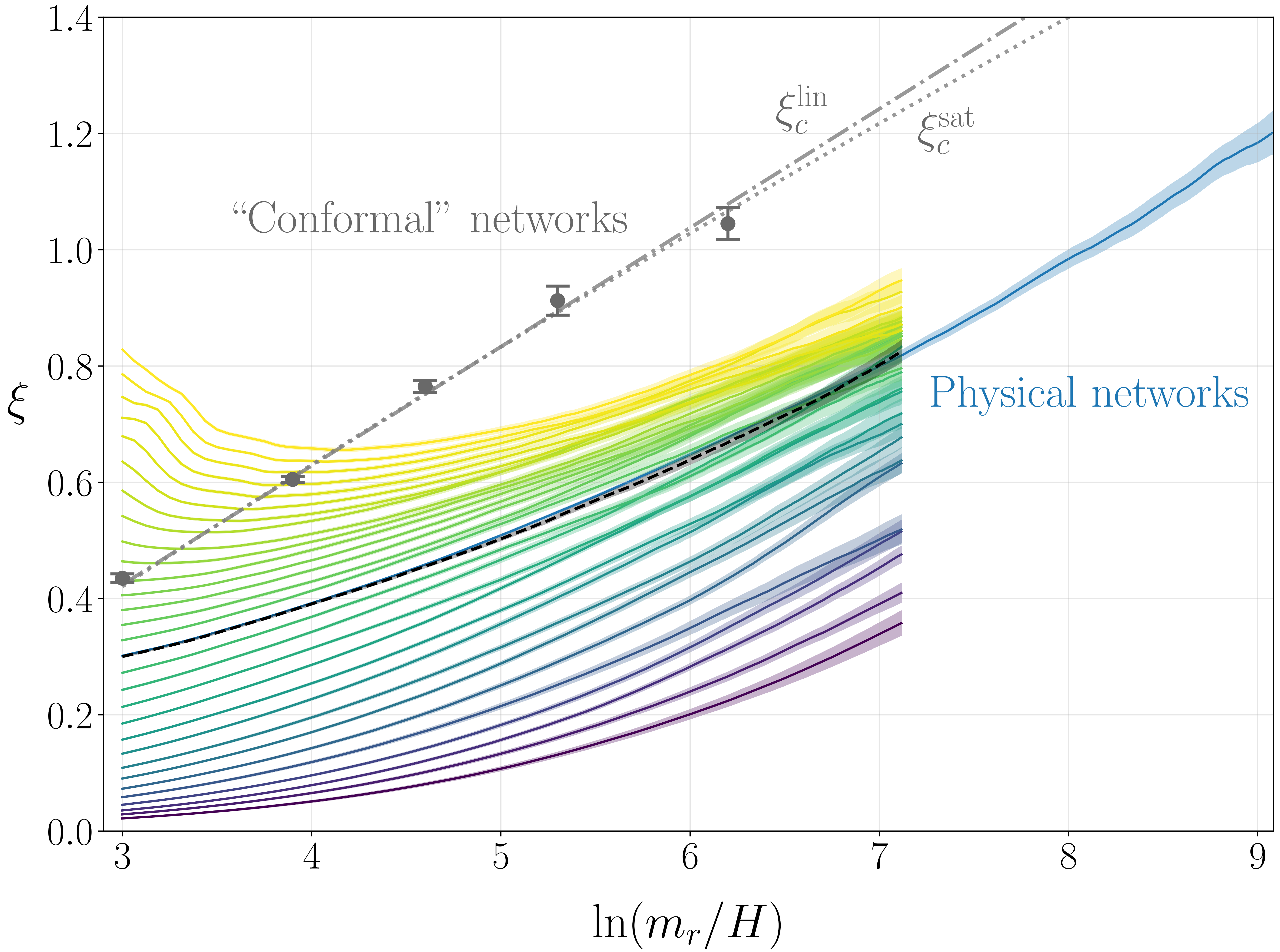
$$\frac{d\xi}{dt} = \frac{C(x)}{t}\left(\xi_c(\ell(t)) - \xi(t)\right)$$

Equilibrium density from "conformal" string network with $\log(m_r/H) = \text{const.}$

Klaer & Moore [1912.08058]

- Reasonable fit to data for $C(x) = \frac{x}{1+\sqrt{x}/c_0}$ with $x = \frac{\xi}{\xi_c}$, $c_0 \sim 1.5^{+0.8}_{-0.4}$
- Admits **attractor** solution, allows for reasonable choice of initial conditions

Evolution of String Density



Fleury & Moore [1509.00026]
Gorghetto+ [1806.04677; 2007.04990]
Kawasaki+ [1806.05566]
Hindmarsh+ [1908.03522]
Buschmann+ [2108.05368; 2412.08699]
Kim+ [2402.00741v3]

Logarithmic growth and **attractor** behaviour compatible with previous findings but behaviour at large $\log(m_r/H)$ still uncertain (**lin**ear growth vs. **satu**ration).

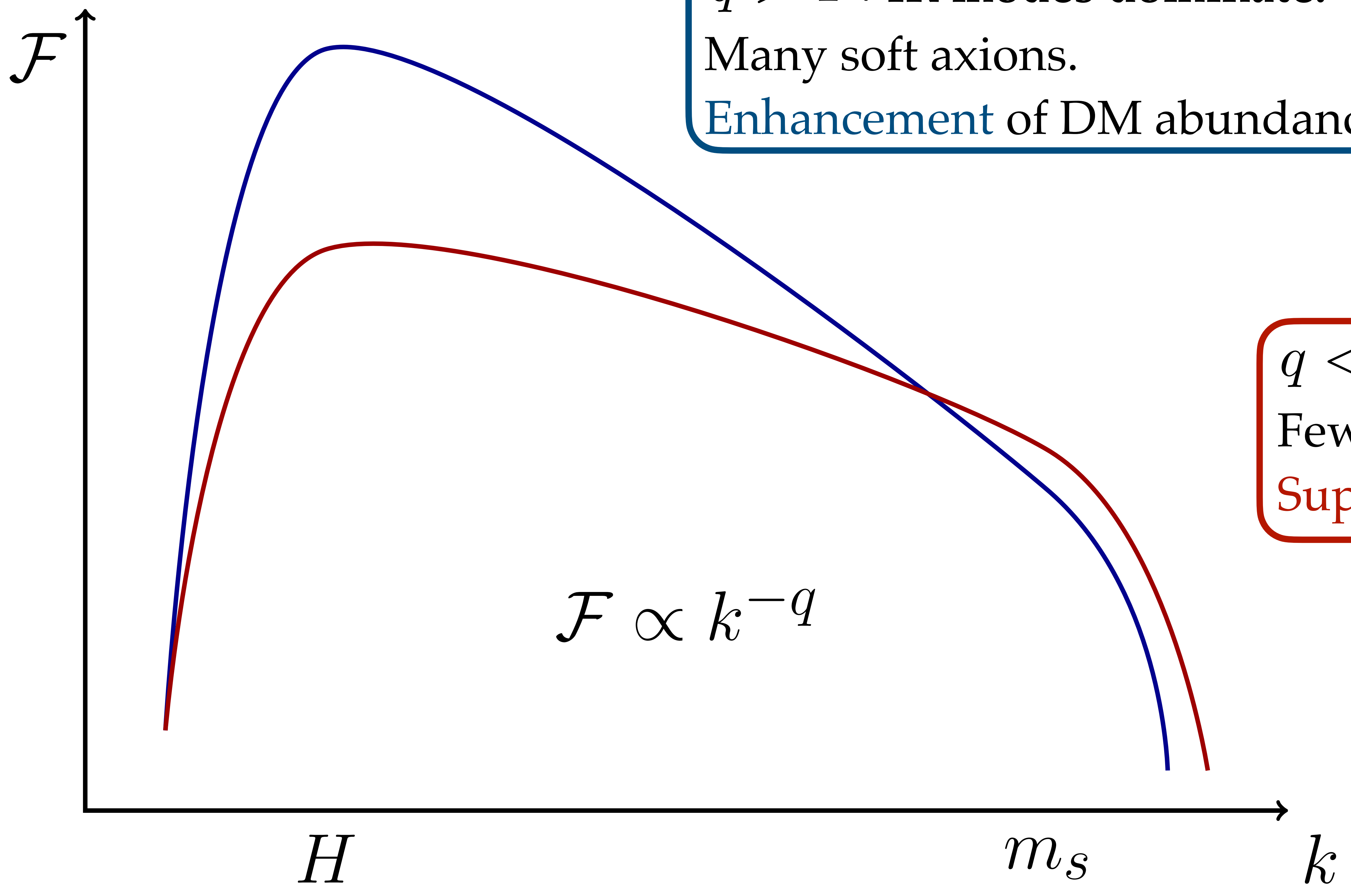
Axion Radiation from Strings

- Differential energy transfer rate:

$$\mathcal{F}\left(\frac{k}{RH}, \frac{m_r}{H}\right) \equiv \frac{1}{(f_a H)^2} \frac{1}{R^3} \frac{\partial}{\partial t}\left(R^4 \frac{\partial \rho_a}{\partial k}\right)$$

(R: scale factor)

- Slope is important! Gorghetto+ [1806.04677, 2007.04990], Buschmann+ [2108.05368, 2412.08699], Saikawa, MK+ [2401.17253]



$q > 1$: **IR modes dominate.**
Many soft axions.
Enhancement of DM abundance.

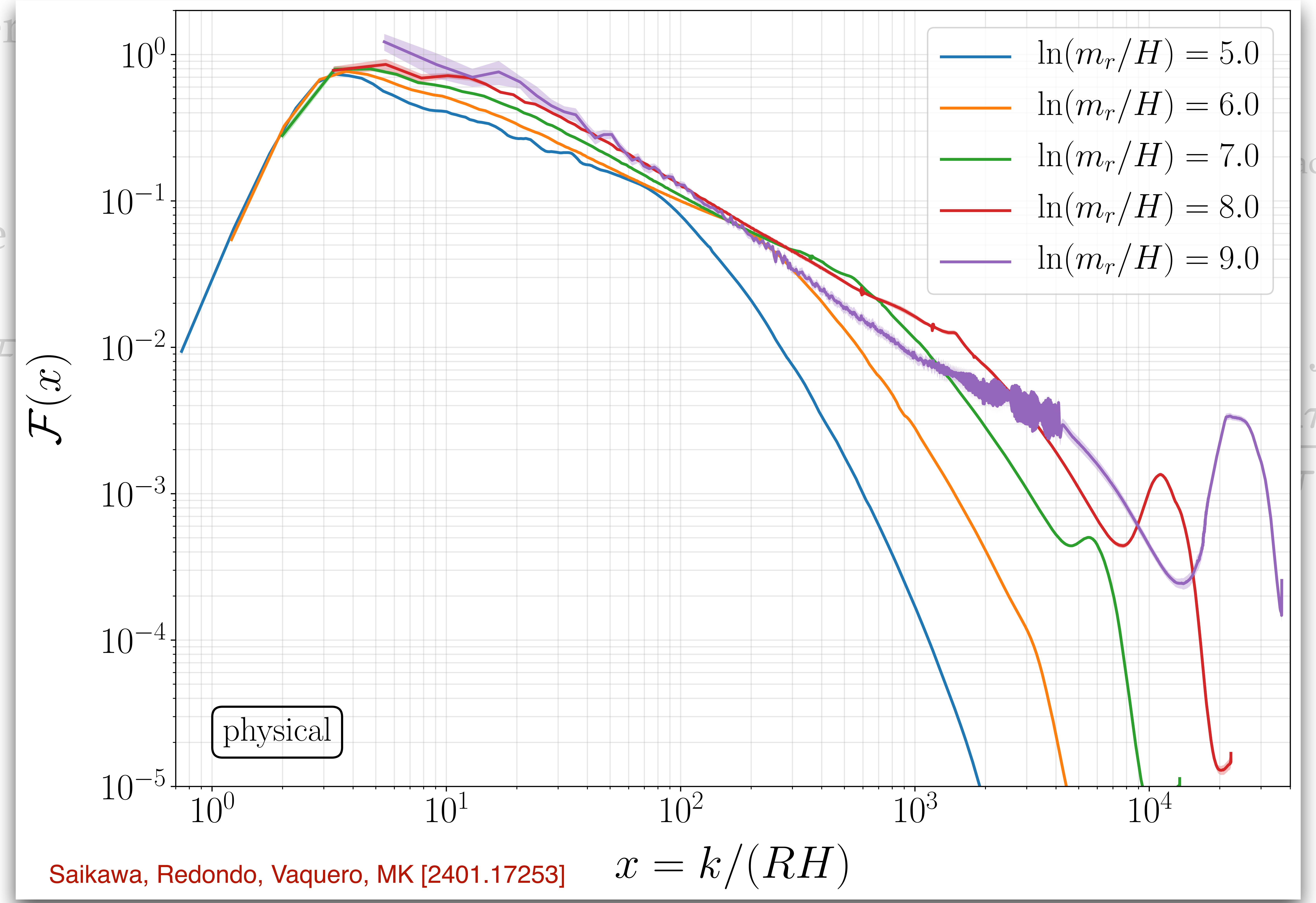
$$\frac{n_a}{f_a^2 H} = \frac{1}{f_a^2 H}\int \mathrm{d}k \frac{1}{\omega}\frac{\partial \rho_a}{\partial k} = \int^{\tau} \frac{\mathrm{d}\tau'}{\tau'} \int \frac{\mathrm{d}x'}{x'} \mathcal{F}$$

$q < 1$: **UV modes dominate.**
Fewer hard axions.
Suppression of DM abundance.

Axion Radiation from Strings

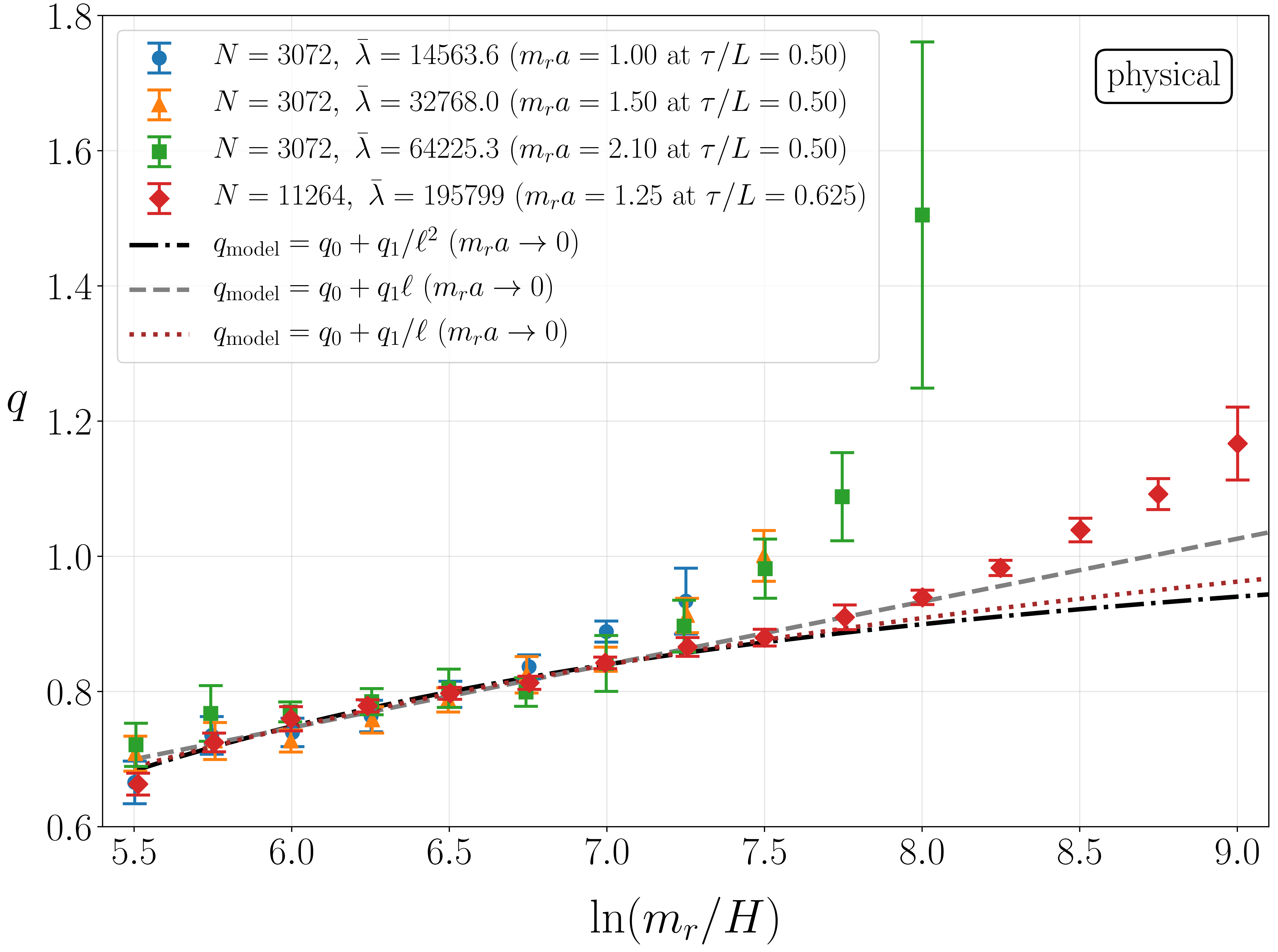
- Differ

- Slope



What can bias the Results?

- There are several systematic effects, that could explain discrepancies in the literature:
  - Initial conditions
  - Axion field oscillations
  - Discretisation effects

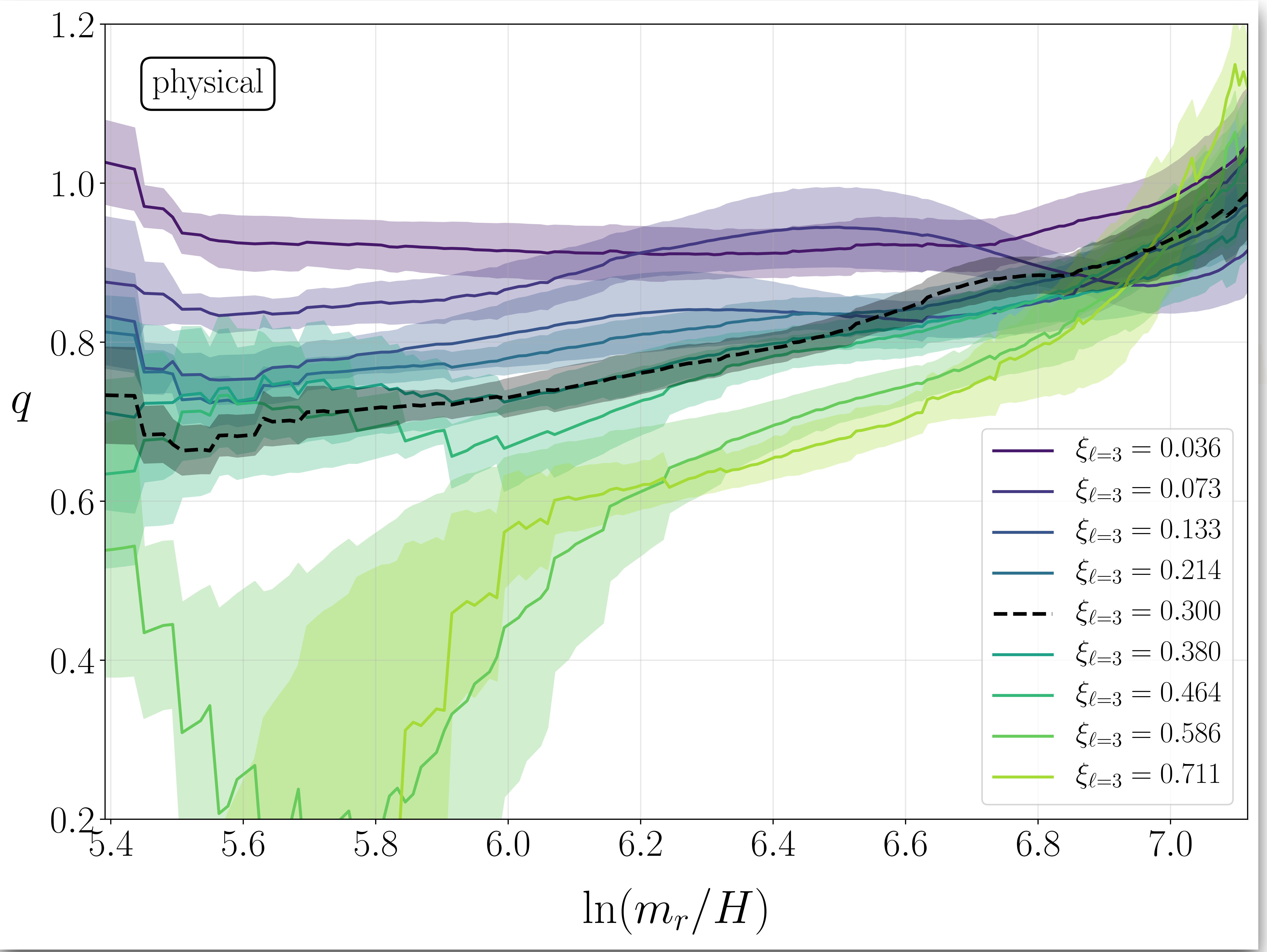


Saikawa, Redondo, Vaquero, MK [2401.17253]

What can bias the Results?

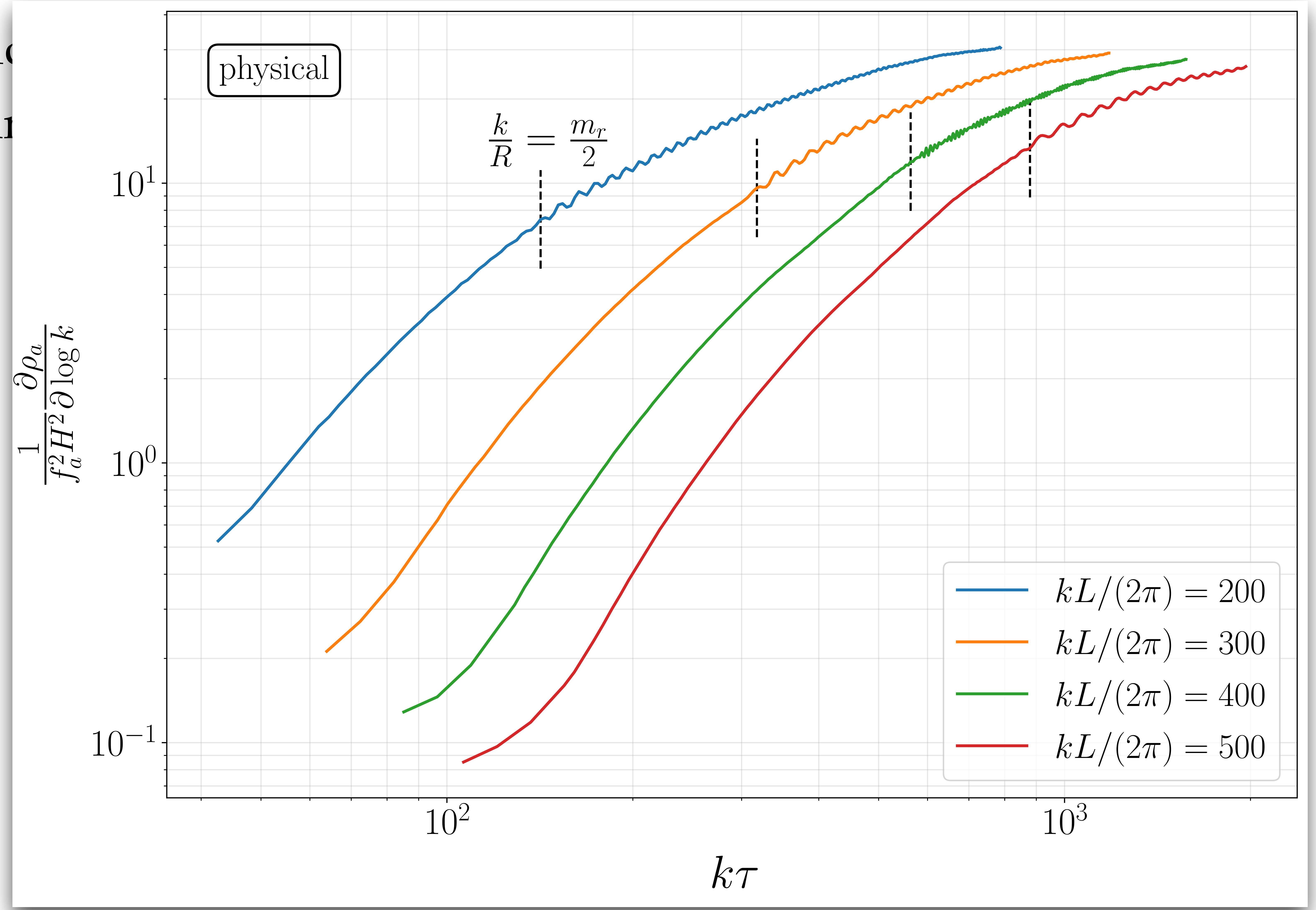
- There are several systematic
  that could explain discrepar
  the literature:

  - Initial conditions
  - Axion field oscillations
  - Discretisation effects



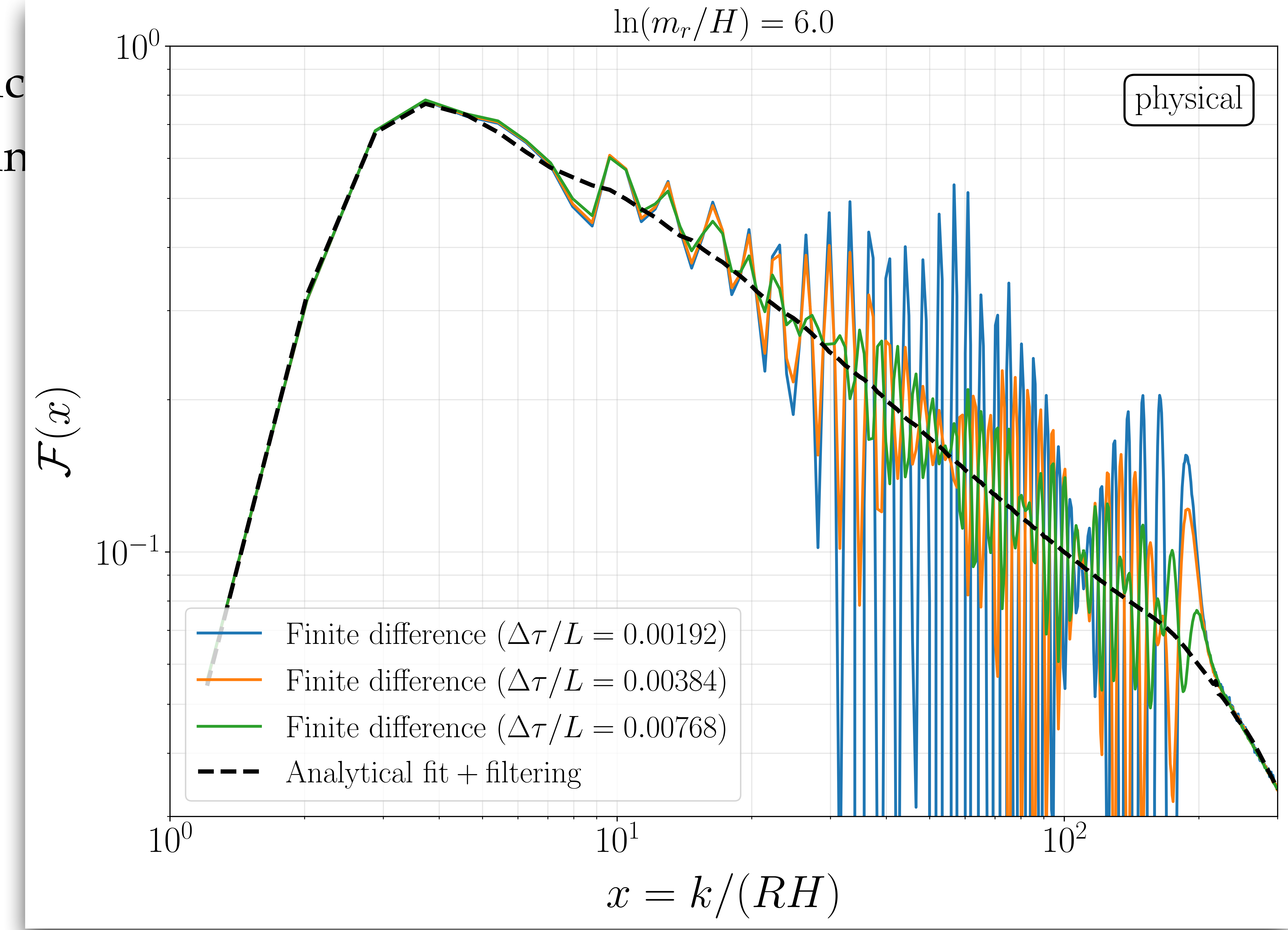
What can bias the Results?

- There are several systemati[c]
  that could explain discrepa[n]
  the literature:
  - Initial conditions
  - Axion field oscillations
  - Discretisation effects



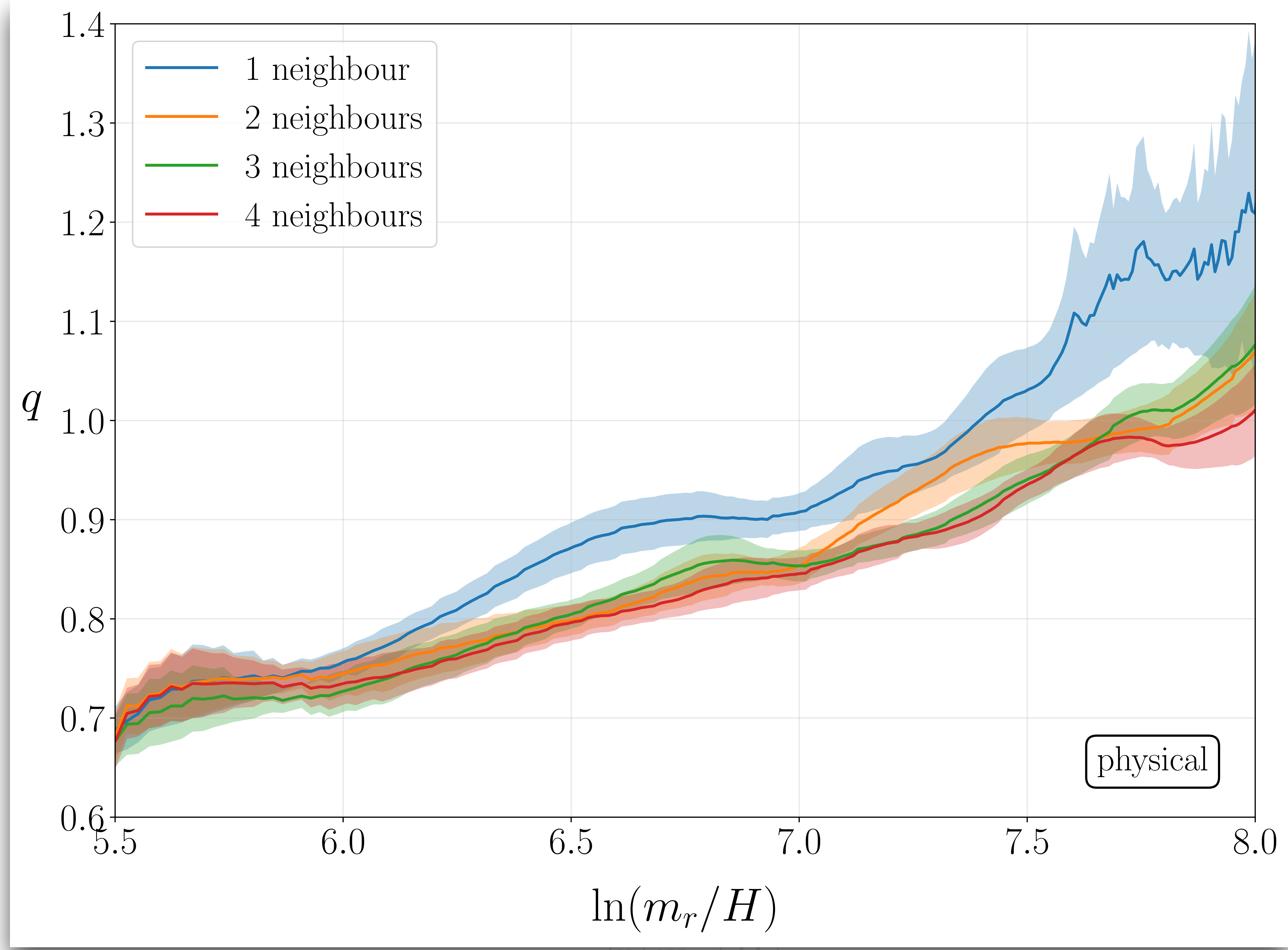
What can bias the Results?

- There are several systematic that could explain discrepan the literature:
  - Initial conditions
  - Axion field oscillations
  - Discretisation effects



What can bias the Results?

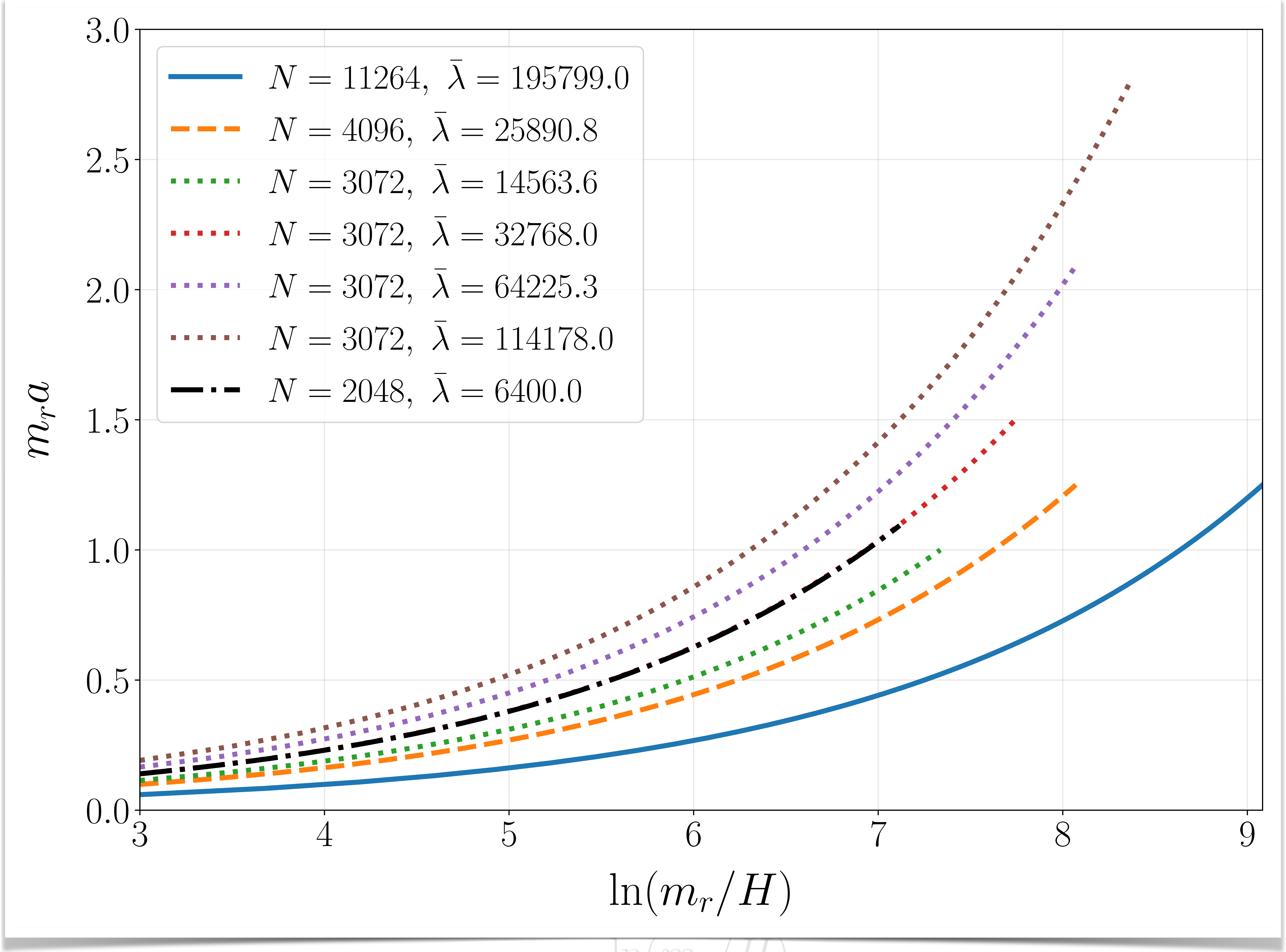
- There are several systematic effects, that could explain discrepancies in the literature:
  - Initial conditions
  - Axion field oscillations
  - Discretisation effects
    - **Laplacian**
    - Resolution of the string core $m_r a$



Saikawa, Redondo, Vaquero, MK [2401.17253]

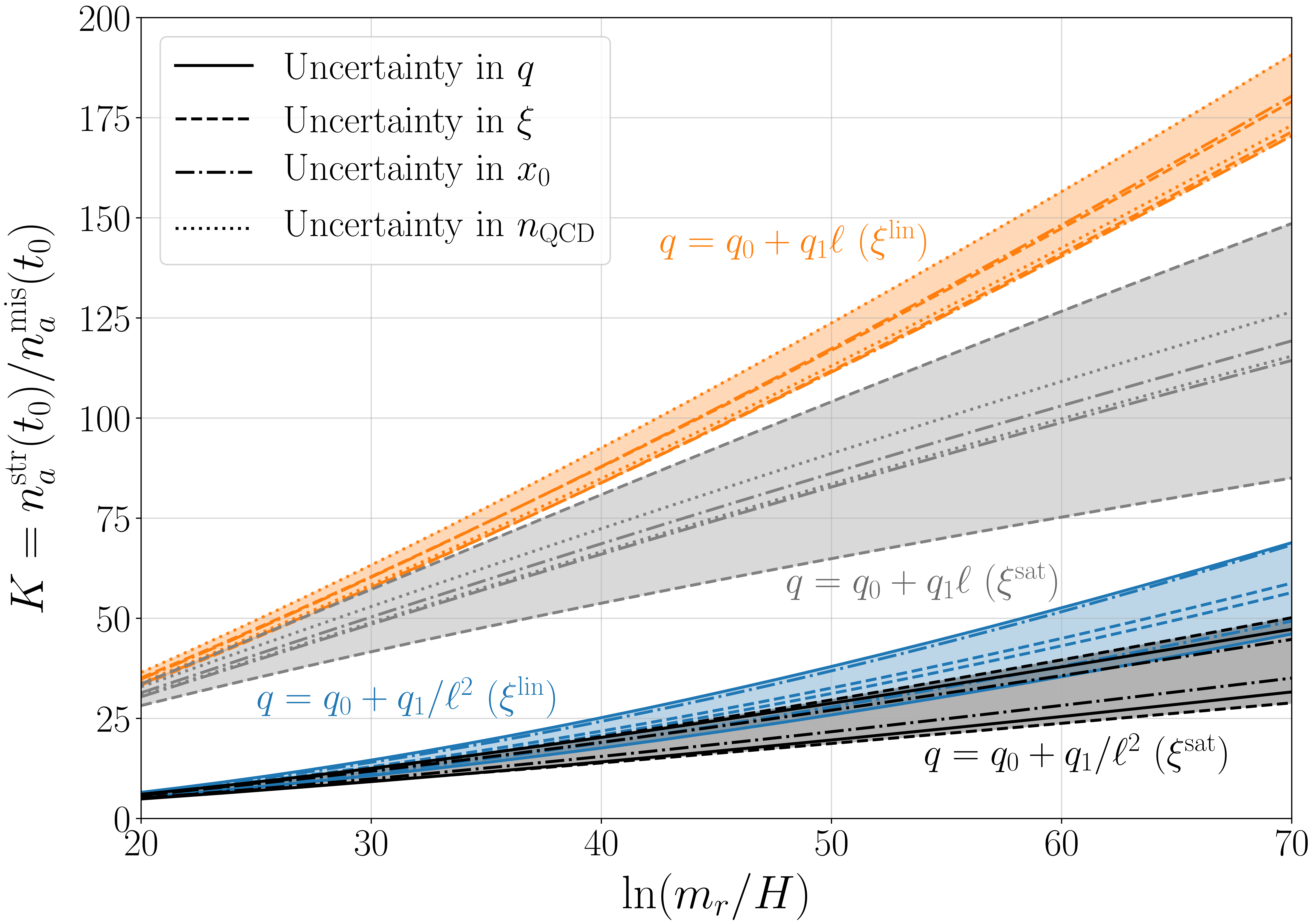
What can bias the Results?

- There are several systematic effects, that could explain discrepancies in the literature:
  - Initial conditions
  - Axion field oscillations
  - Discretisation effects
    - Laplacian
    - **Resolution of the string core $m_r a$**



Saikawa, Redondo, Vaquero, MK [2401.17253]

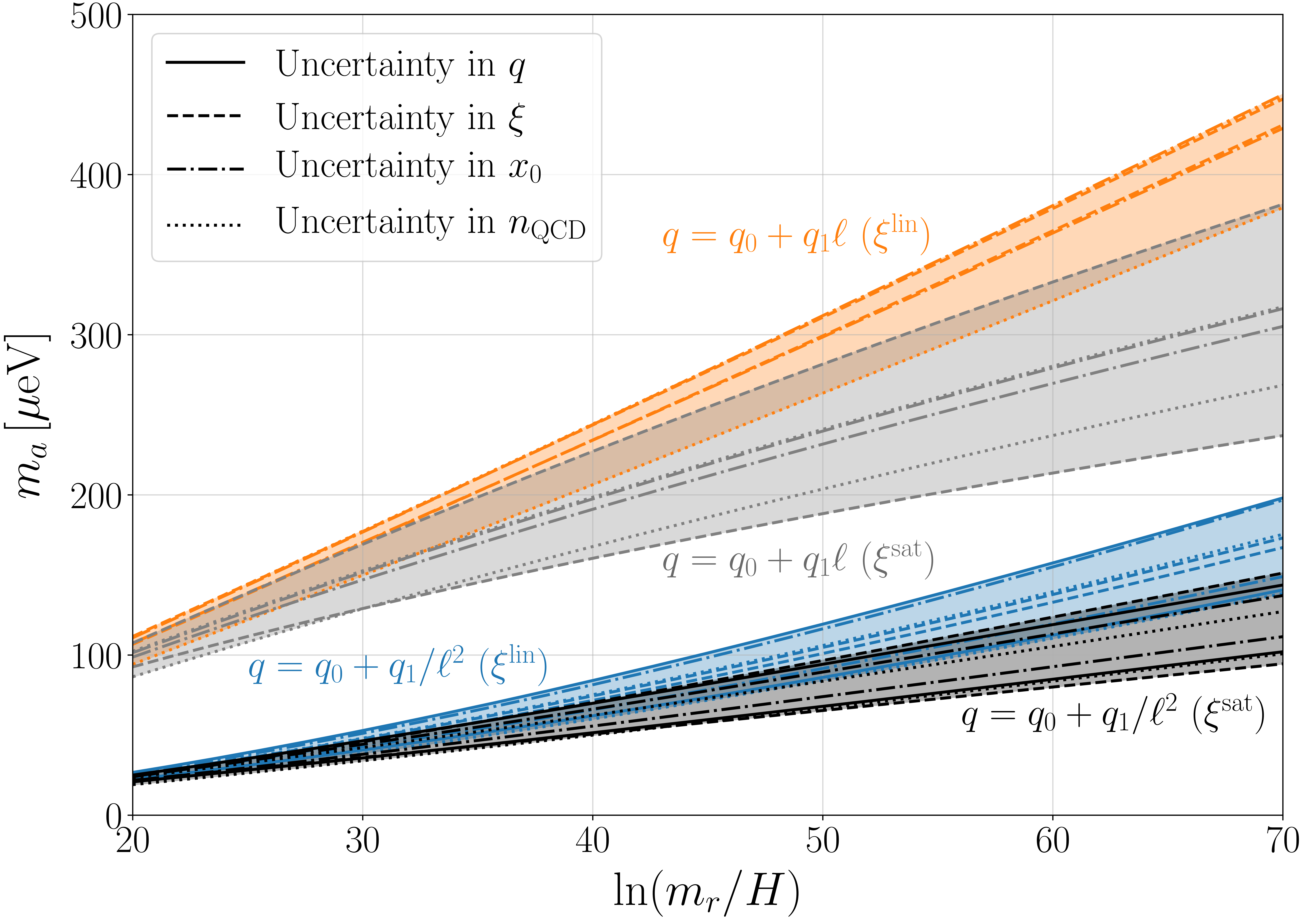
Axions from Strings vs. Misalignment



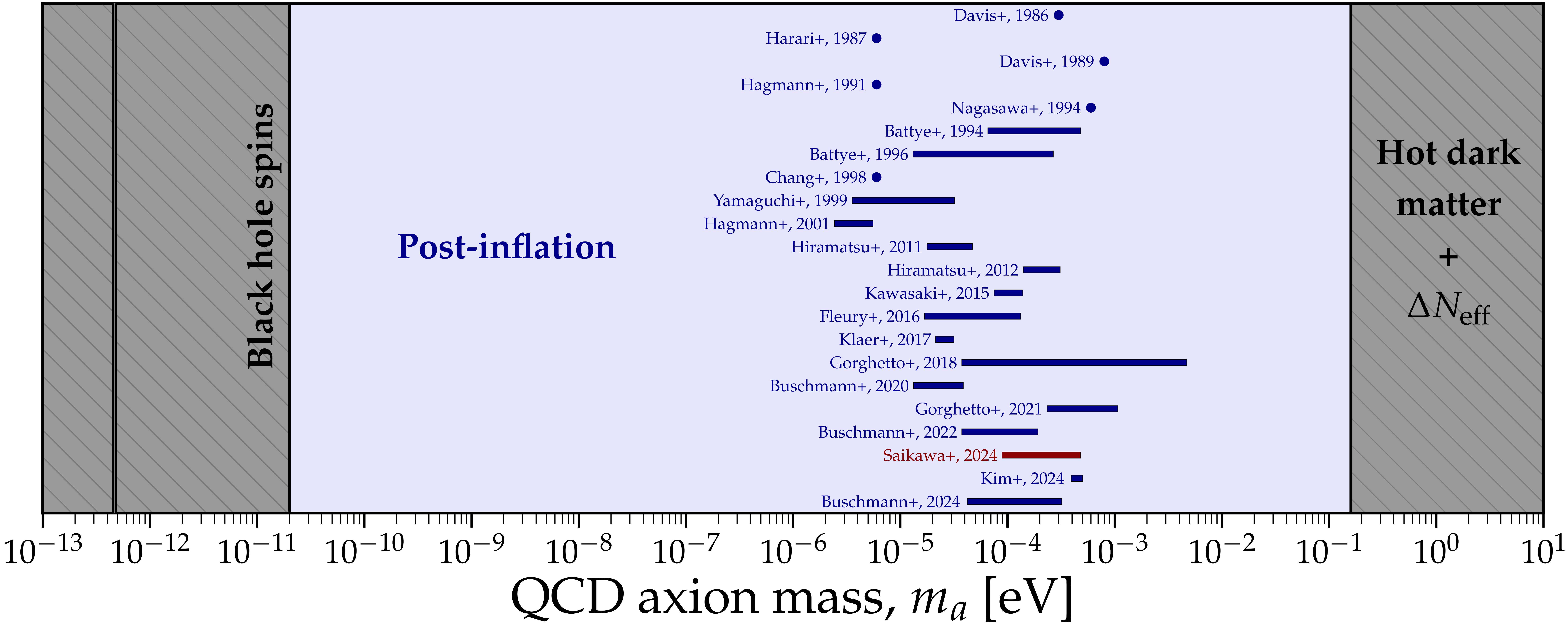
Known for "standard" angle-averaged misalignment:

$$\Omega_a h^2 = K \Omega_a^{\mathrm{mis}} h^2$$

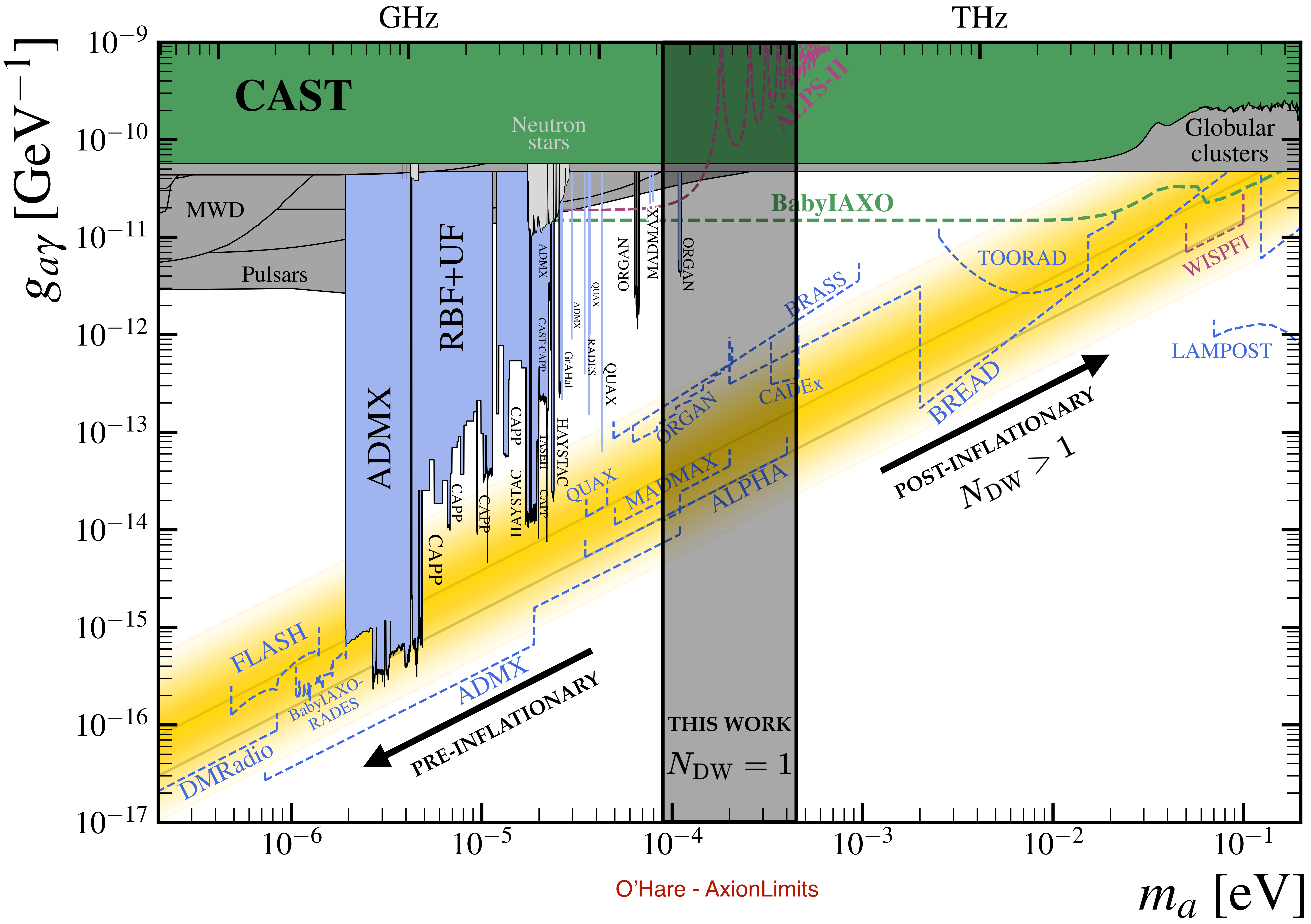
Axion Dark Matter Mass Prediction



Axion DM Mass Predictions from String Simulations

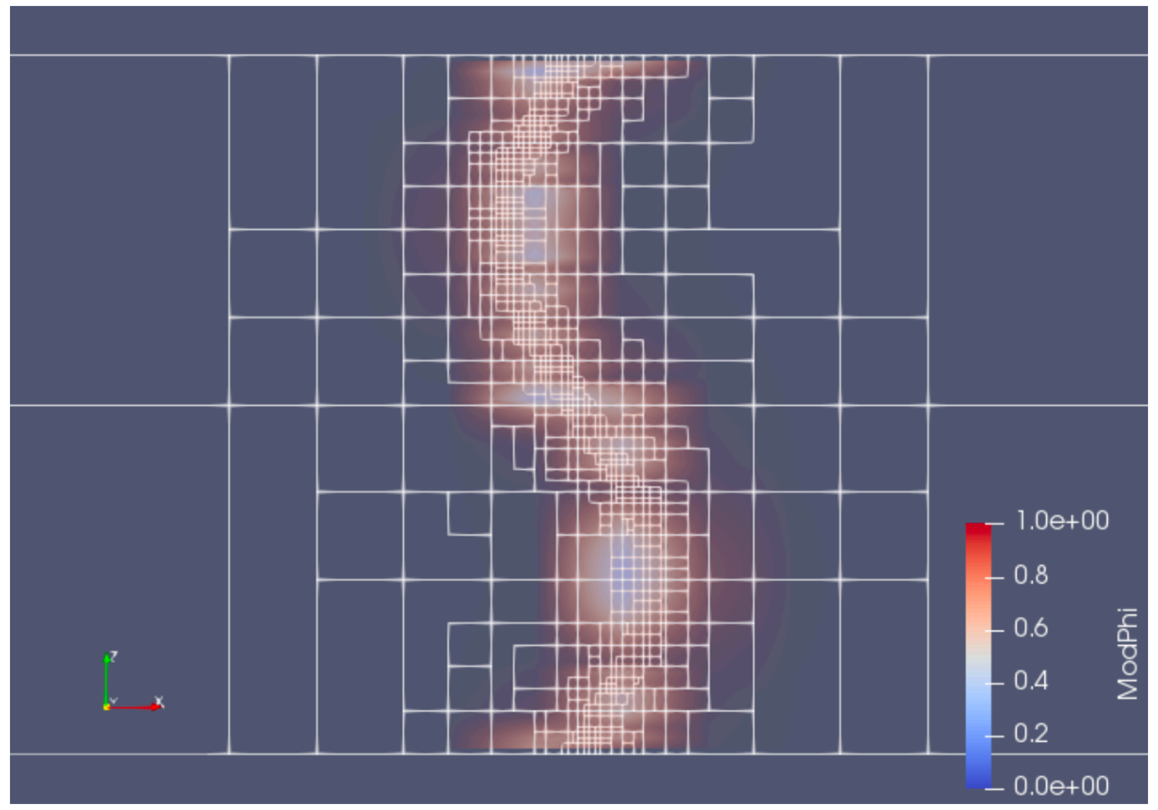


Buschmann, MK+ [to appear]

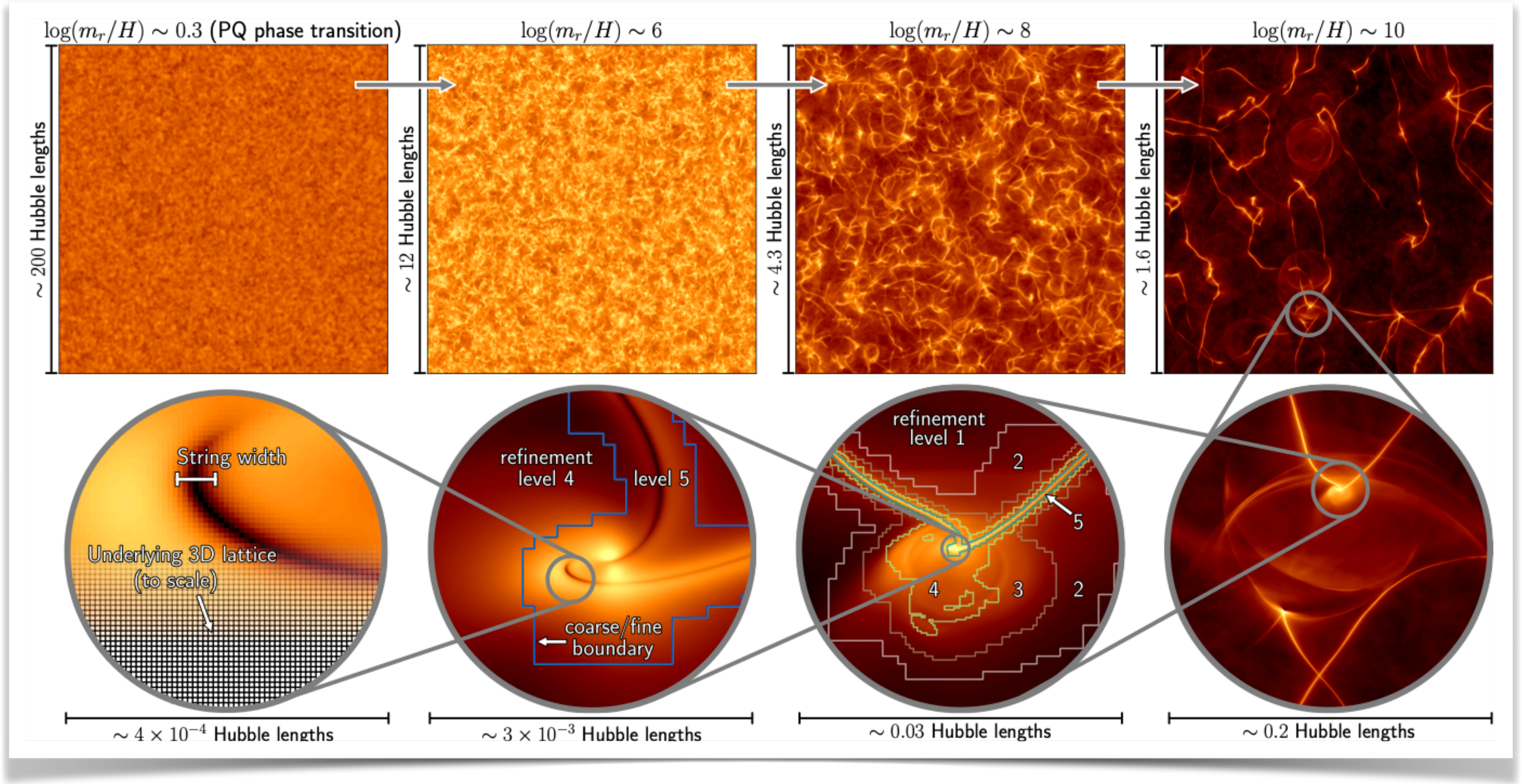

GHz
THz
CAST
Neutron stars
ALPS-II
Globular clusters
MWD
Pulsars
BabyIAXO
TOORAD
WISPFI
BRASS
LAMPOST
ADMX
RBF+UF
CAPP
HAYSTAC
ORGAN
MADMAX
QUAX
RADES
GrAHal
CAST-CAPP
TASEH
CADEx
BREAD
ALPHA
FLASH
BabyIAXO-RADES
DMRadio
POST-INFLATIONARY
$N_{DW} > 1$
PRE-INFLATIONARY
THIS WORK
$N_{DW} = 1$
$g_{a\gamma}$ [GeV^{-1}]
m_a [eV]
O'Hare - AxionLimits

Outlook: Adaptive Mesh Refinement (AMR)

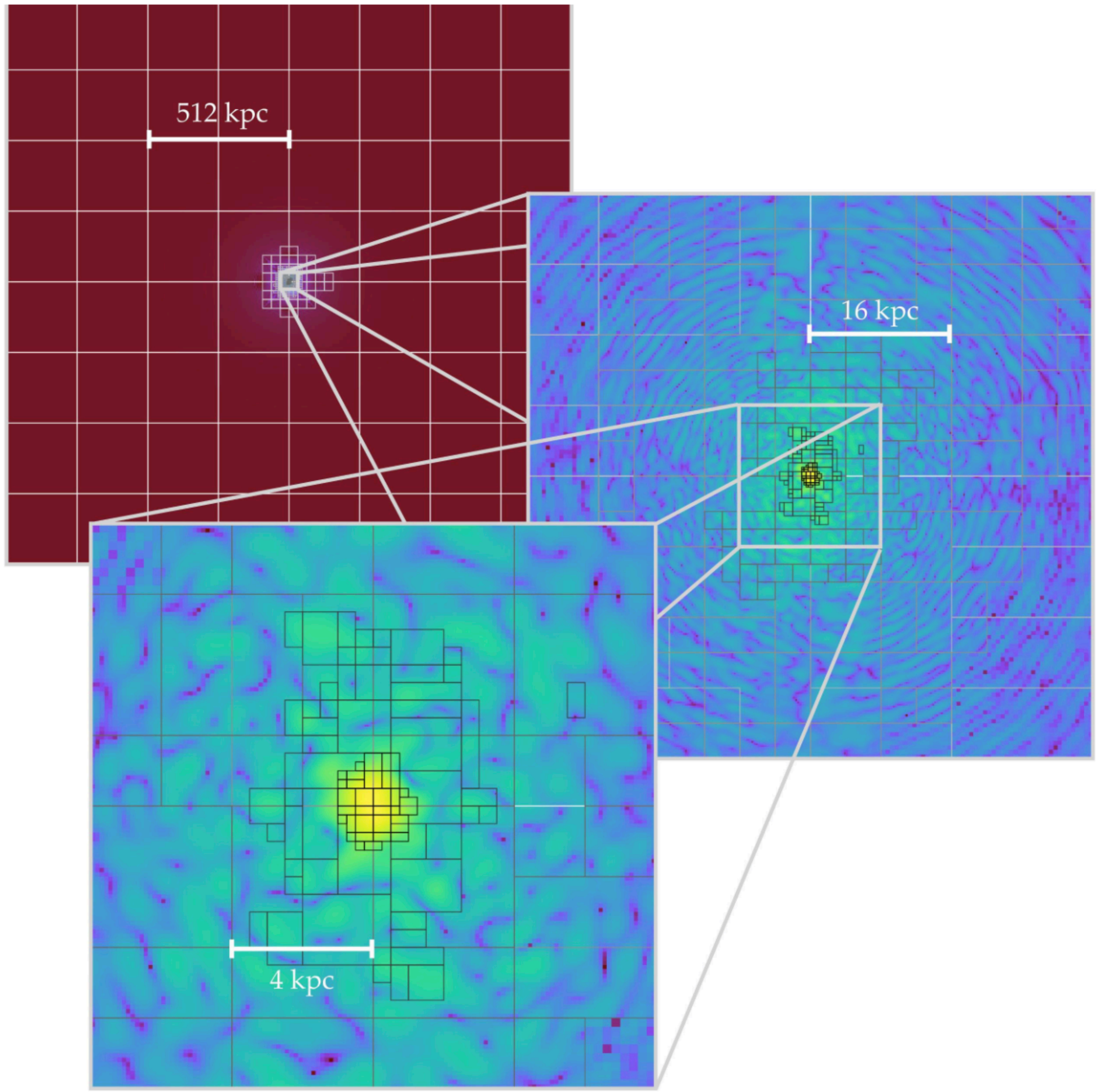
- **Idea:** Focus computational power on specific parts of the grid
- Nowadays widely used in cosmological simulation codes, numerical relativity **and** in axion string simulations
- Current codes mostly based on AMReX



Drew & Shellard [1910.01718]
"GRChombo"



Buschmann+ [2412.08699]
"sledgehamr"

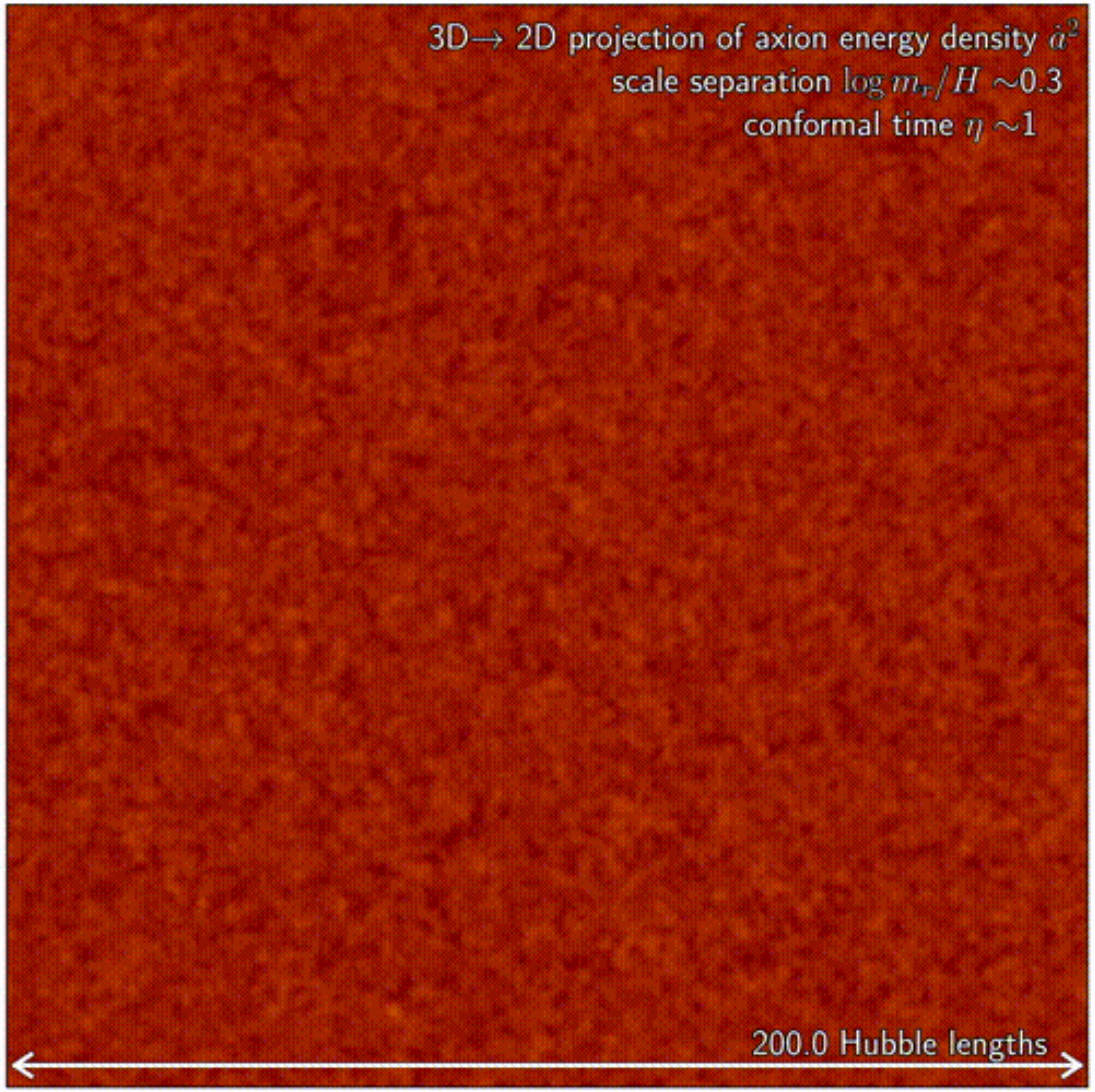


Schwabe+ [2007.08256]
"axioNyx"

Outlook: Adaptive Mesh Refinement (AMR)

- **Idea:** Focus co… the grid
- Nowadays wid… des, numerical relat…
- Current codes r…

Drew & Shellard [1910.01718]
"GRChombo"



"sledgehamr"

Buschmann [2404.02950]

Schwabe+ [2007.08256]
"axioNyx"

Summary

- Understanding of global string dynamics is very important for a precise prediction of the axion dark matter mass in the post-inflationary scenario.
- Our simulations predict $95\mu\text{eV} \lesssim m_a \lesssim 450\mu\text{eV}$.
- Fast developments in recent simulations allow us to have a better understanding, albeit serious discrepancies, this work identifies some of the major problems in the interpretation of results.
- There are several systematic effects that could bias the result, that could explain these discrepancies:
    - Initial conditions
    - Axion field oscillations
    - Discretisation effects
- Further improvement in the dynamical range would be helpful to make the extrapolation trustworthy, can be achieved for example with AMR.

Summary

- Understanding of the global string dynamics is very important for a precise prediction of the axion dark matter mass in the post-inflationary scenario.
- Our simulations predict $95\mu\mathrm{eV} \lesssim m_a \lesssim 450\mu\mathrm{eV}$.
- Fast developments in recent simulations allow us to have a better understanding, albeit serious discrepancies.
- There are several systematic effects that could bias the result, that could explain these discrepancies:
  - Initial conditions
  - Axion field oscillations
  - Discretisation effects
- Further improvement in the dynamical range would be helpful to make the extrapolation trustworthy, can be achieved for example with AMR.

¡Gracias!

Any Questions?

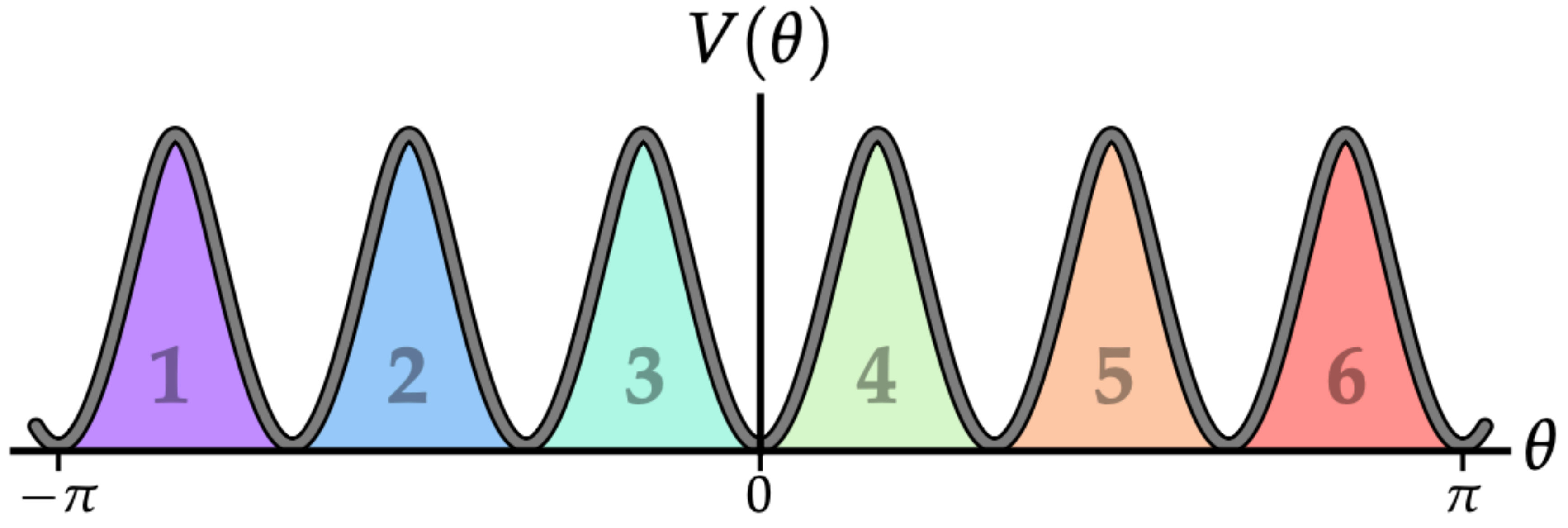
Backup Slides

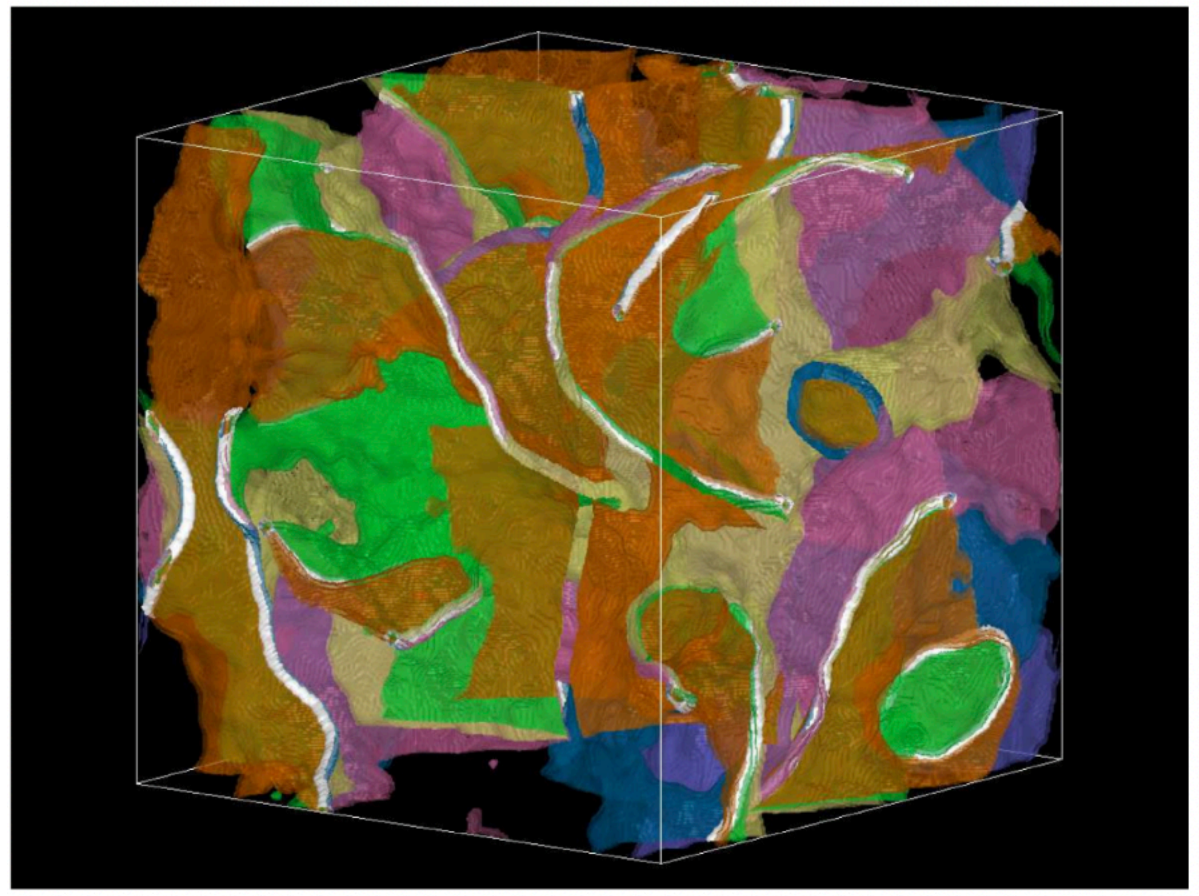
General

$N_{DW} > 1$: Axion Domain Wall Problem

- Axion cycles around N_{DW} times between $(-\pi, \pi)$
- In general we get more axions from wall decay, so preferred m_a is higher.
- Phenomenologically difficult. Domain wall network gets stuck and overwhelms the cosmic energy density.
- Must have some preferred minimum!

$$V(\theta) \approx -\chi(T)\cos(N_{DW}\theta)$$





(e) $N_{DW} = 6$ Hiramatsu+ [1207.3166]

More details on the recent Paper

Simulation Overview

- More than 1500 simulations performed at
  - RAVEN and COBRA supercomputers at Max Planck Computing and Data Facility (MPCDF)
  - SQUID supercomputer at Cybermedia Center, Osaka University
- Box sizes of up to 11.264^3 (256 CPU nodes)

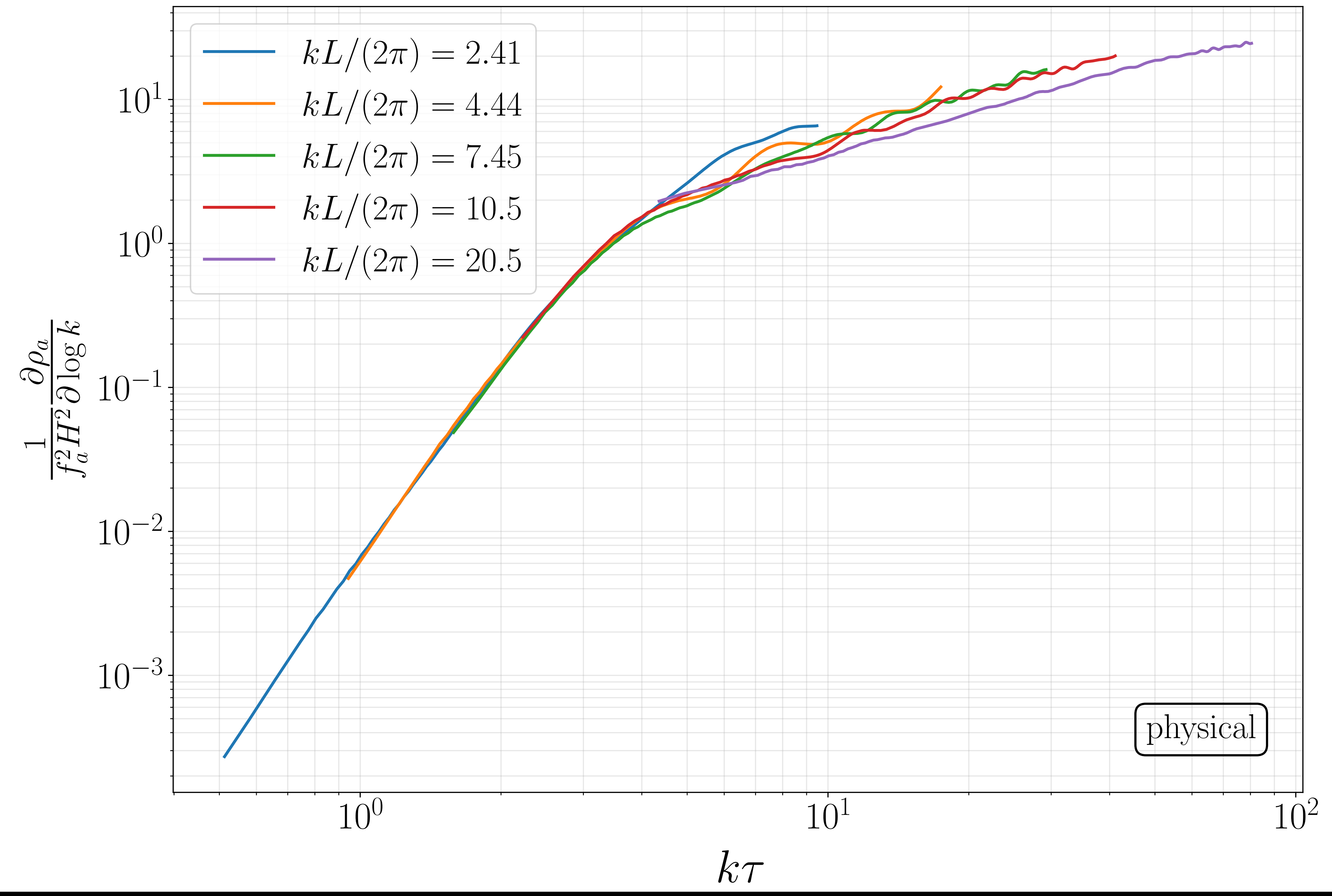
Type[a]	Grid size (N^3)	Laplacian	Final time (τ_f/L)	$\ln(m_r/H)$ at τ_f	Parameter	Number of simulations
Physical	11264^3	4-neighbours	0.625	9.08	$\bar{\lambda} = 195799$	20
Physical	4096^3	1-neighbour	0.625	8.07	$\bar{\lambda} = 25890.8$	30
Physical	4096^3	2-neighbours	0.625	8.07	$\bar{\lambda} = 25890.8$	30
Physical	4096^3	3-neighbours	0.625	8.07	$\bar{\lambda} = 25890.8$	30
Physical	4096^3	4-neighbours	0.625	8.07	$\bar{\lambda} = 25890.8$	30
Physical	3072^3	4-neighbours	0.5	7.34	$\bar{\lambda} = 14563.6$	30
Physical	3072^3	4-neighbours	0.5	7.74	$\bar{\lambda} = 32768$	30
Physical	3072^3	4-neighbours	0.5	8.08	$\bar{\lambda} = 64225.3$	30
Physical	3072^3	4-neighbours	0.5	8.37	$\bar{\lambda} = 114178$	30
Physical	2048^3	4-neighbours	0.55	7.12	$\bar{\lambda} = 6400$	30×30[b]
Physical	1024^3	4-neighbours	0.5	6.23	$\bar{\lambda} = 1600$	30×4[c]
Physical	3072^3	4-neighbours	0.458367	7.5	$\bar{\lambda} = 28571.2$	30
Physical	2560^3	4-neighbours	0.550042	7.5	$\bar{\lambda} = 13778.5$	30
Physical	2048^3	4-neighbours	0.687552	7.5	$\bar{\lambda} = 5643.68$	30
Physical	1536^3	4-neighbours	0.916735	7.5	$\bar{\lambda} = 1785.69$	30
Physical	1024^3	4-neighbours	1.3751	7.5	$\bar{\lambda} = 352.73$	30
PRS	8192^3	4-neighbours	0.55	6.80	$m_r a = 0.2$	20
PRS	8192^3	4-neighbours	0.55	7.21	$m_r a = 0.3$	20
PRS	8192^3	4-neighbours	0.55	7.72	$m_r a = 0.5$	20
PRS	8192^3	4-neighbours	0.55	8.06	$m_r a = 0.7$	20
PRS	8192^3	4-neighbours	0.55	8.41	$m_r a = 1.0$	20
PRS	8192^3	4-neighbours	0.55	8.82	$m_r a = 1.5$	20
PRS	4096^3	4-neighbours	0.55	7.72	$m_r a = 1.0$	30
PRS	2048^3	4-neighbours	0.55	7.03	$m_r a = 1.0$	30
PRS	1024^3	4-neighbours	0.55	6.33	$m_r a = 1.0$	30
PRS	2048^3	4-neighbours	0.5	6.93	$m_r a = 1.0$	1

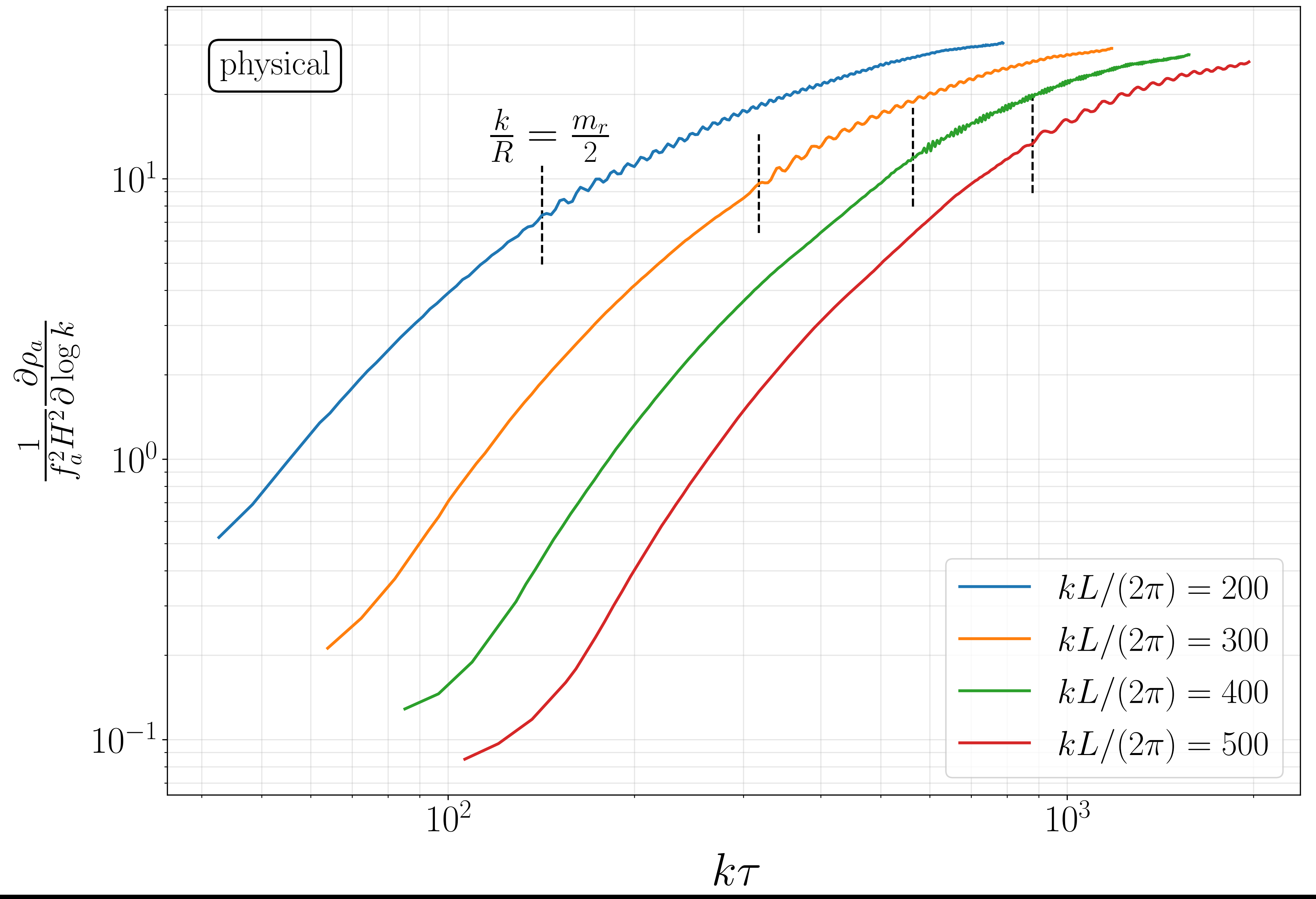
Axion Mode Evolution

- To calculate the differential spectrum, we need to know the time evolution of one mode:

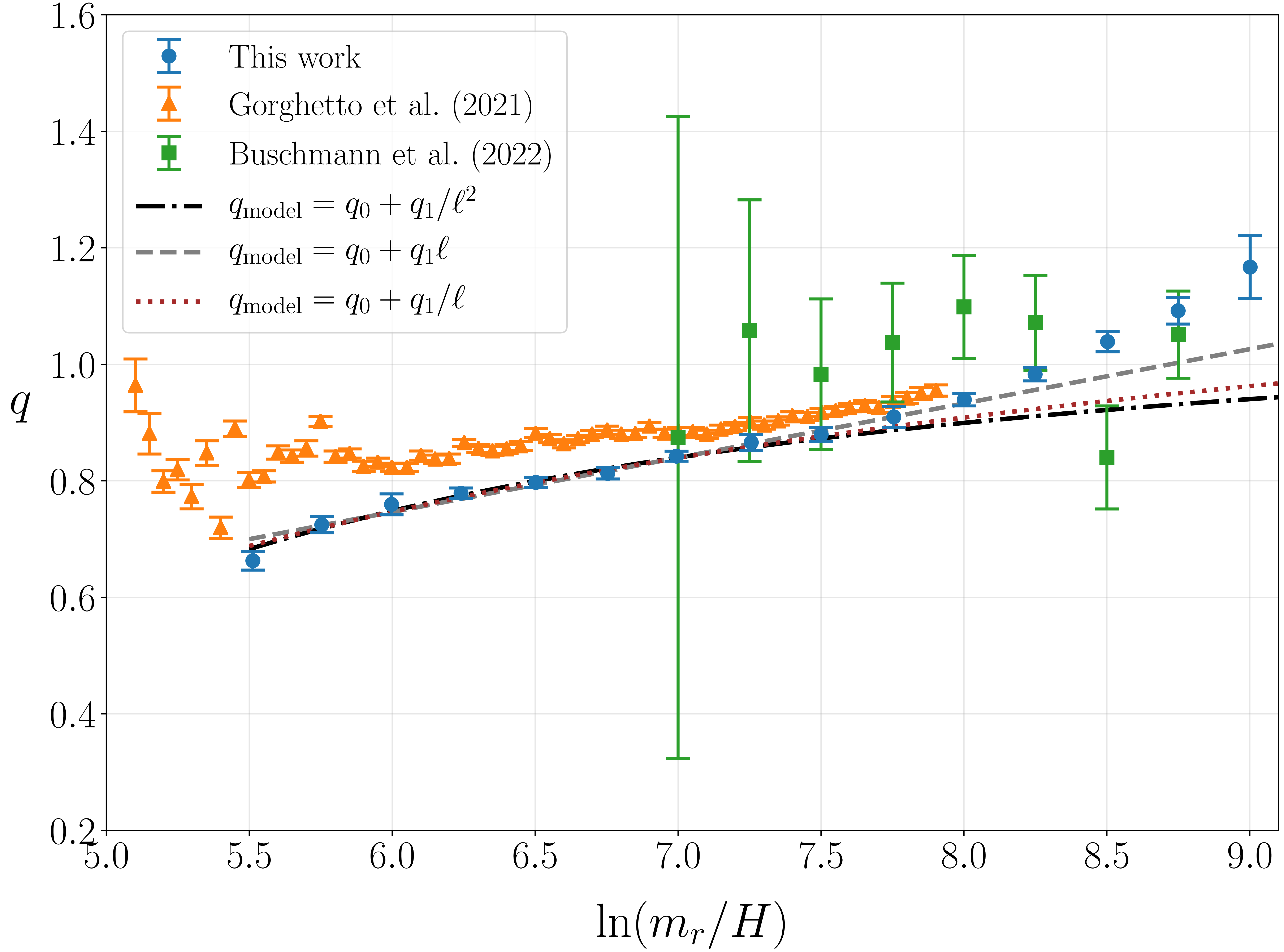
$$\mathcal{F} = \frac{1}{(f_a H)^2} \frac{1}{R^3} \frac{\partial}{\partial t} \left(R^4 \frac{\partial \rho_a}{\partial k} \right)$$

- Contains oscillating components with frequency $\sim 2k$, interpreted as axion field oscillations after the horizon entry or production from the radial field.

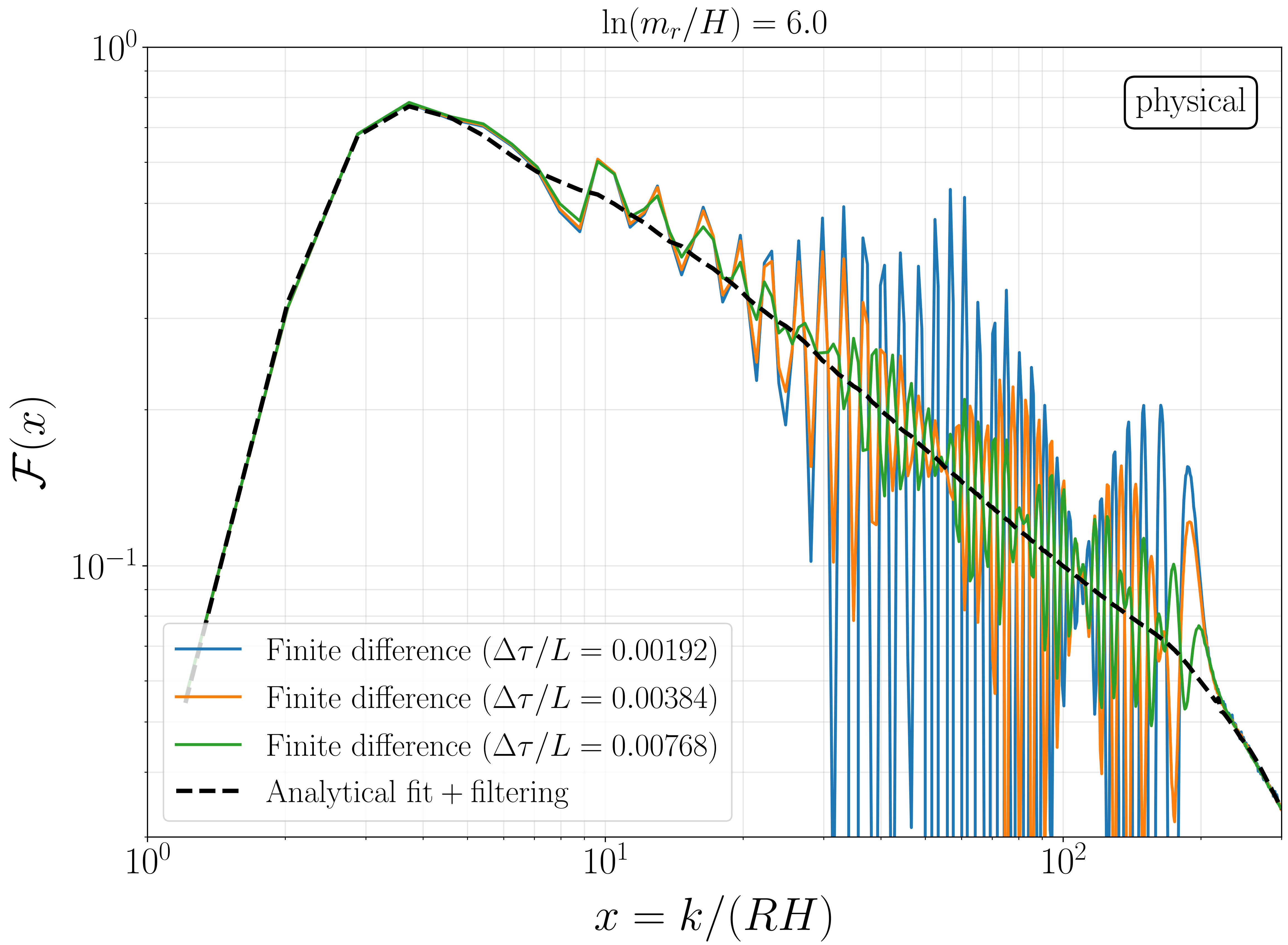




Comparison with recent Results



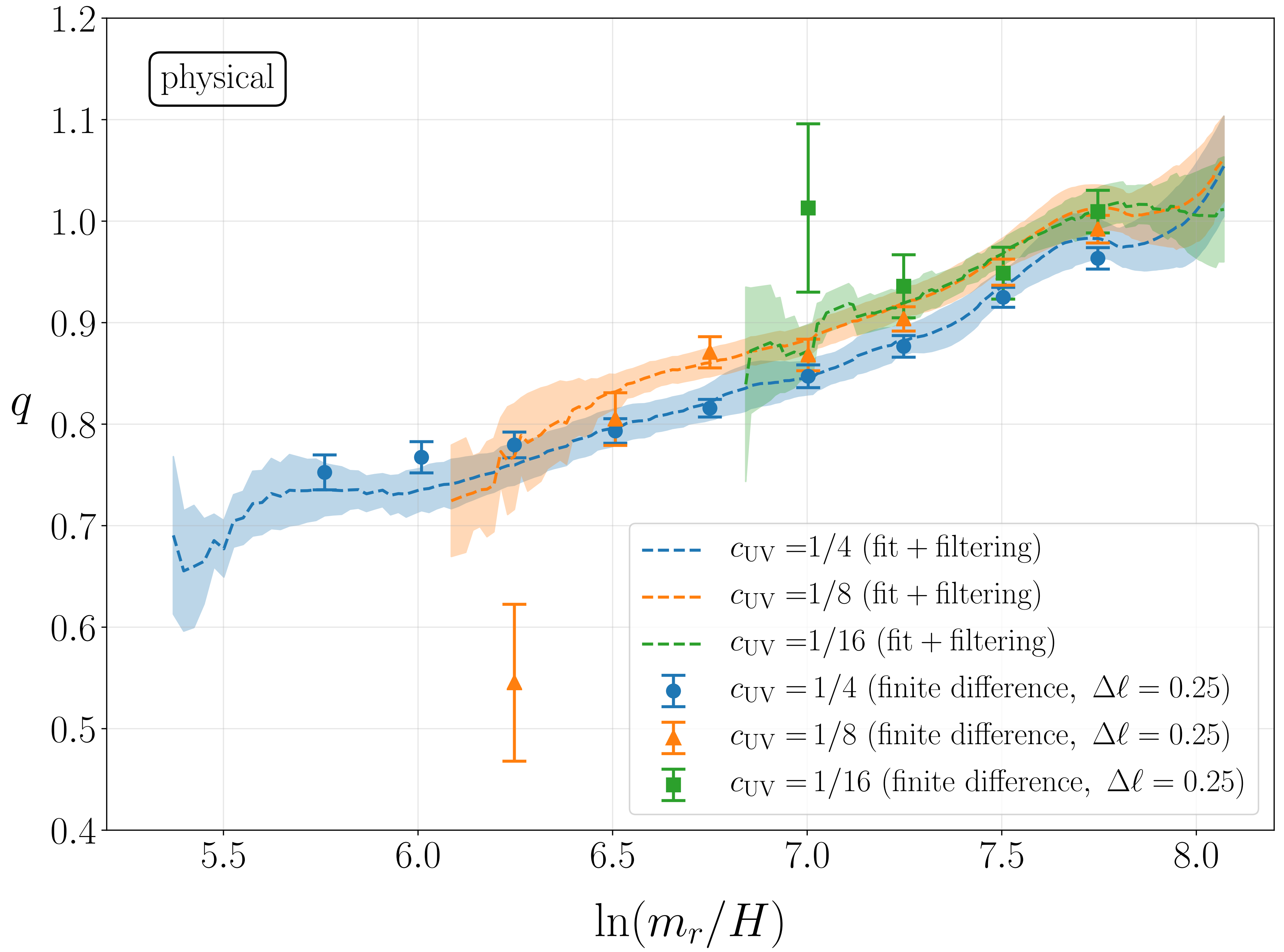
Calculation of the Instantaneous Spectrum



- Simple finite difference leads to a lot of contaminations from axion field oscillations.
- One can reduce them by applying a filter to remove high frequency components in the mode evolution data.

Axion Field Oscillations

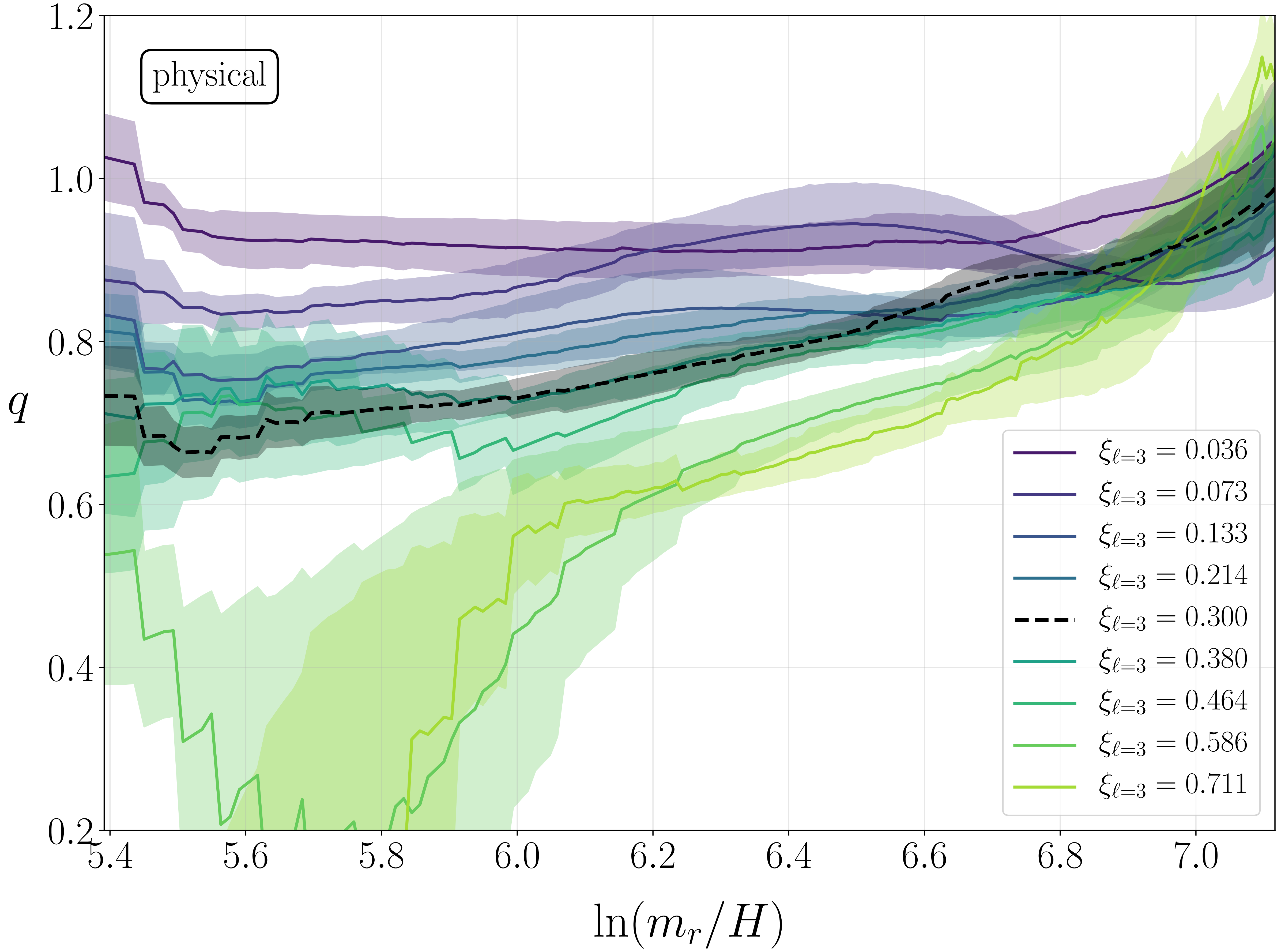
- Fit power law $\mathcal{F} \sim k^{-q}$ to the data in the range $c_{\rm IR} H < k/R < c_{\rm UV} m_r$



- The oscillations in the IR modes have an impact on the measurement of q.

- The effect can be alleviated by taking a broader range for the fit.

Initial Conditions

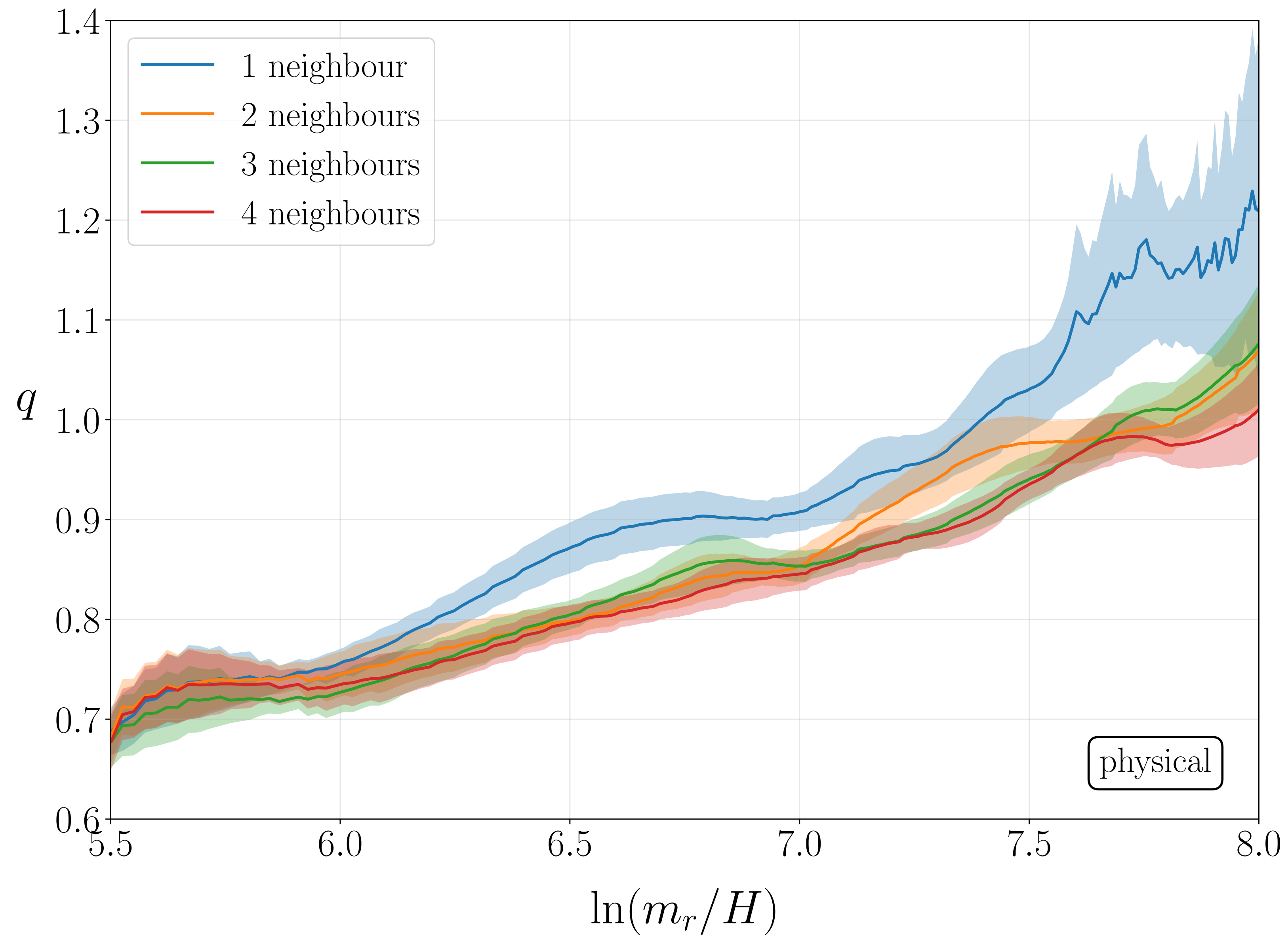


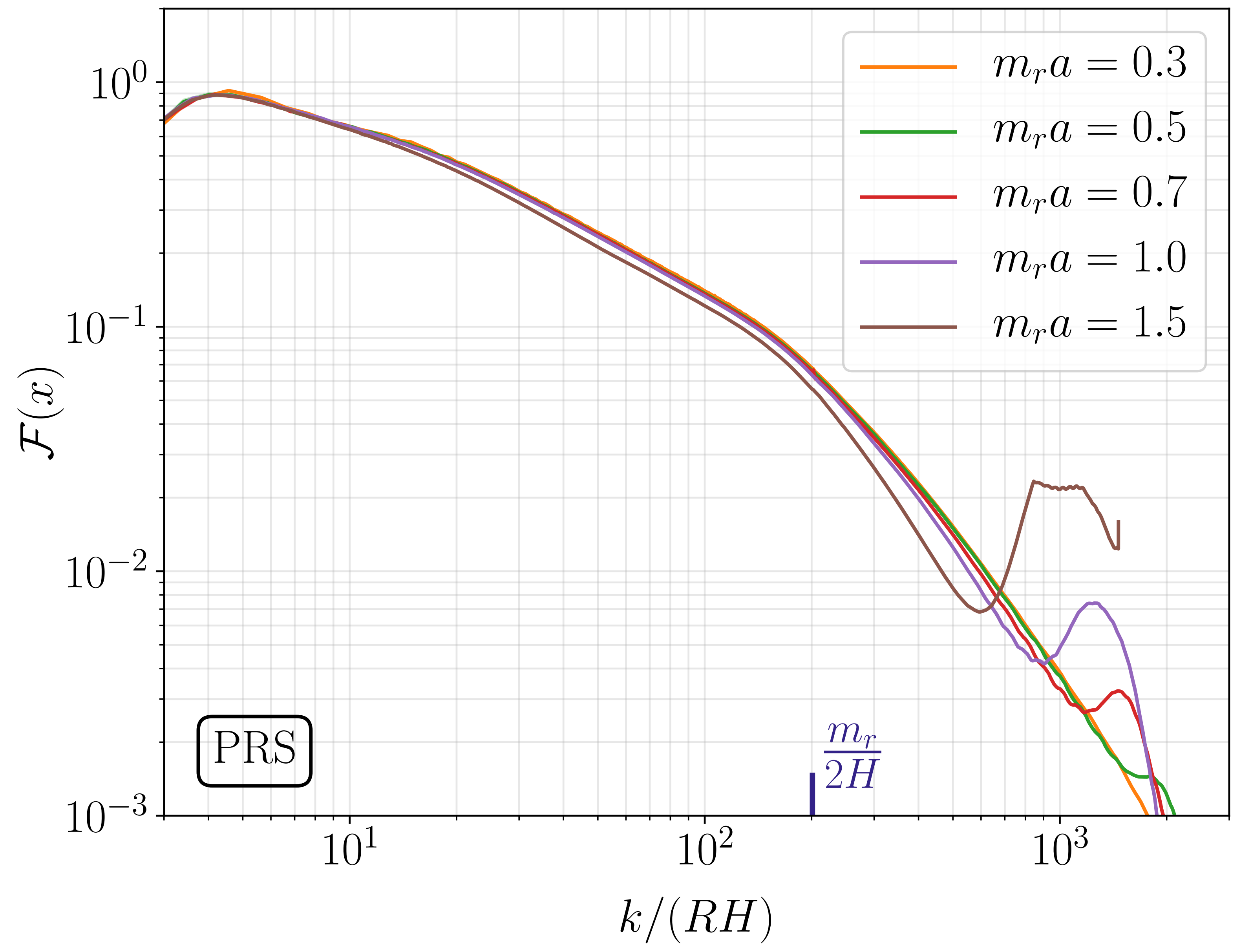
- Differences in the initial string density affect the slope of the radiation spectrum.
- Overdense (underdense) initial conditions could bias the estimation of q towards lower (higher) values.

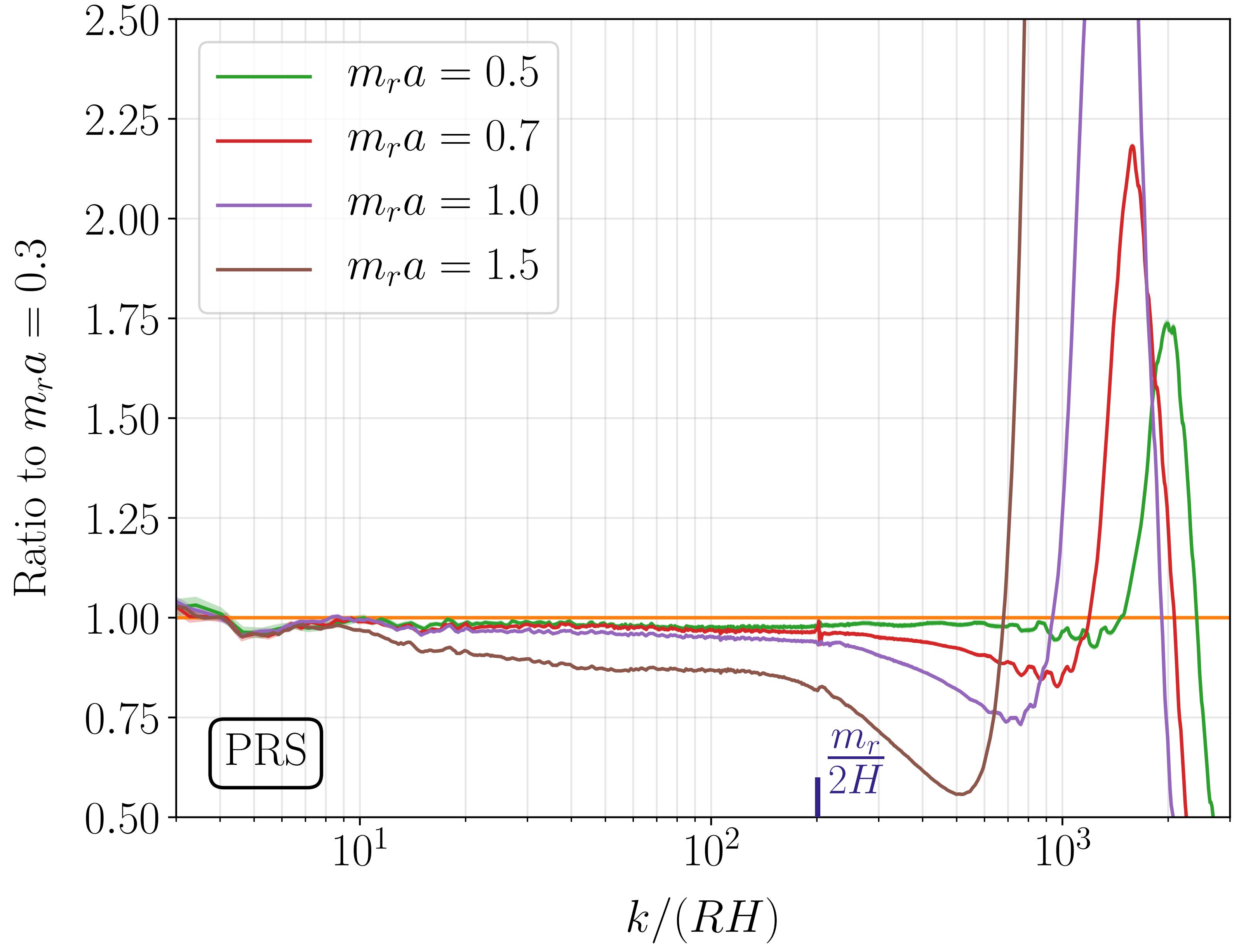
Discretisation Effects

Effects that can bias q towards larger values:

- Discretisation scheme of the Laplacian
- Resolution of the string core (parametrised by $m_r a$)



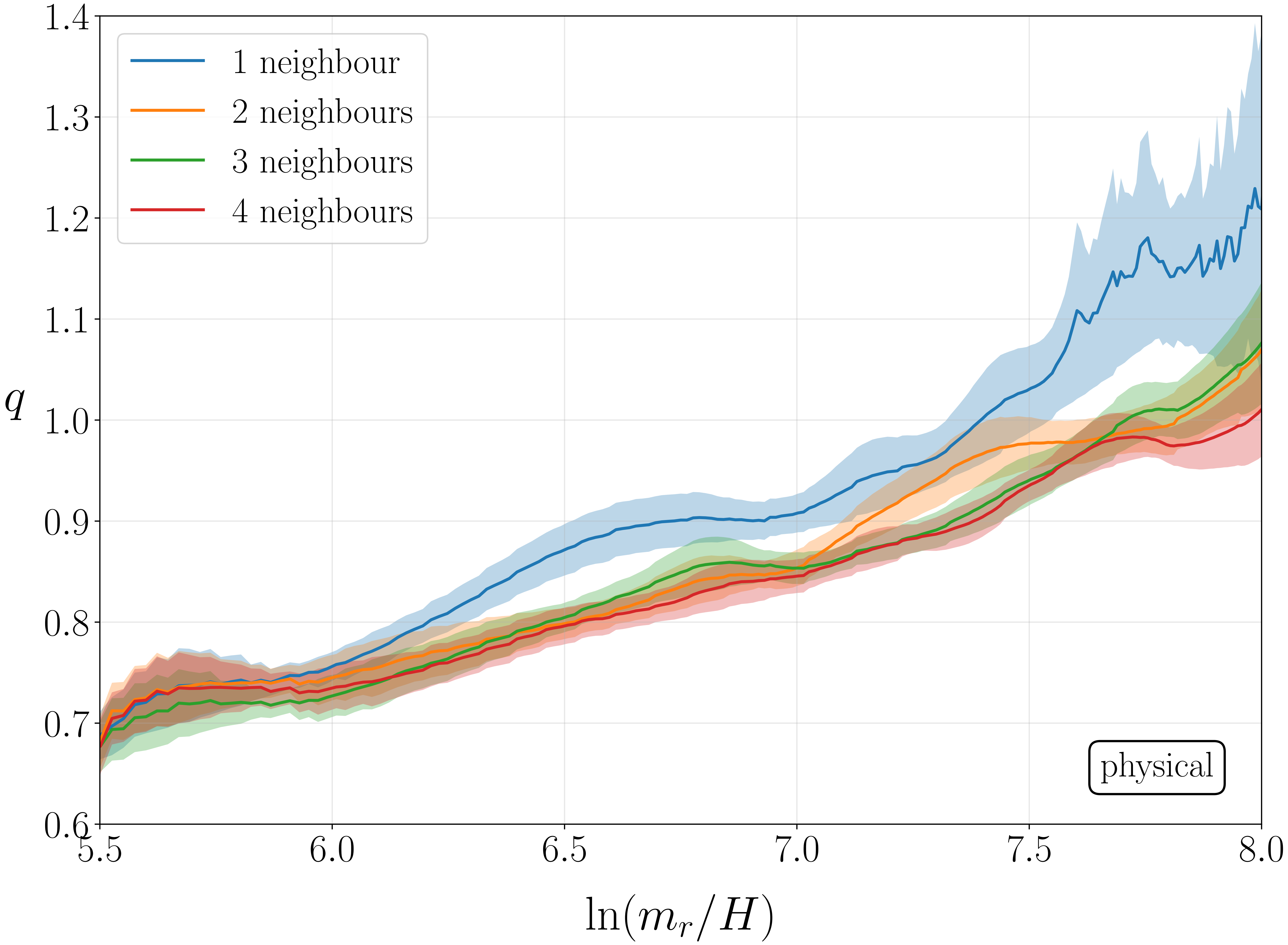




Discretisation Effects

Effects that can b

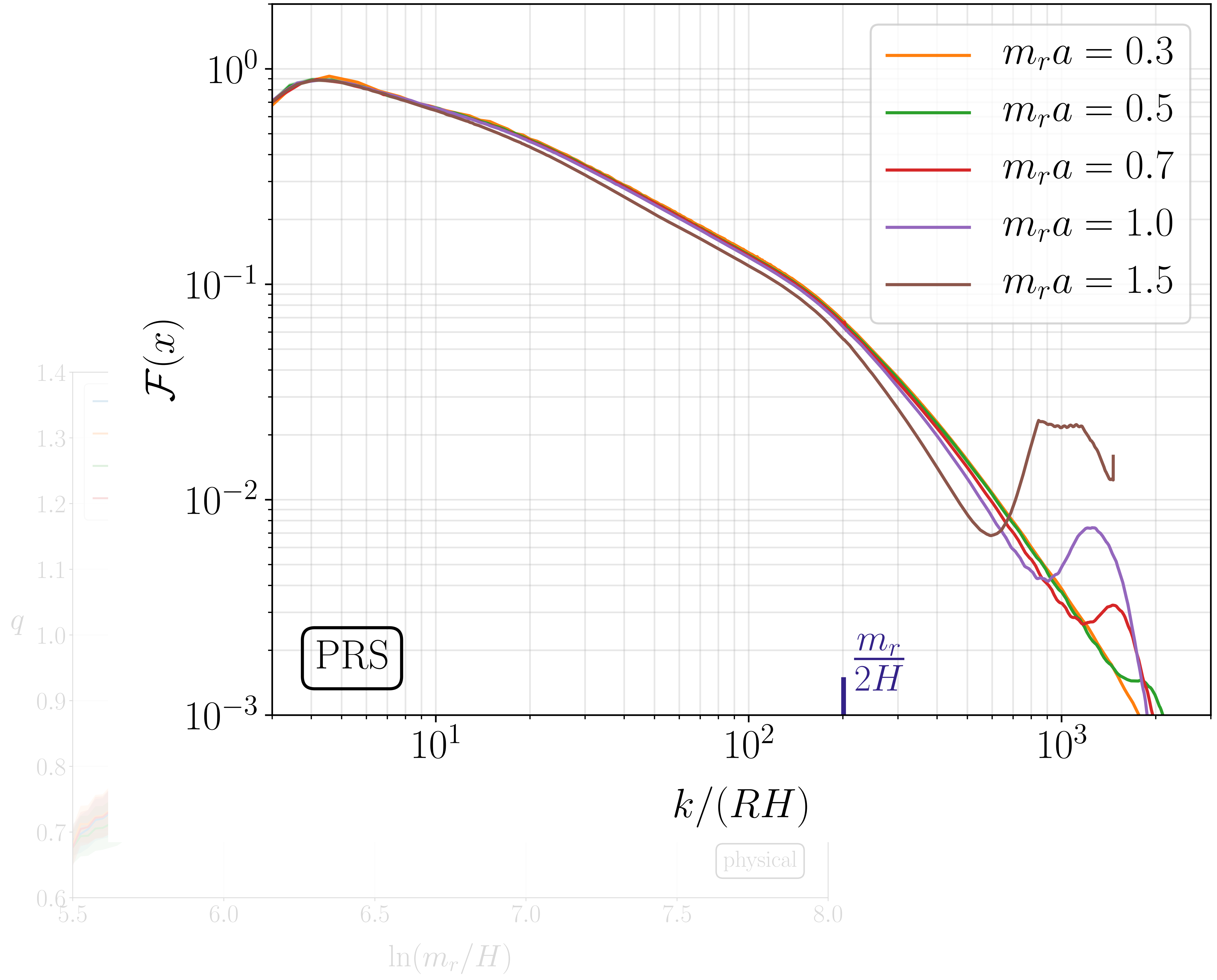
- Discretisati
- Resolution

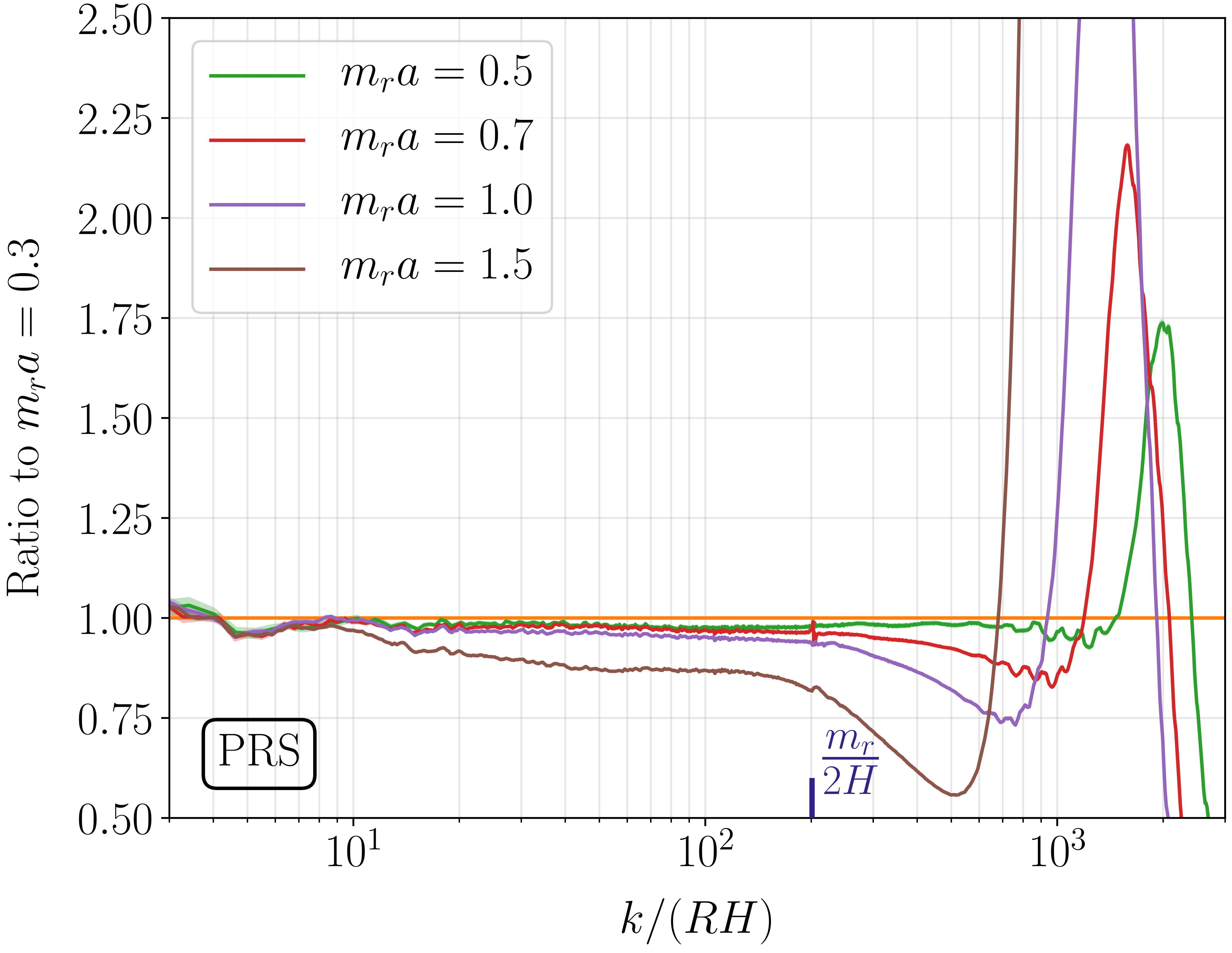


We observe that the value of q can be **overestimated** for smaller N_g!

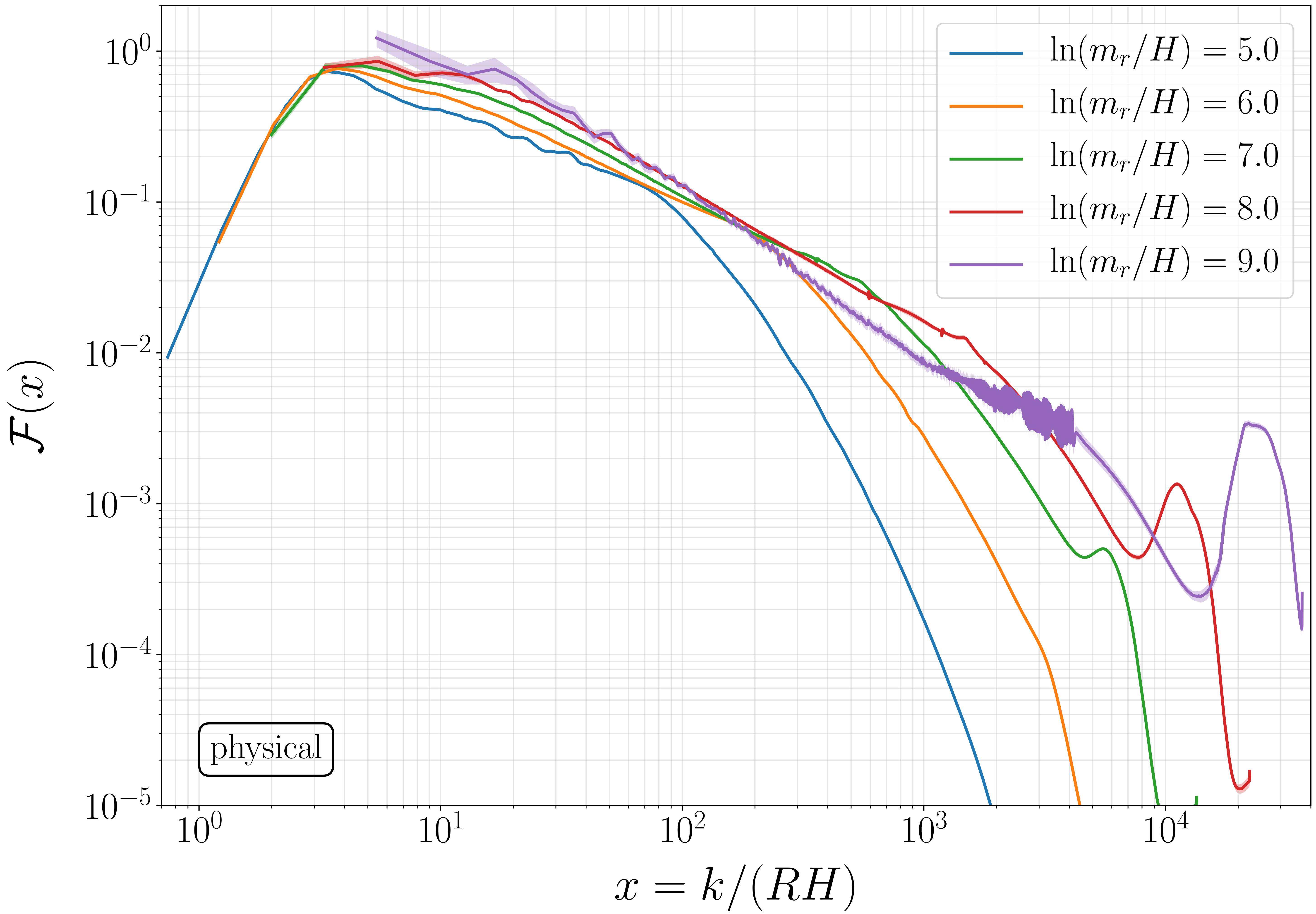
Discretisation Effects

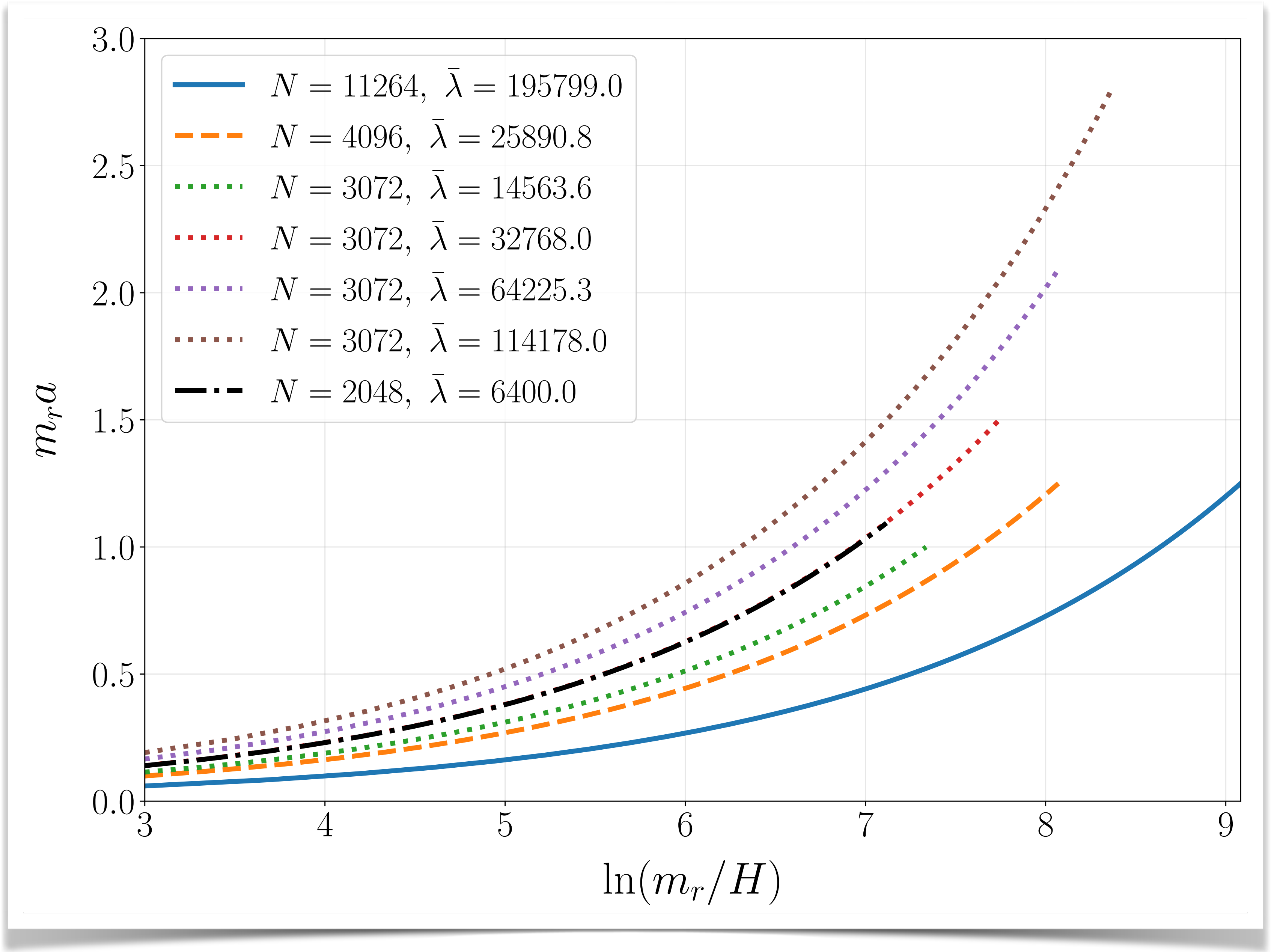
Effects that can bias q towards larger values:





Discretisation Effects



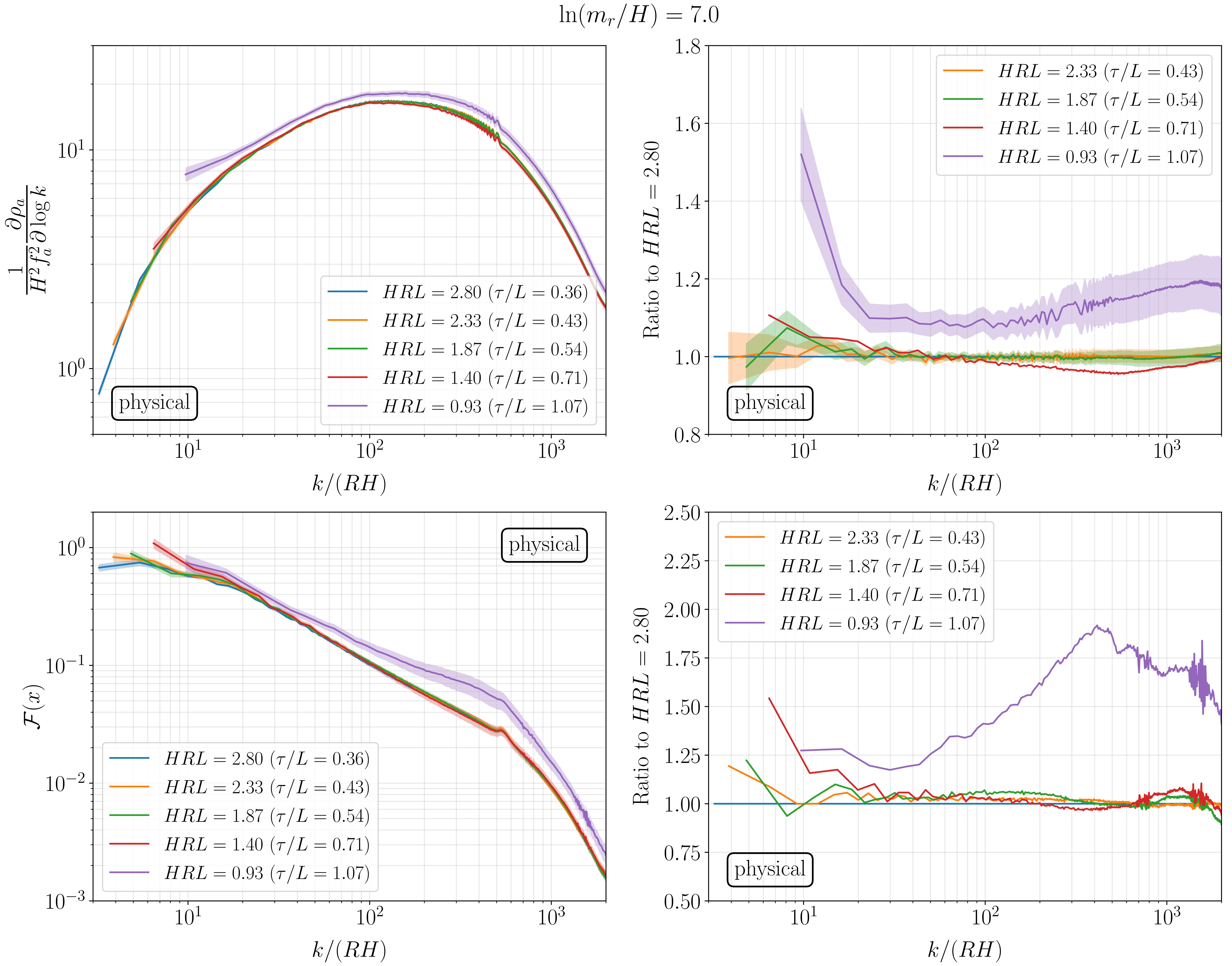


Effects increases drastically at larger $\log(m_r/H)$, leading to a significant distortion of the spectrum.

Finite Volume Effects

- Fix $m_r a = 1.0$ and vary ratio of phys. box size RL to Hubble radius H^{-1} at $\ln(m_r/H) = 7$
- Results converge for $HRL \gtrsim 1.4$ (or $\tau/L \lesssim 0.7$)
- We terminate the simulations at $\tau/L \leq 0.625$

Should **not** be a problem!



Technicalities

Masking the Spectrum

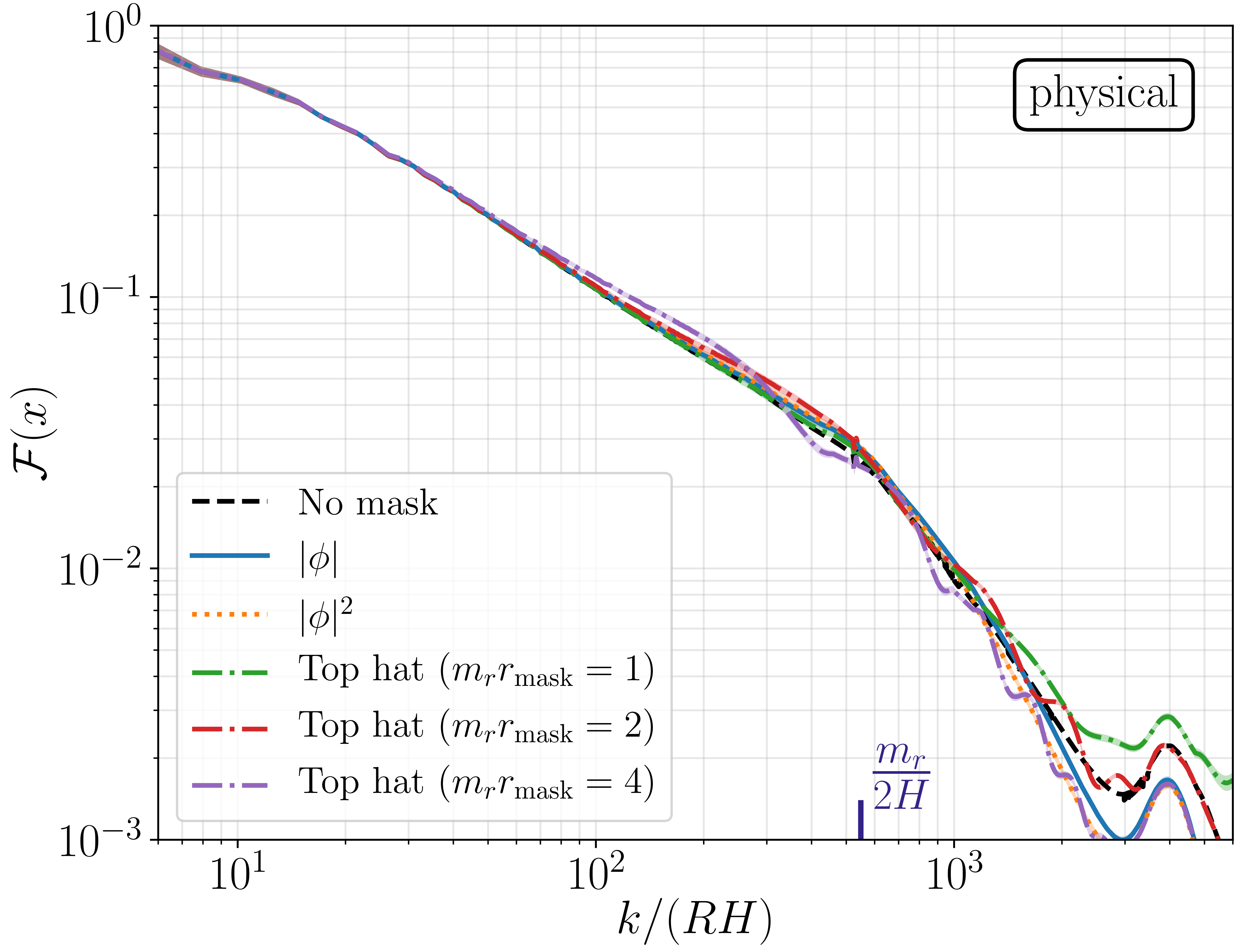
- To try to mitigate the contamination from the string core, we can introduce masks to to compute derivatives:

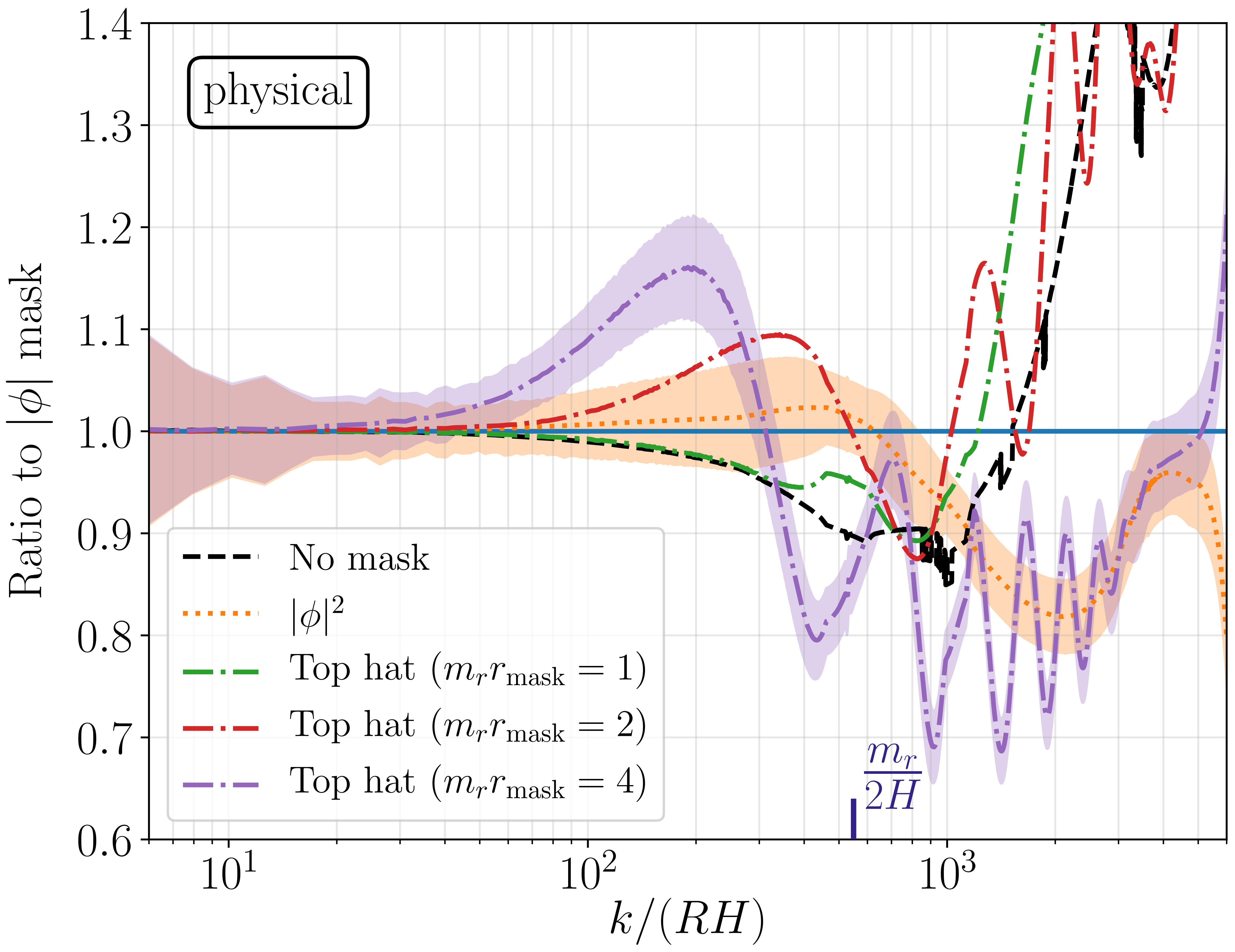
$$\dot{X}^{\text{mask}}(\boldsymbol{x}) = M(\boldsymbol{x})\dot{X}(\boldsymbol{x})$$

- Simple choice is to use the fact that the value of the radial field $|\phi|$ is zero inside the core.

$$M(x) = \left(\frac{|\phi(x)|}{f_a}\right)^k$$

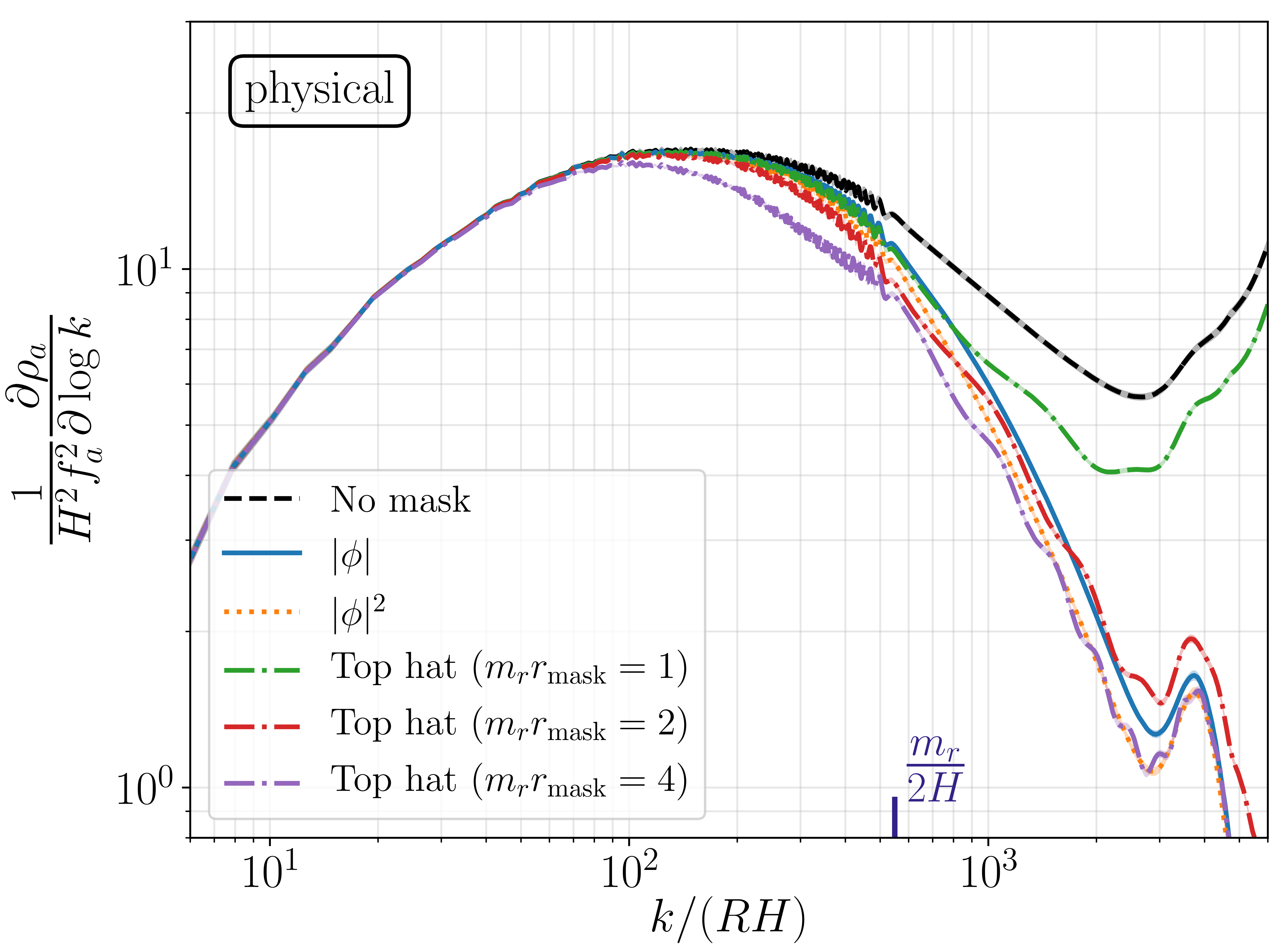
Masking the Spectrum

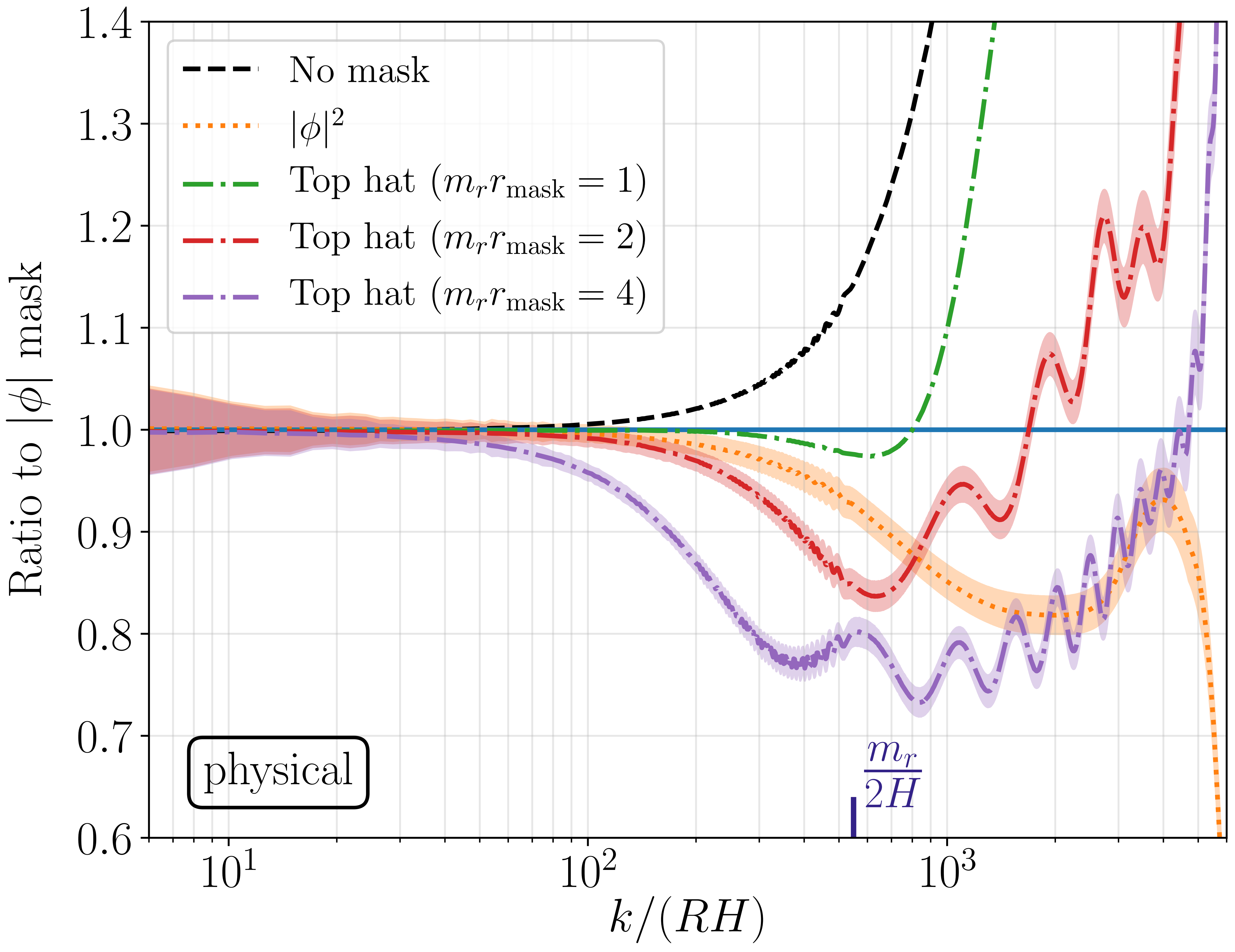




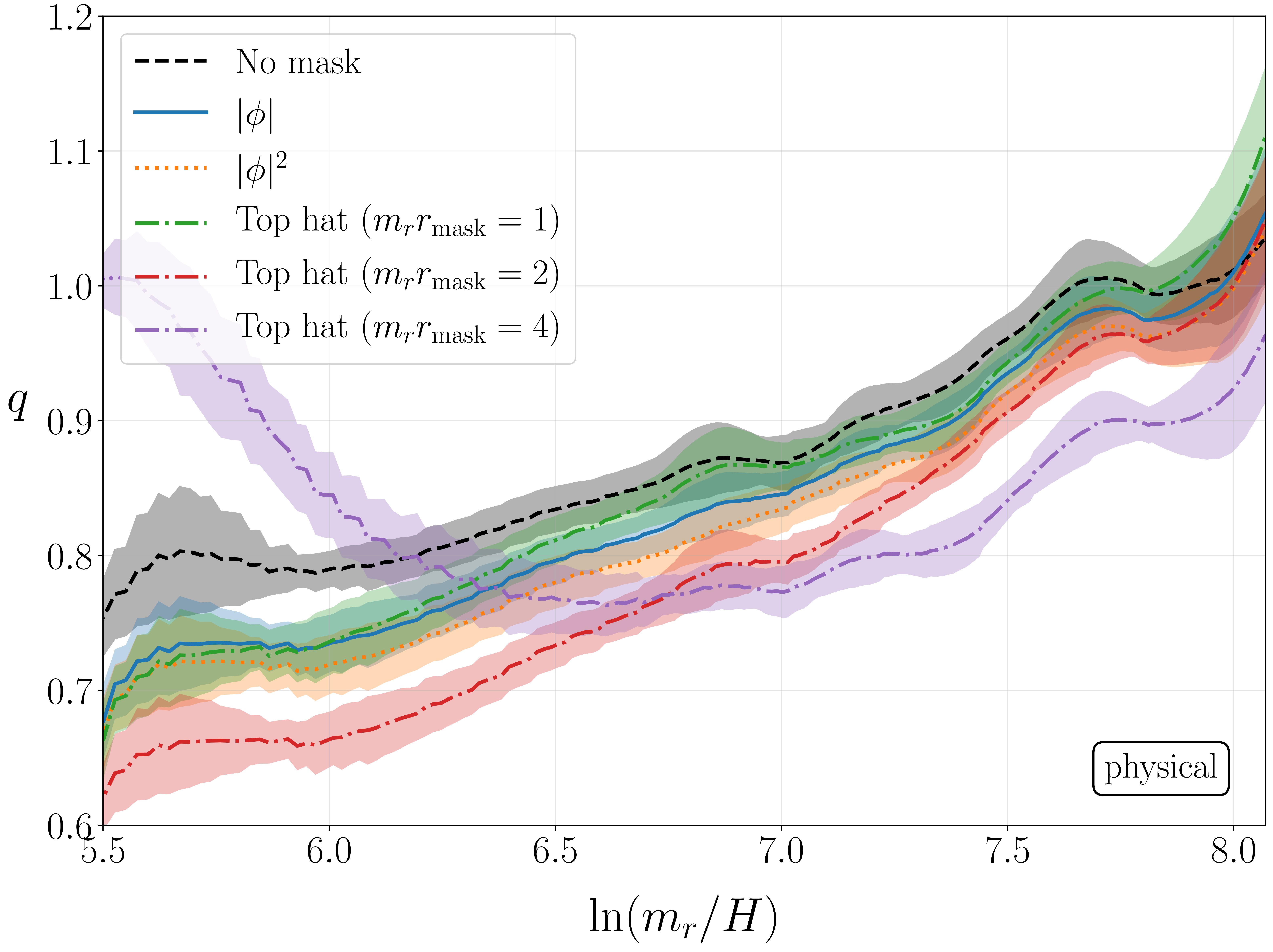
Masking the Spectrum

$\ln(m_r/H) = 7.0$





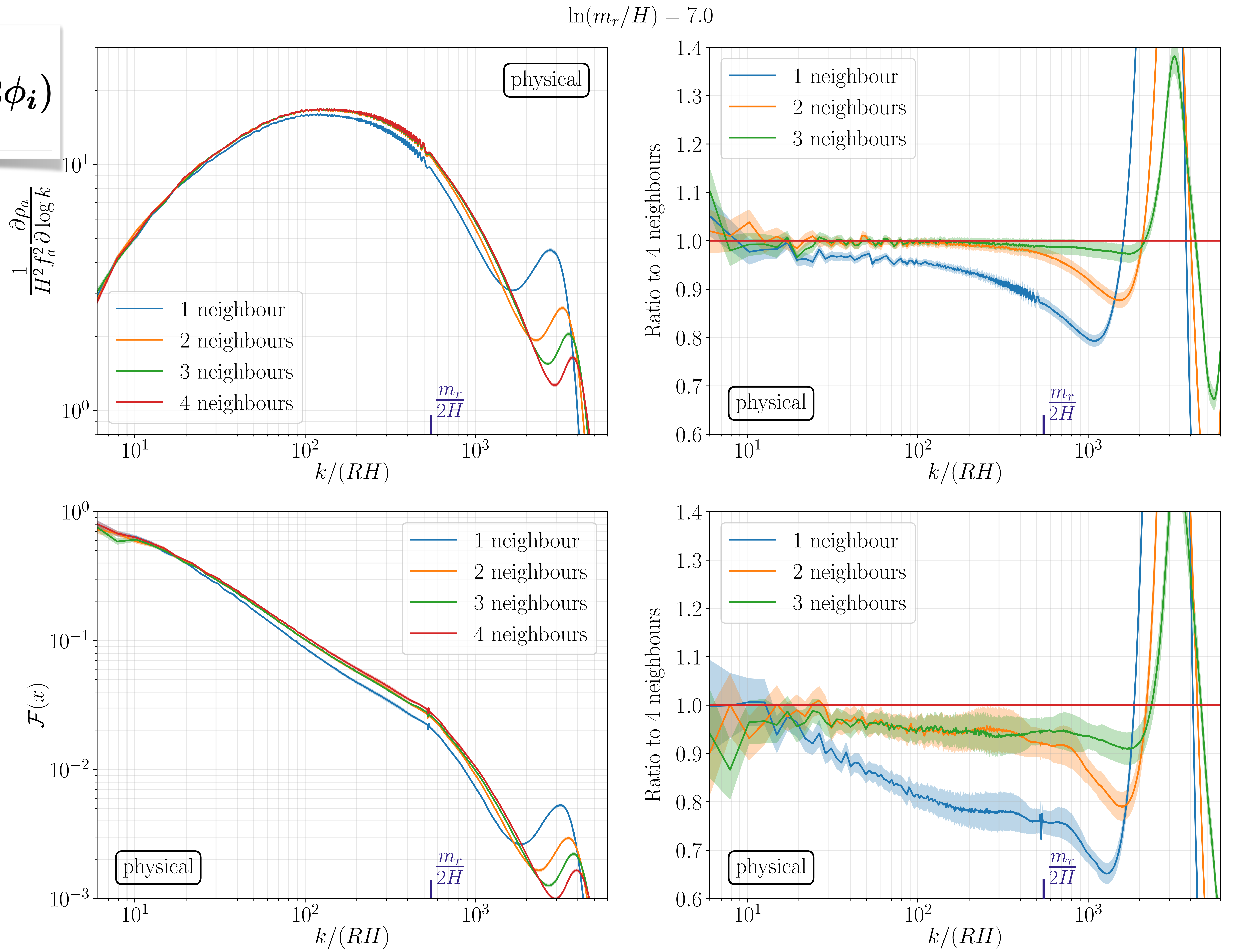
Masking the Spectrum



Discretisation of the Laplacian

$$(\nabla^2\phi)_{\boldsymbol{i}} = \frac{1}{\delta^2}\sum_{u=x,y,z}\sum_{n=1}^{N_g} C_n(\phi_{\boldsymbol{i}+n\boldsymbol{n}_u} + \phi_{\boldsymbol{i}-n\boldsymbol{n}_u} - 2\phi_{\boldsymbol{i}})$$

- Spectrum **underestimated** at intermediate momenta for smaller N_g
- Observation of peak-like structure in the UV, height related to N_g

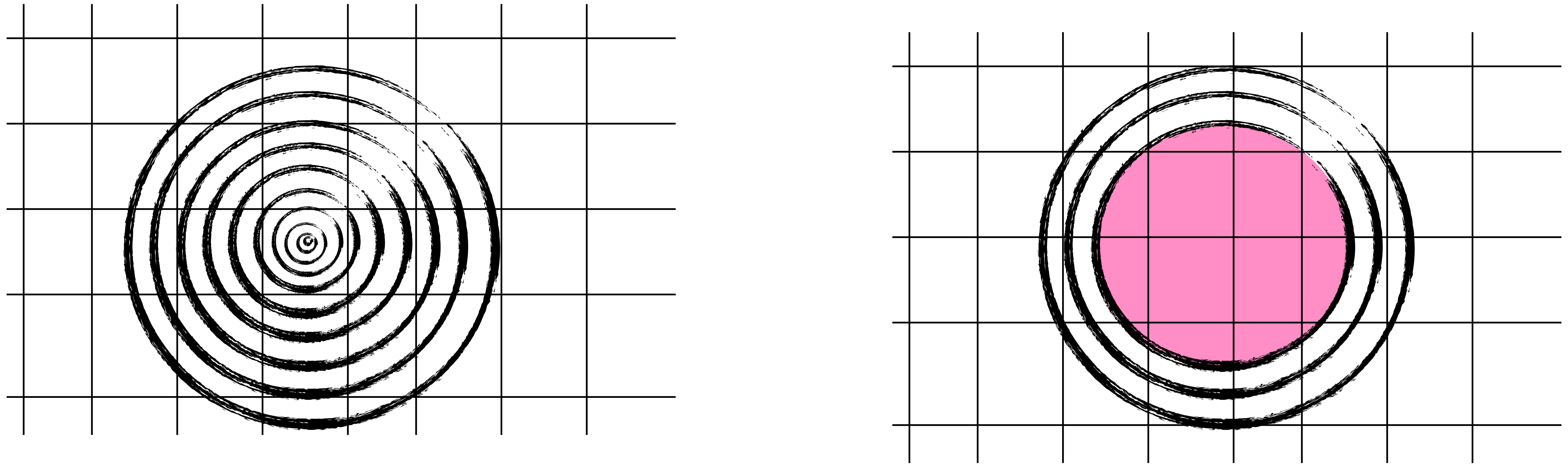


Dynamical Range + AMR

How can we reach a larger dynamical range?

- **Brute Force**: Larger simulations on more powerful supercomputers
- **Better:** Use the given computational power more efficiently: **AMR!**
- **In addition:** Study effective models that allow us to study the network dynamics at high tension (**Moore strings**) with 2+3 extra degrees of freedom (two additional complex scalars + one vector field)

Klaer, Moore [1707.05566, 1708.07521, 1912.08058]



$$\log \sim 2(q_1^2 + q_2^2)$$

Potential Improvement with AMR

- We can estimate the RAM needed to perform an AMR complex scalar simulation:

$$\mathrm{RAM} = 2 \times 2 \times 4\ \mathrm{bytes}\ \times \left(N_0^3 + \frac{\pi n_c n_r^2}{4} \frac{r^\ell - 1}{r - 1} N_p \right)$$

$N_p = \xi \times 6(L/(N_0\tau))^2 \times N_0^3$
Fleury & Moore [1509.00026]

- This takes into account, that we refine only around the strings and that we want to balance the RAM between the root and the refined grids

- Suggests time-dependent number of refinement levels:.

$$\ell + 7 \simeq \log_2\left(N_0^3/(\pi N_p)\right) = \log_2\left(N_0^2\tau^2/(\pi 6 \xi L^2)\right)$$

- Results in log ~ 13,16,18 for base grids of $N_0 = 2048,\ 4096,\ 8192$ with $\ell = 9,\ 11,\ 13.$ In practice not so trivial ..

Potential Improvement with AMR