

Warmth & Noise in Cosmological Fields

Mustafa A. Amin



RICE



2211.09775, 2408.05591, 2503.20881, 2506.12131, 2510.15046, 2510.17977



Mirbabayi



Ling  RICE



Delos



May



Yang  RICE

How light can dark matter be ?

Mustafa A. Amin



RICE



How light can dark matter be ? $m \gtrsim 10^{-19} \text{eV}^{*,**}$

Mustafa A. Amin



with M. Mirbabayi

2211.09775

* for dark matter density dominated by sub-Hubble modes
** by coincidence, the number is same more generally

Warmth & Noise in Cosmological Fields

Mustafa A. Amin



RICE



2211.09775, 2408.05591, 2503.20881, 2506.12131, 2510.15046, 2510.17977



Mirbabayi



Ling  RICE



Delos

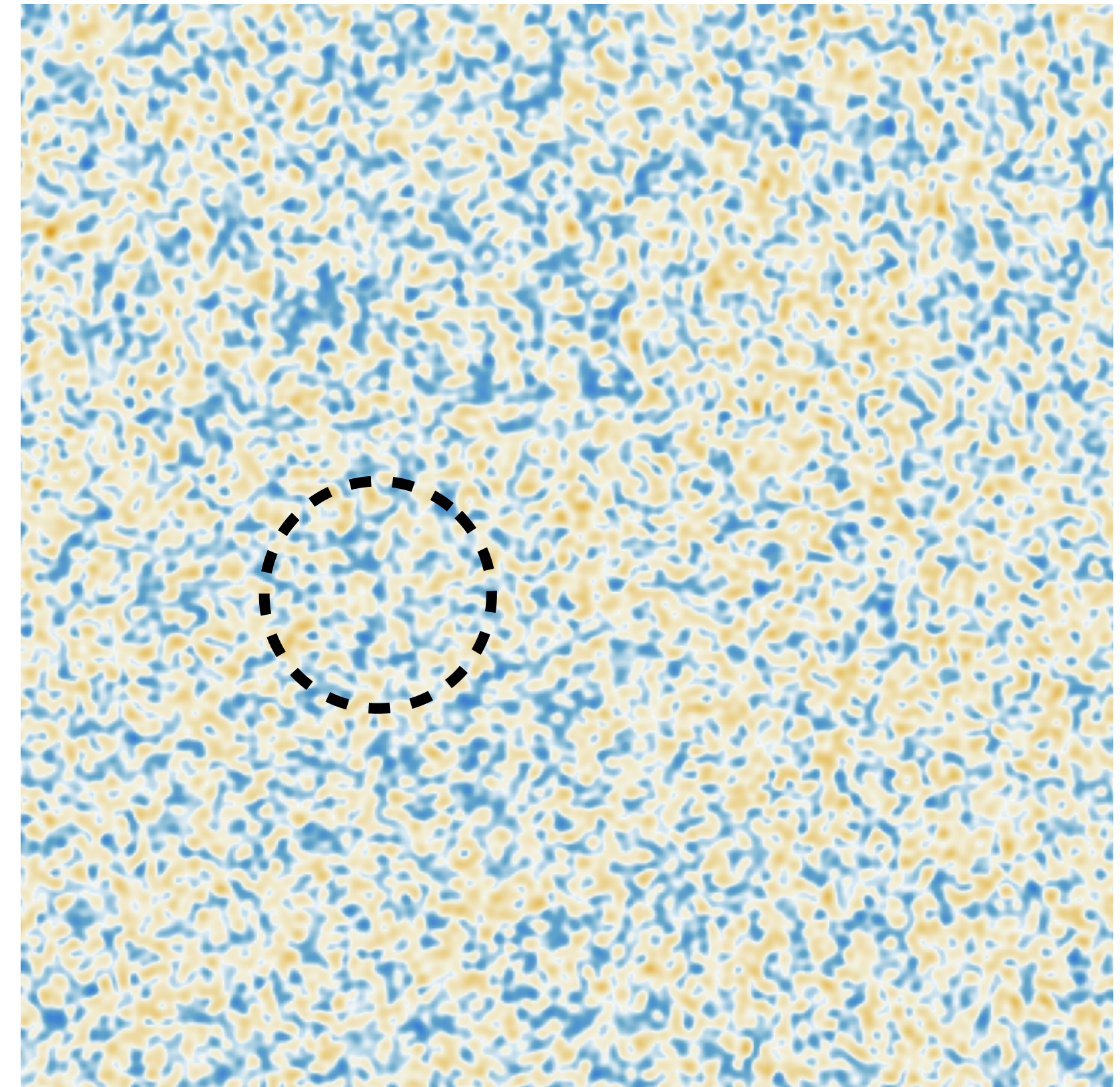
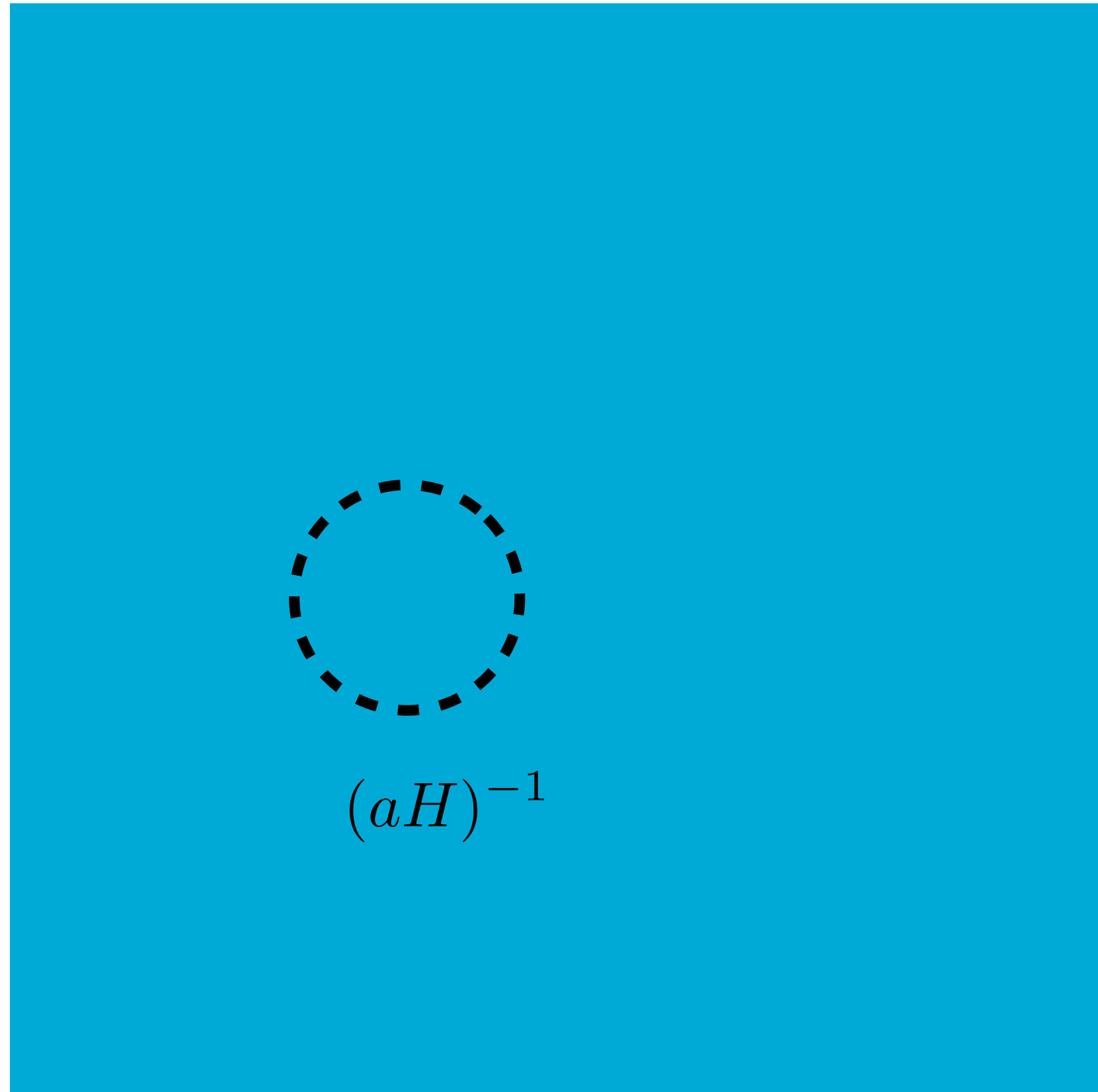


May



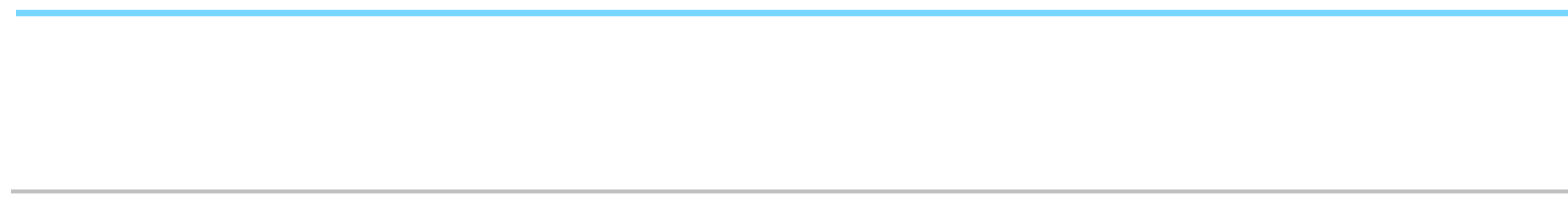
Yang  RICE

Cold & Quiet vs. Warm & Noisy Fields



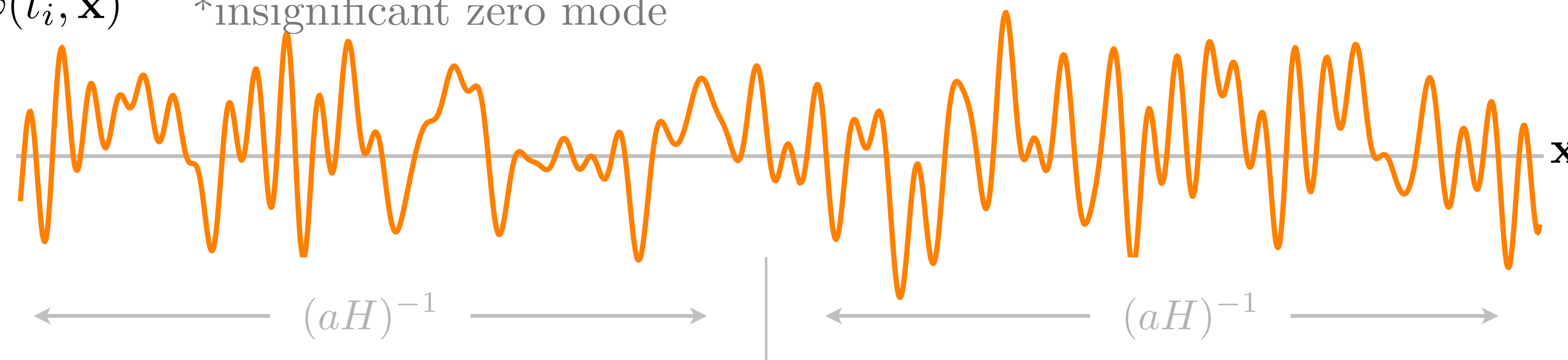
Quiet vs. Noisy Fields

$\varphi(t_i, \mathbf{x})$ * significant zero mode



eg. axions with $\mathbb{PQ}$ before inflation
inflaton just at end of inf.

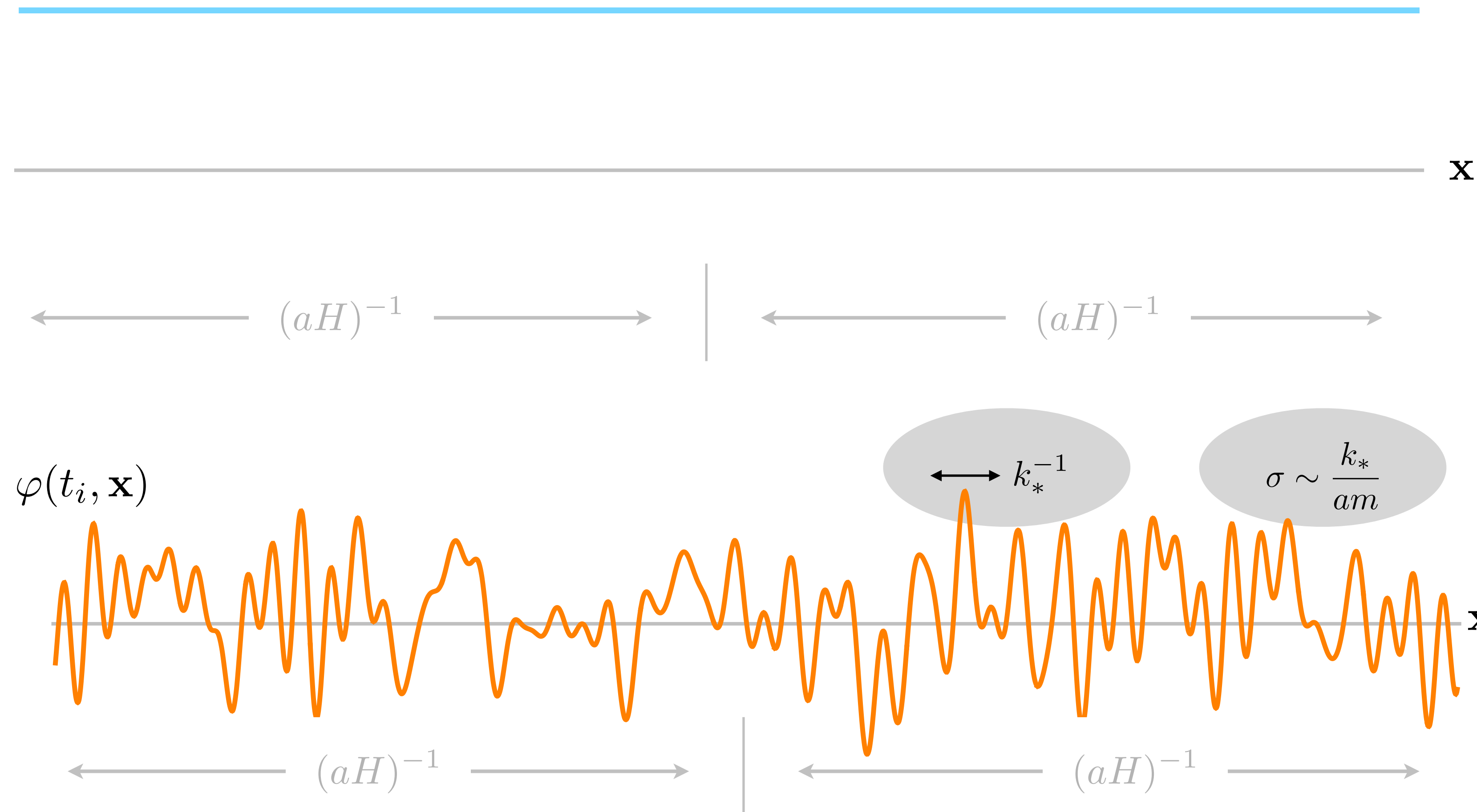
$\varphi(t_i, \mathbf{x})$ * insignificant zero mode



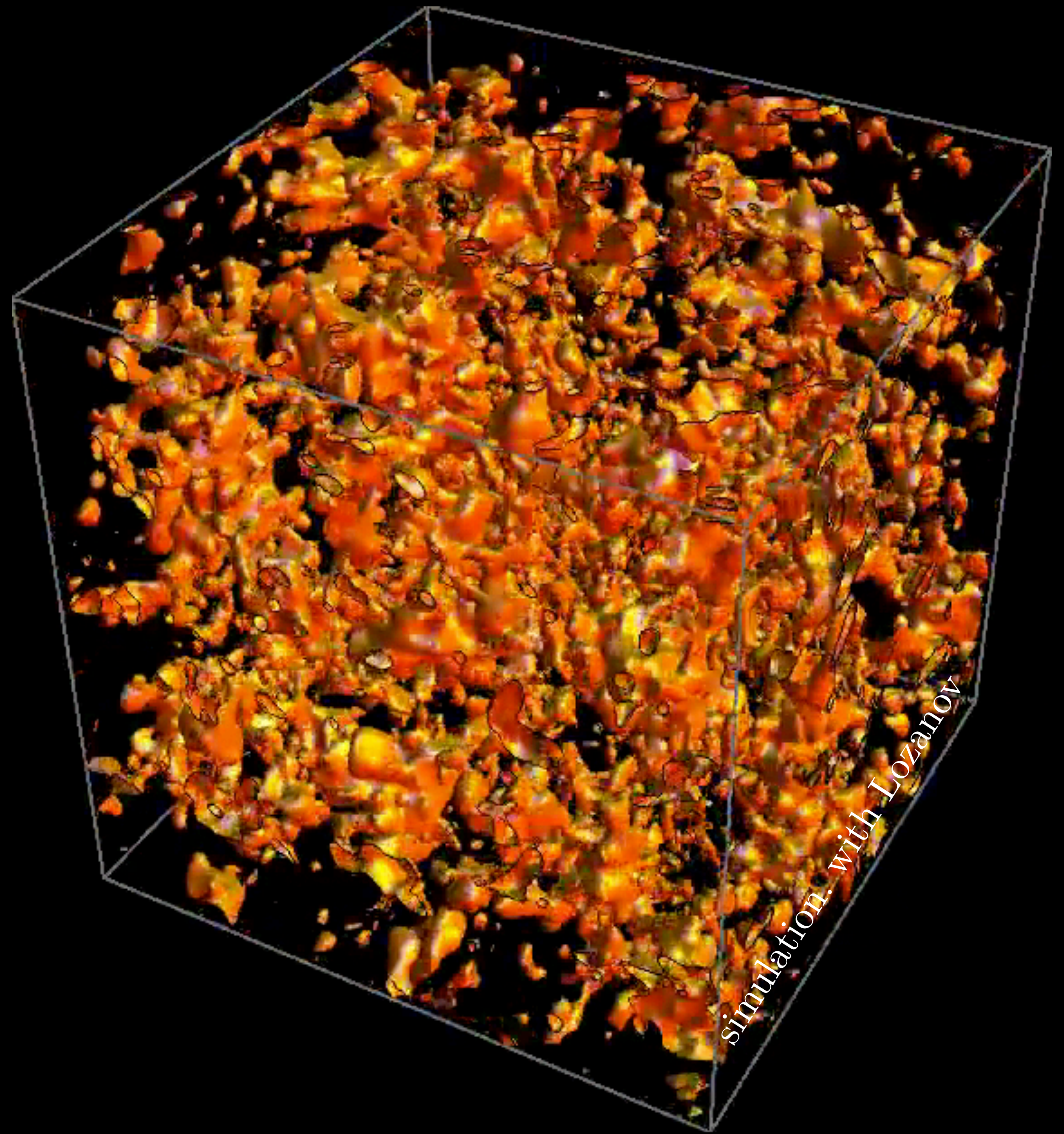
eg. axions with $\mathbb{PQ}$ after inflation
field after inflation fragmentation

Cold & Quiet vs. Warm & Noisy Fields

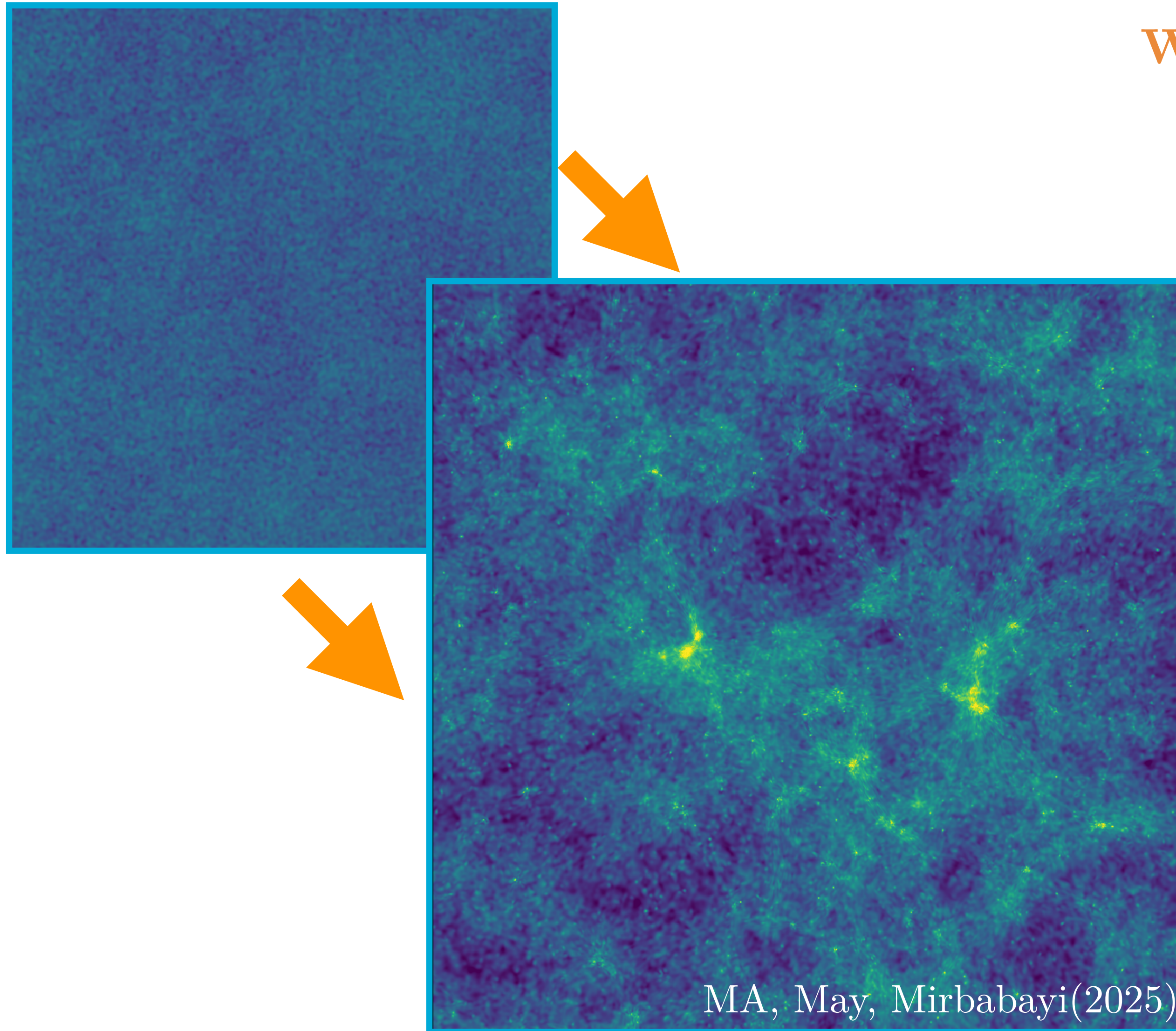
$\varphi(t_i, \mathbf{x})$



gravitational clustering?



main goal: gravitational clustering ?



Warm and Noisy Field 
 $\longleftrightarrow k_*^{-1}$ $\sigma \sim \frac{k_*}{am}$

density perturbations

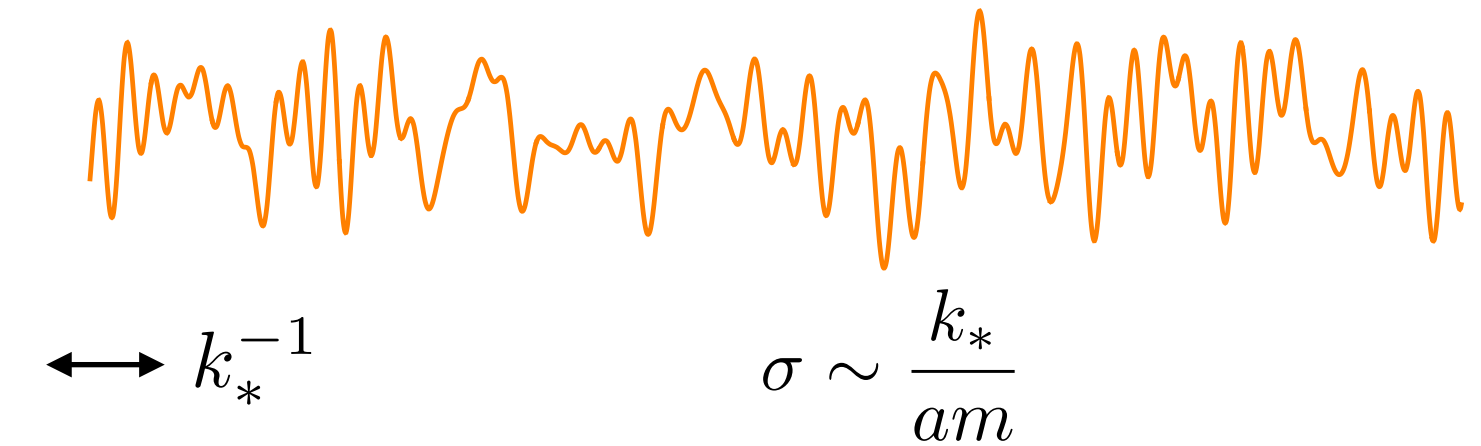
- suppression
 - free-streaming + Jeans
- enhancement
 - Poisson-like fluctuations

scale dependent evolution

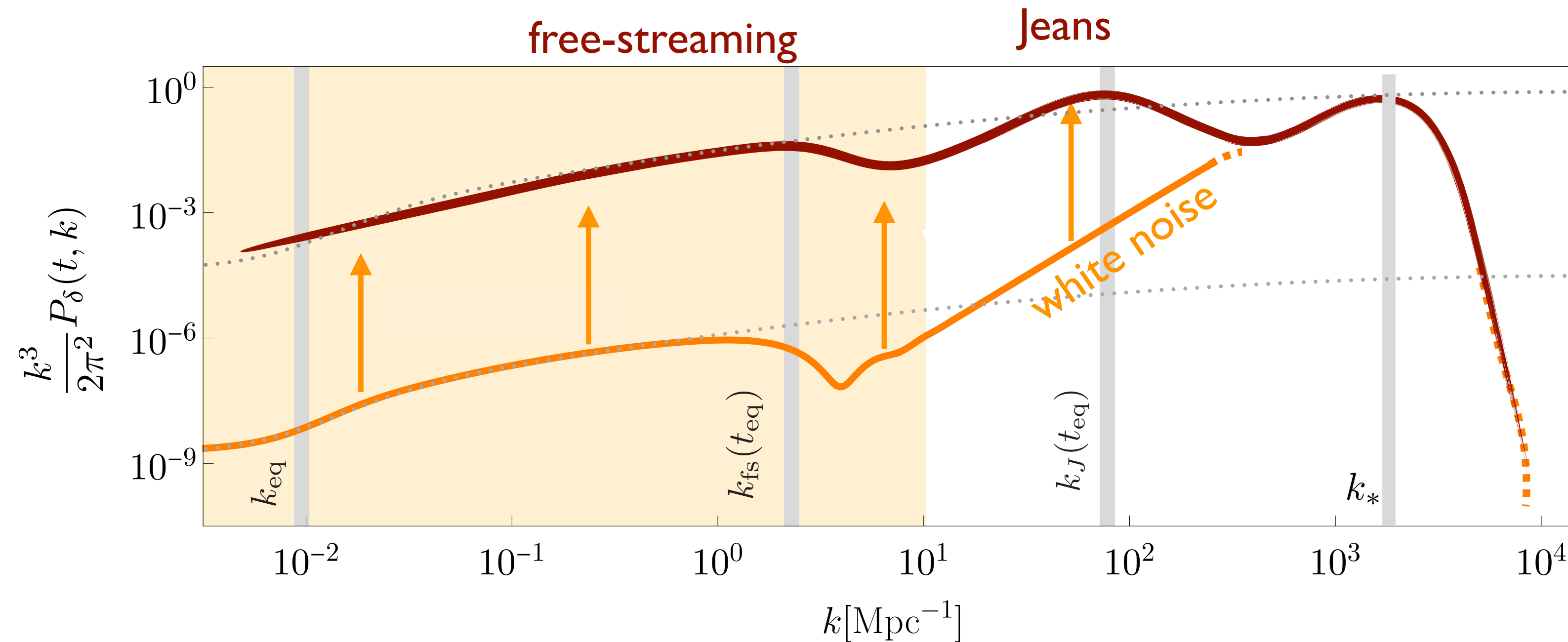
- but no zero field mode to perturb

Warm & Noisy Wave Dark Matter

Dark matter density dominated by **sub-Hubble** field modes



$\longleftrightarrow k_*^{-1}$ $\sigma \sim \frac{k_*}{am}$



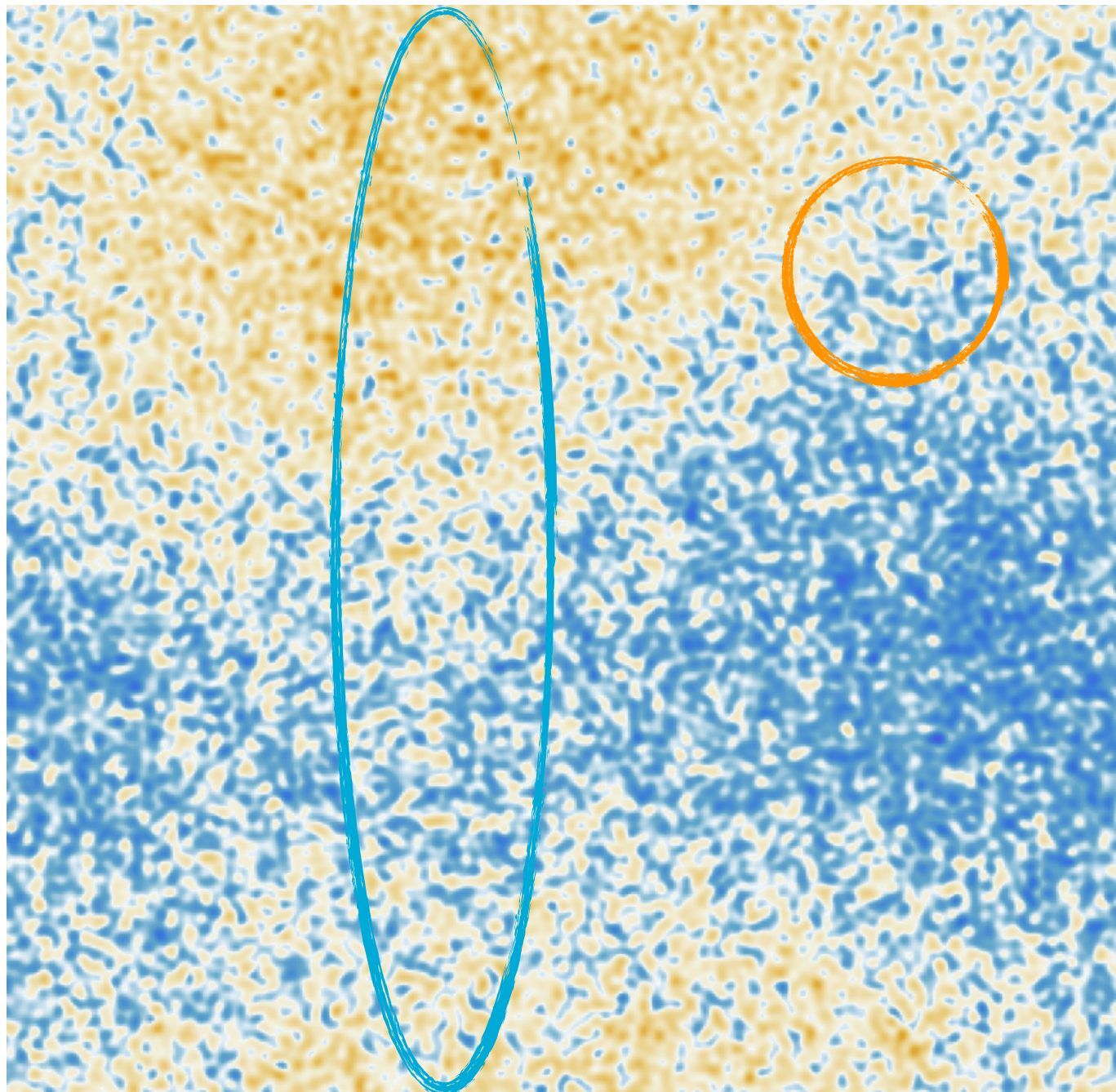
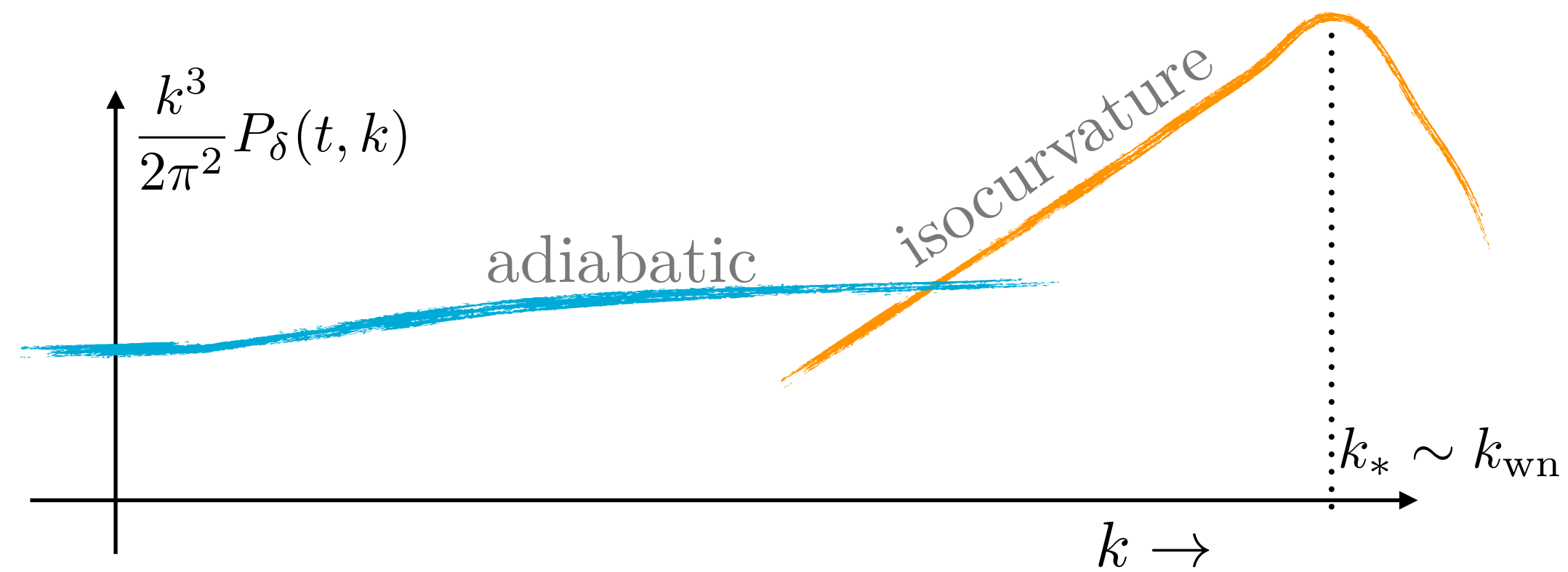
Observationally accessible*,
scale-dependent features
in the density power spectrum

k_{fs}
free-streaming

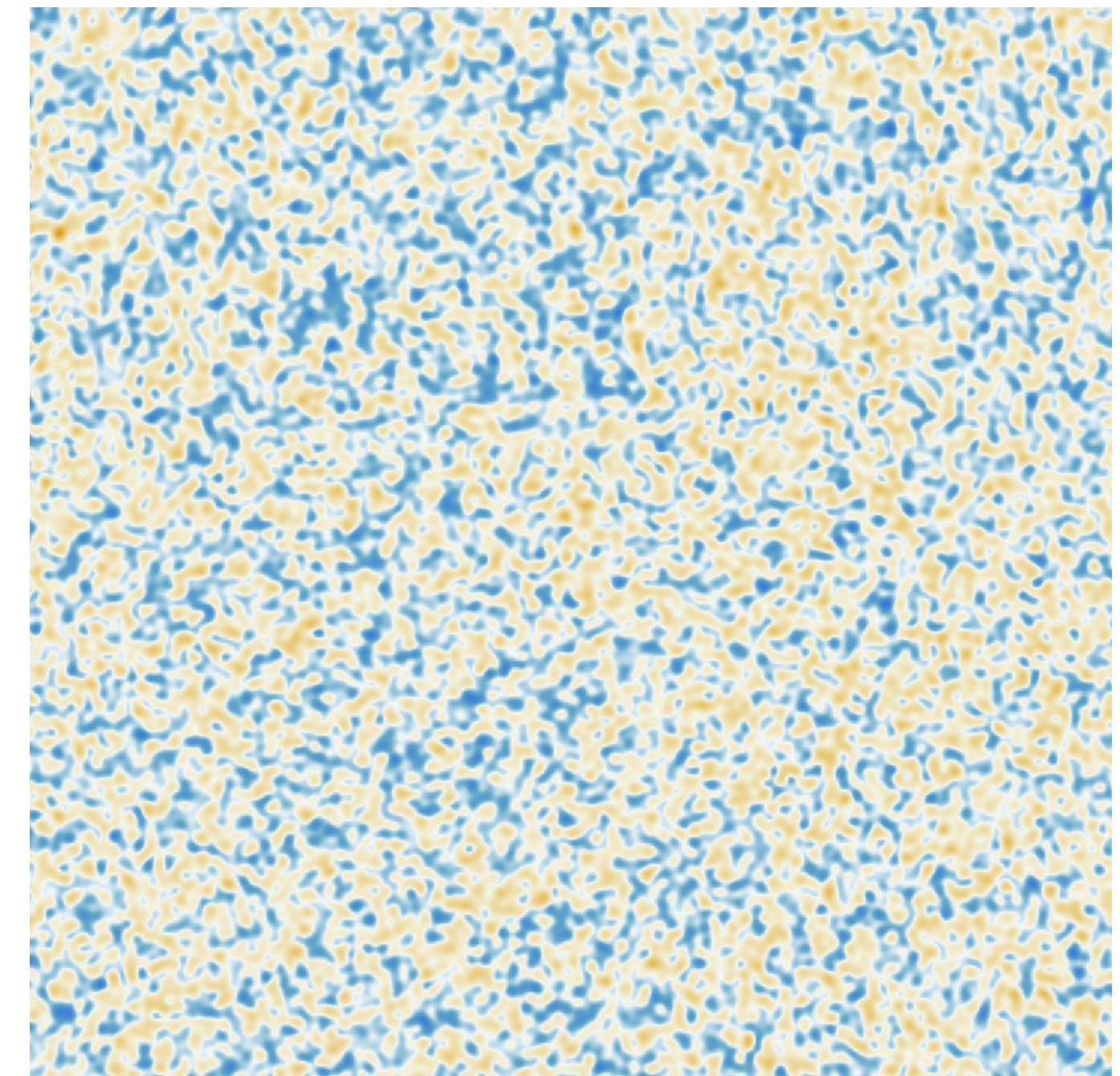
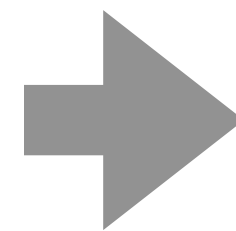
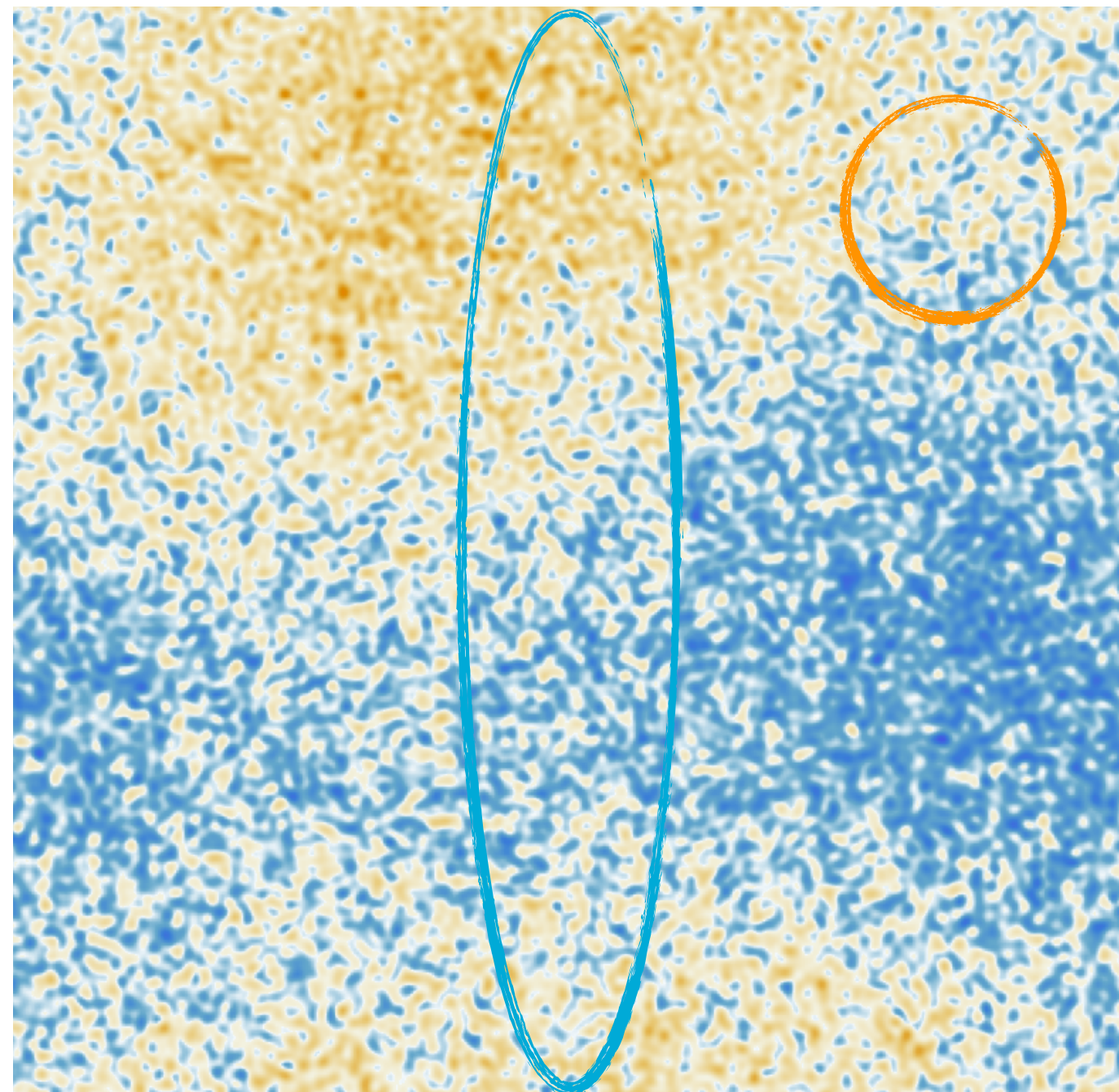
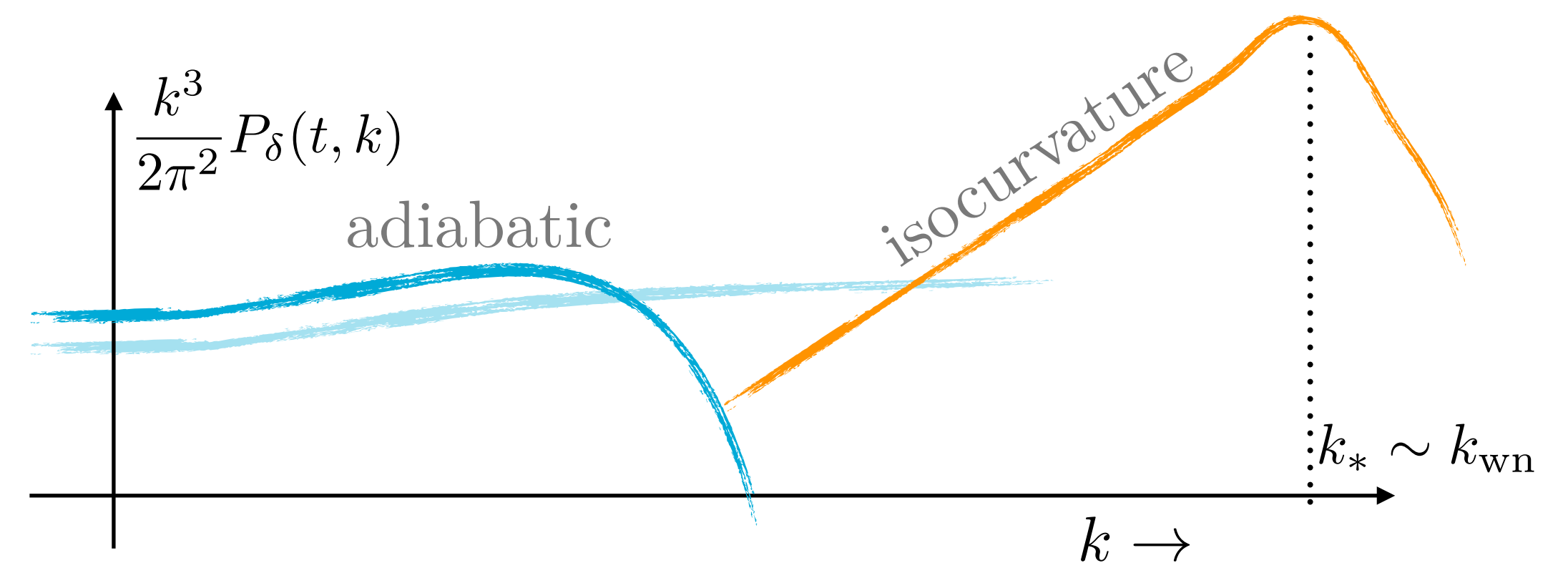
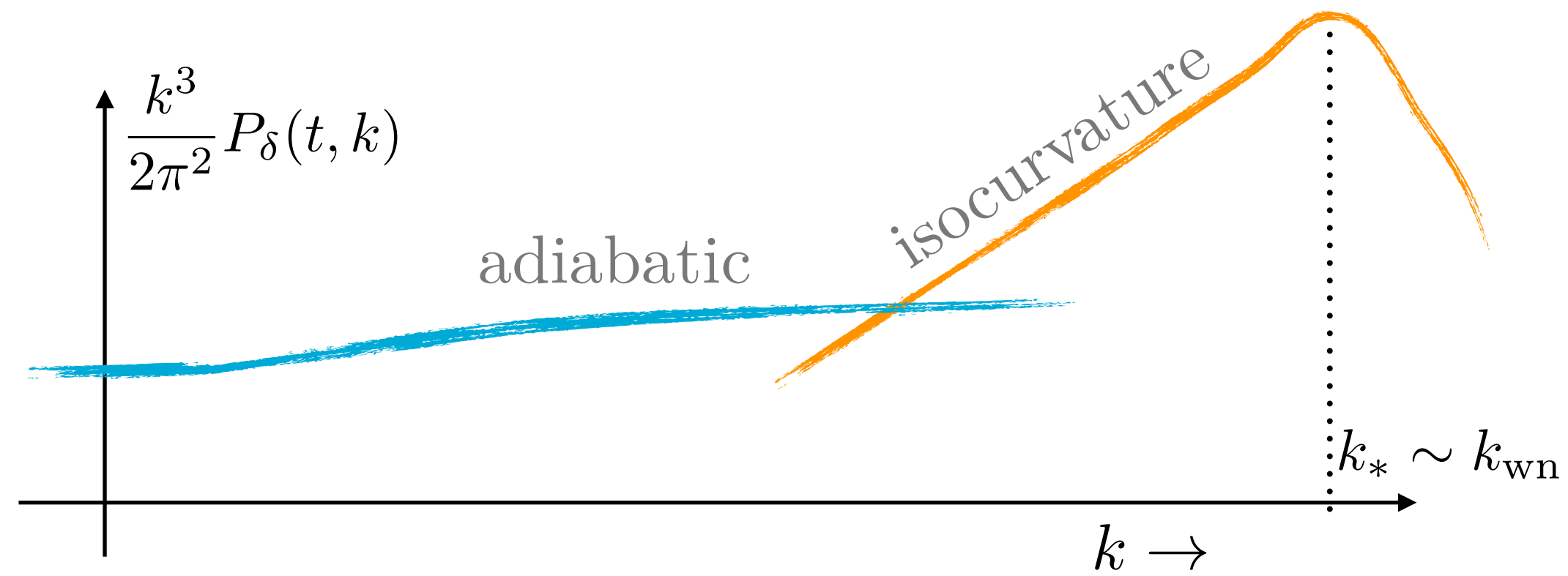
k_J
Jeans

k_*
white noise

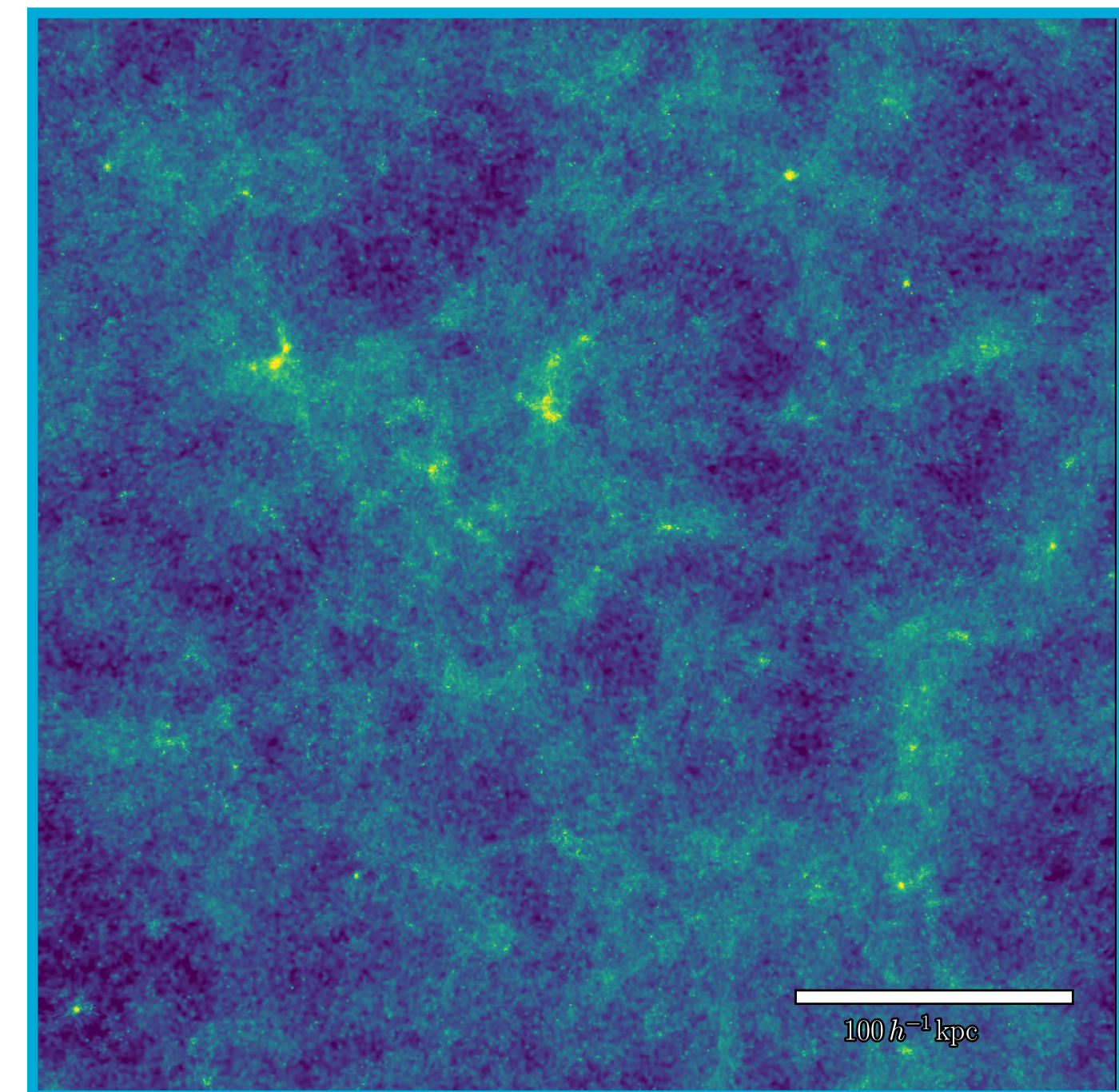
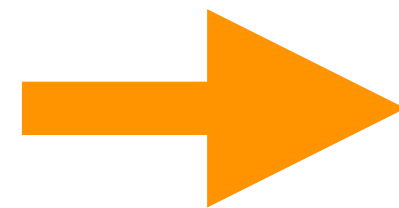
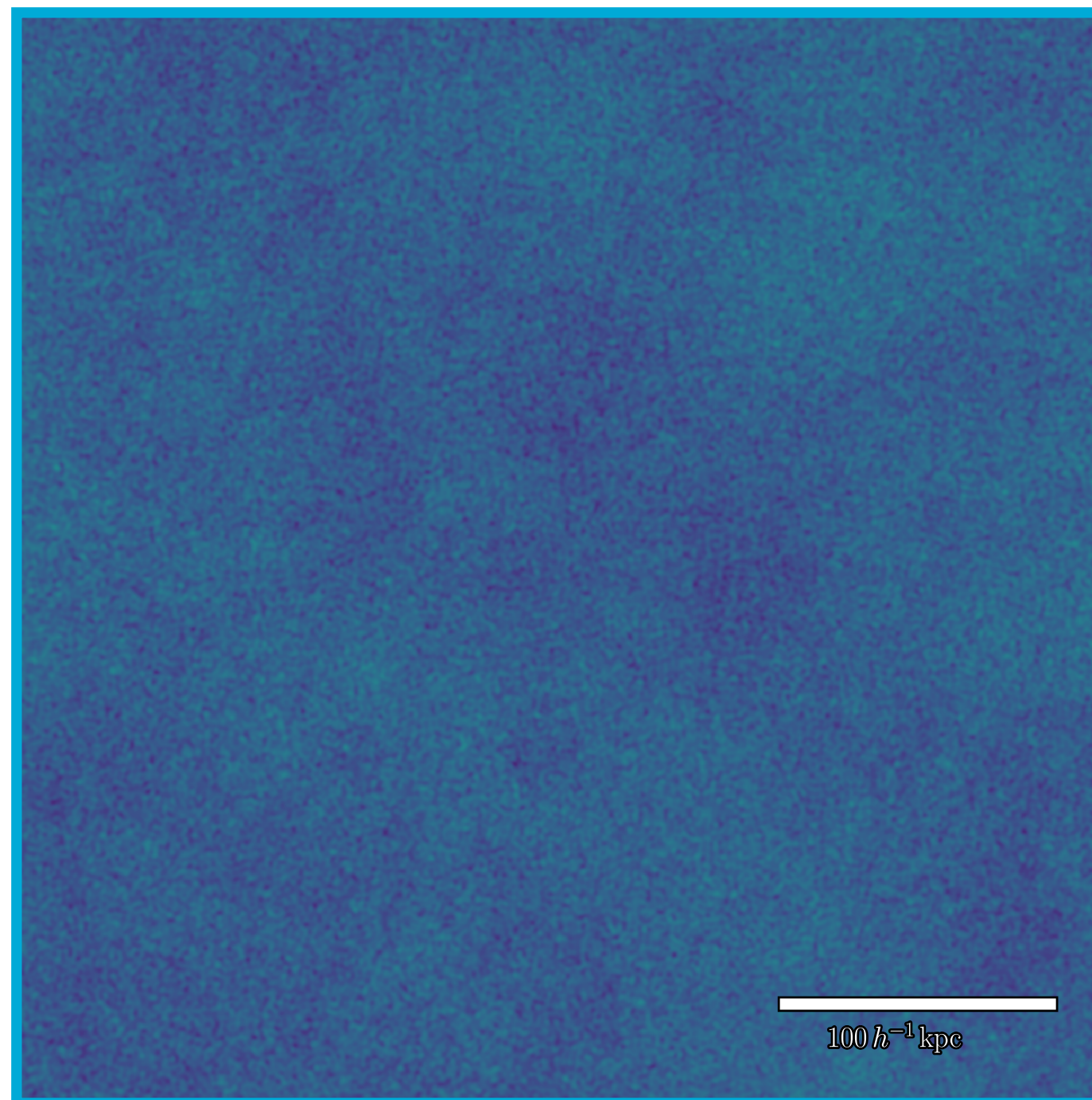
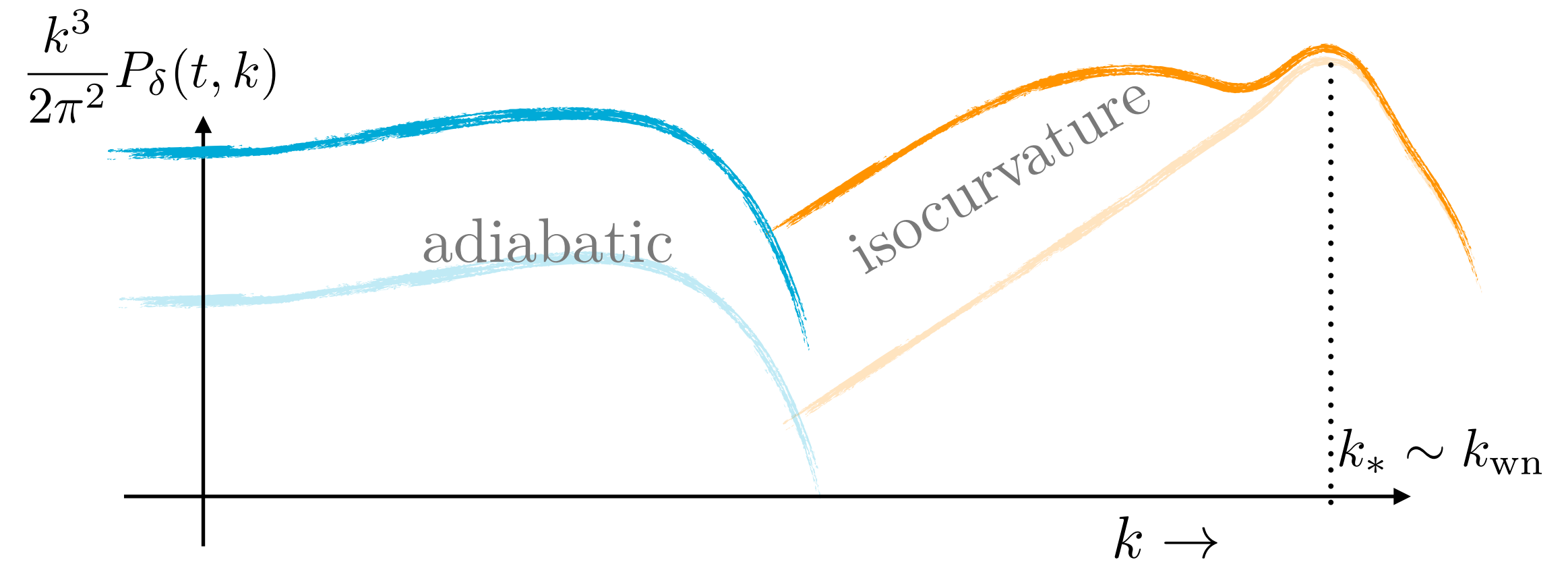
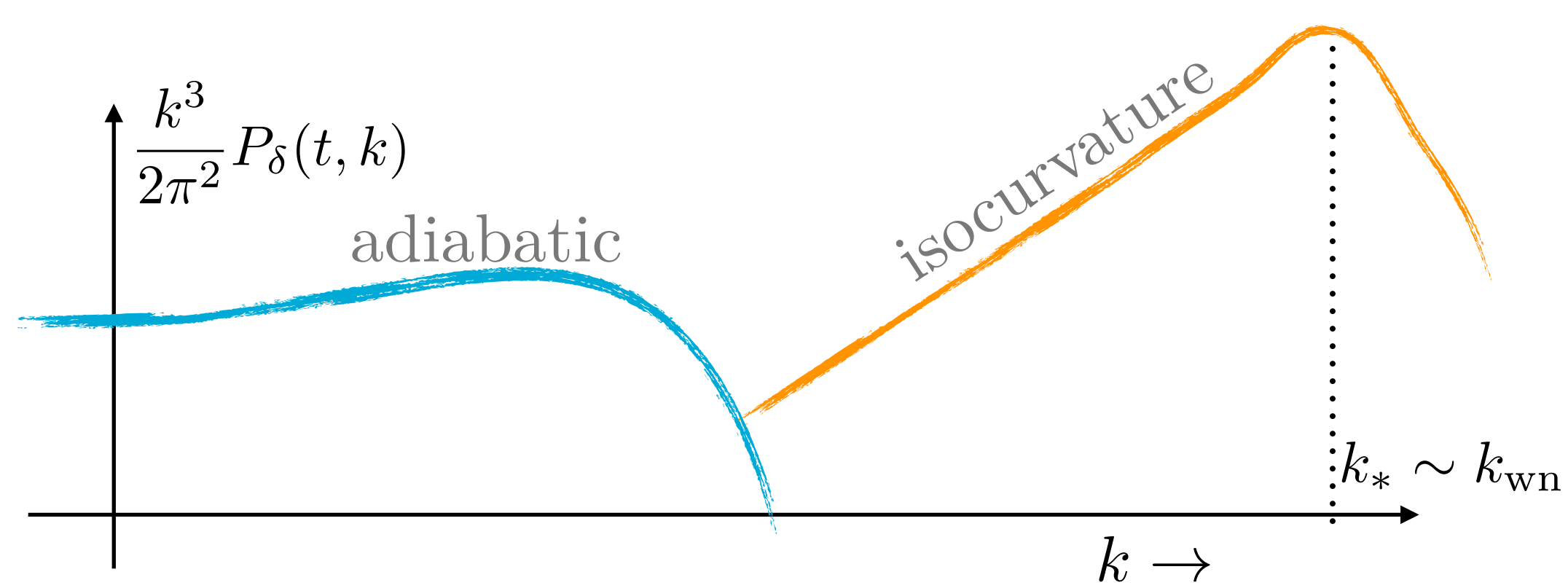
density evolution during radiation domination



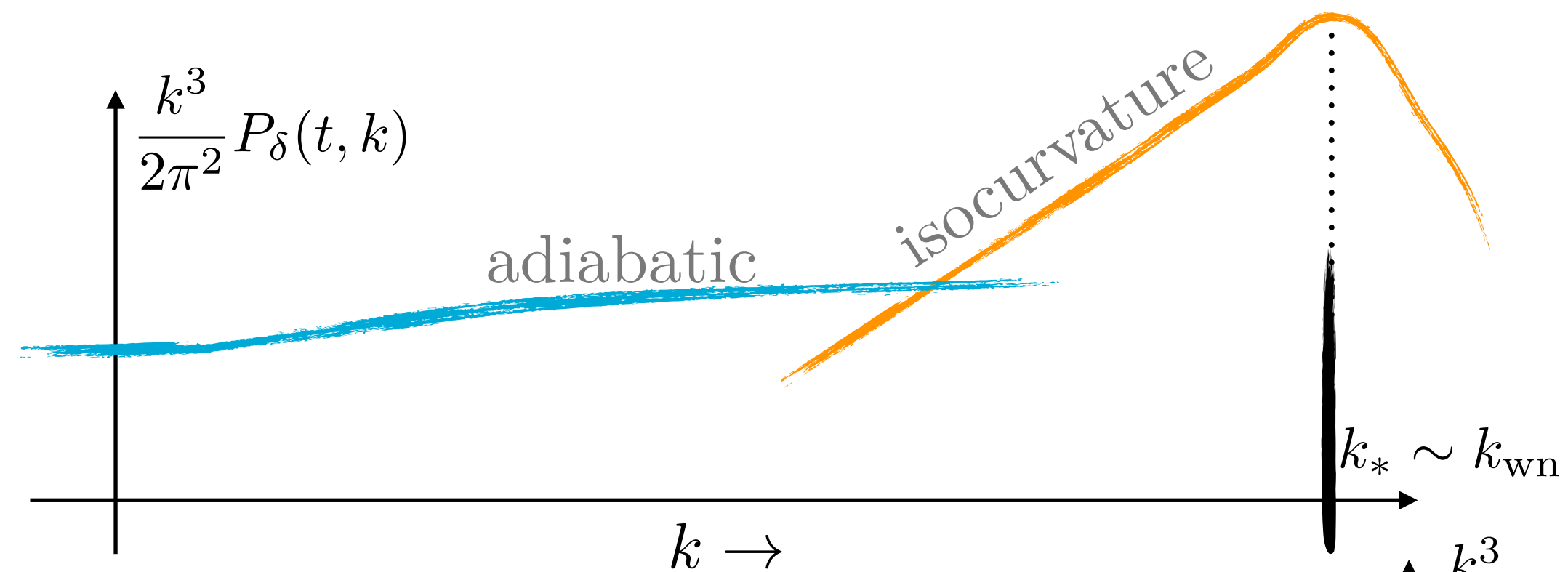
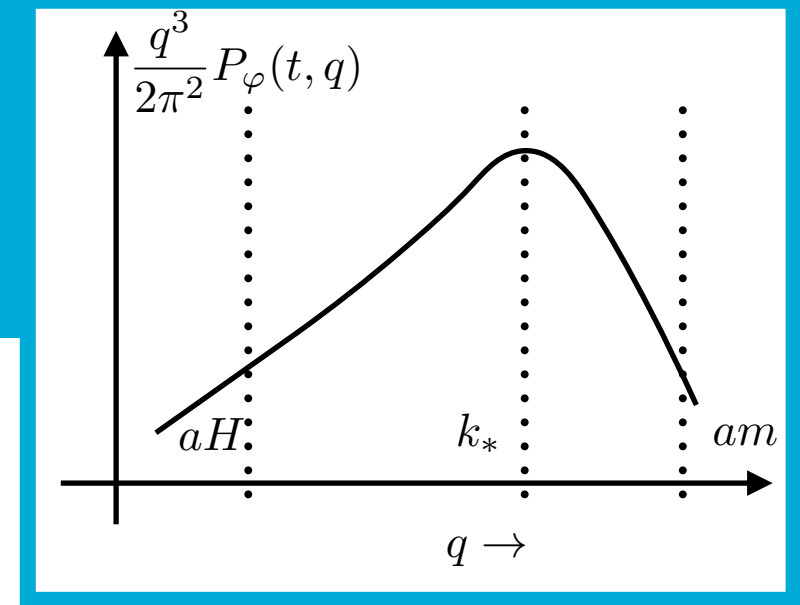
density evolution during radiation domination



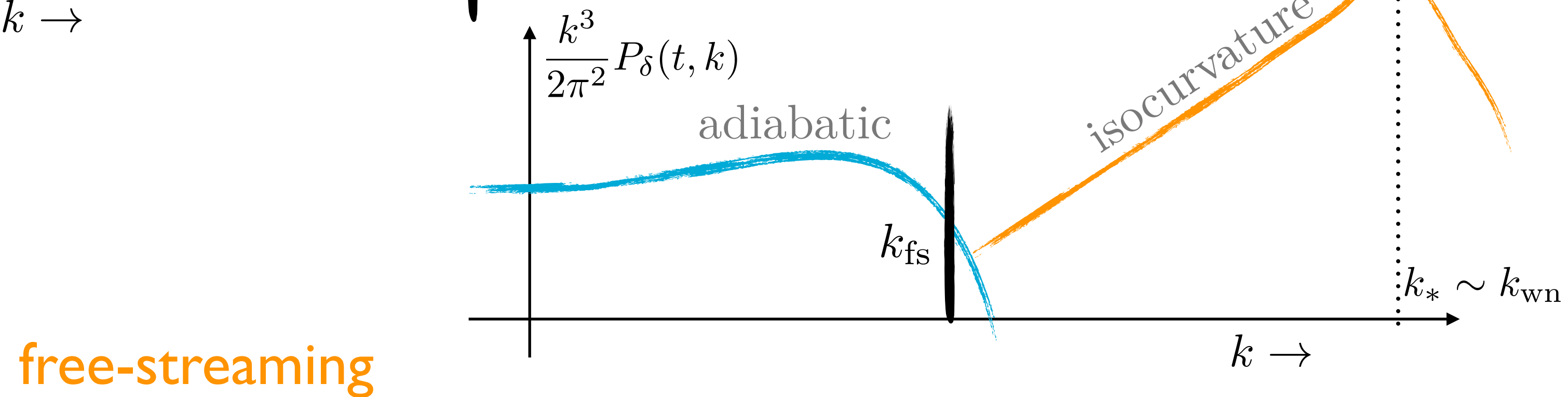
density evolution during matter domination



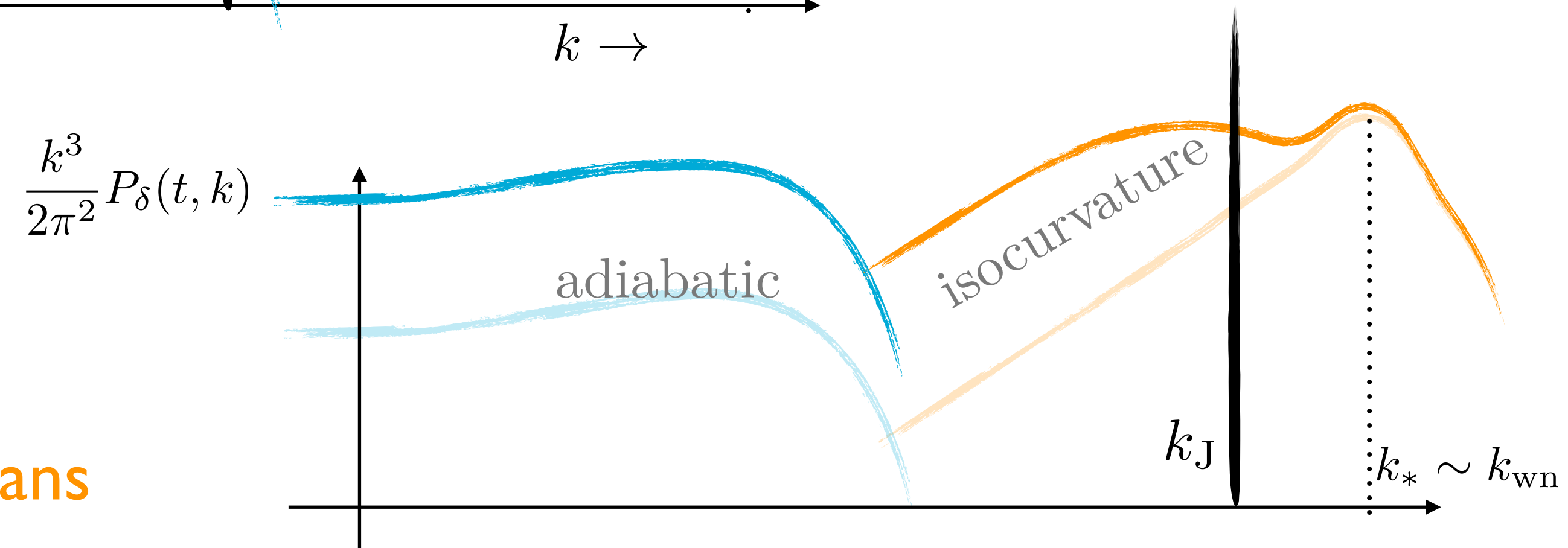
evolution of the matter power spectrum



white noise

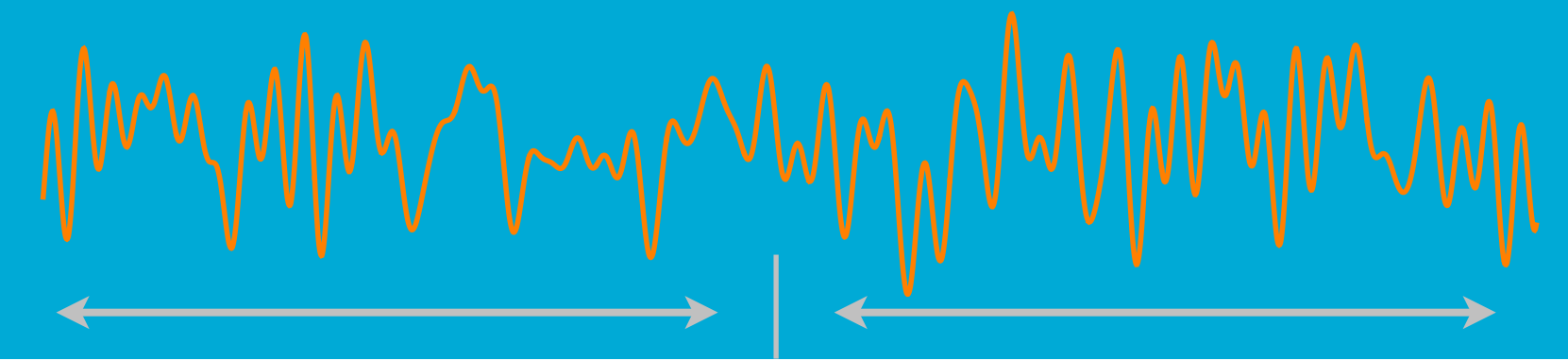


free-streaming

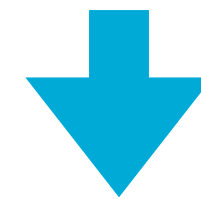


Jeans

the lower bound on wave DM mass

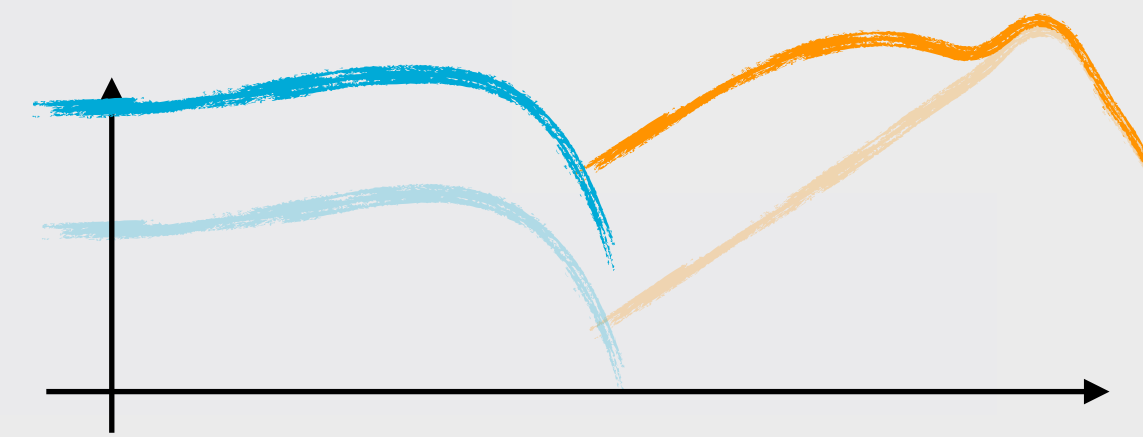


Dark matter density dominated by **sub-Hubble** field modes

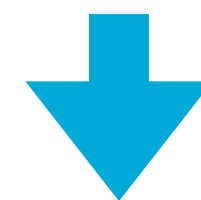


Then in density power spectrum:

1. **white-noise** excess
2. **free-streaming** suppression



1. and 2. not seen for $k < k_{\text{obs}} \sim 10 \text{ Mpc}^{-1}$



$$m \gtrsim 10^{-19} \text{ eV}$$

MA & Mirbabayi (2022/24)

comparison with literature

$$m \gtrsim 1 \times 10^{-21} \text{ eV}$$

Banik et. al (2019) — Streams

$$m \gtrsim 3 \times 10^{-21} \text{ eV}$$

Nadler et. al (2021) — MW satellites

$$m \gtrsim 2 \times 10^{-20}$$

Rogers and Peiris (2021) — $\text{Ly}\alpha$

$$m \gtrsim 4 \times 10^{-21} \text{ eV}$$

Powell et. al (2023) — lensing

$$m \gtrsim 3 \times 10^{-19} \text{ eV}$$

Dalal & Kravtsov (2022) — dynamical heating
+May, Dalal & Kravtsov (2025)

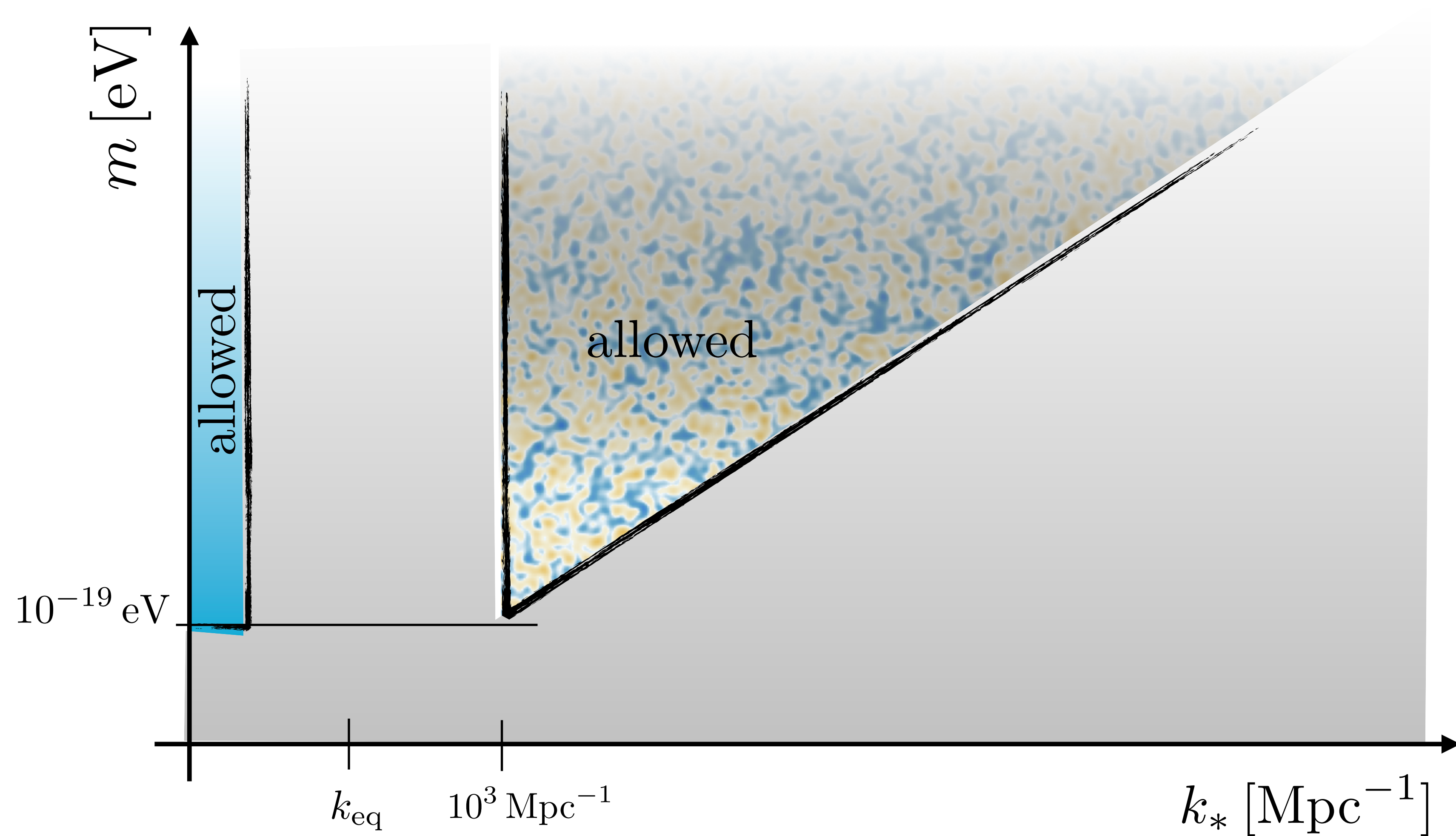
$$m \gtrsim 10^{-19} \text{ eV}$$

MA & Mirbabayi (2022/24)

* constraints come from different observations and assumptions, compare with care.

* For thermal production, bound is $\sim$ few keV — why the difference ? — Ask me.

the lower bound on wave DM mass

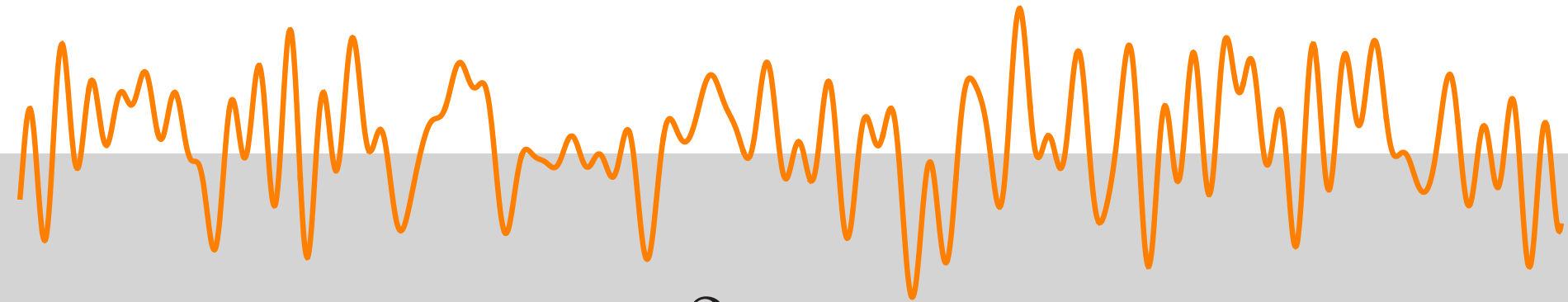




technical details of the calculation

MA, May, Mirbabayi (2025) MA, Delos (2025)

non-relativistic fields in an expanding universe

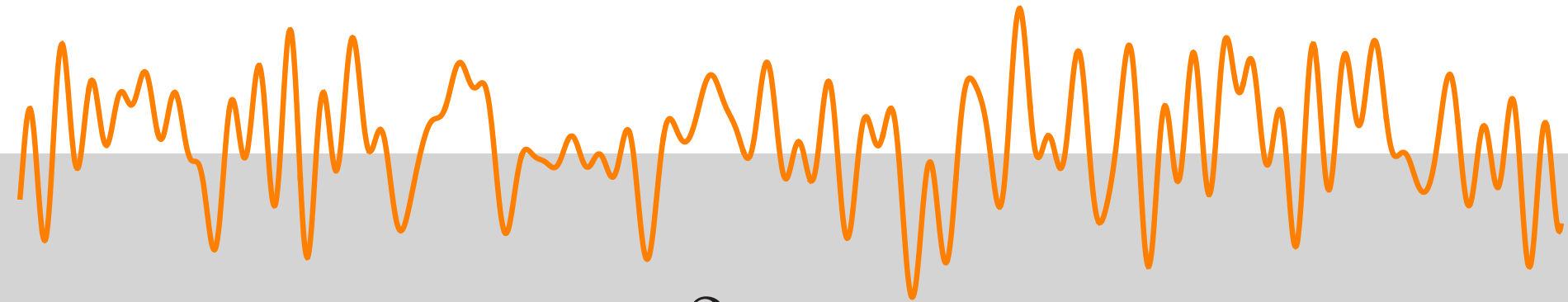


$$i\partial_{\eta}\psi = -\frac{\nabla^2}{2m}\psi + ma^2\Phi\psi$$

$$\nabla^2\Phi = \frac{4\pi G}{a}(\rho - \bar{\rho}), \quad \rho = m|\psi|^2$$

$$d\eta = a^{-2}dt$$

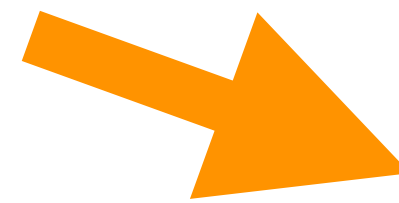
2-pt density correlation function



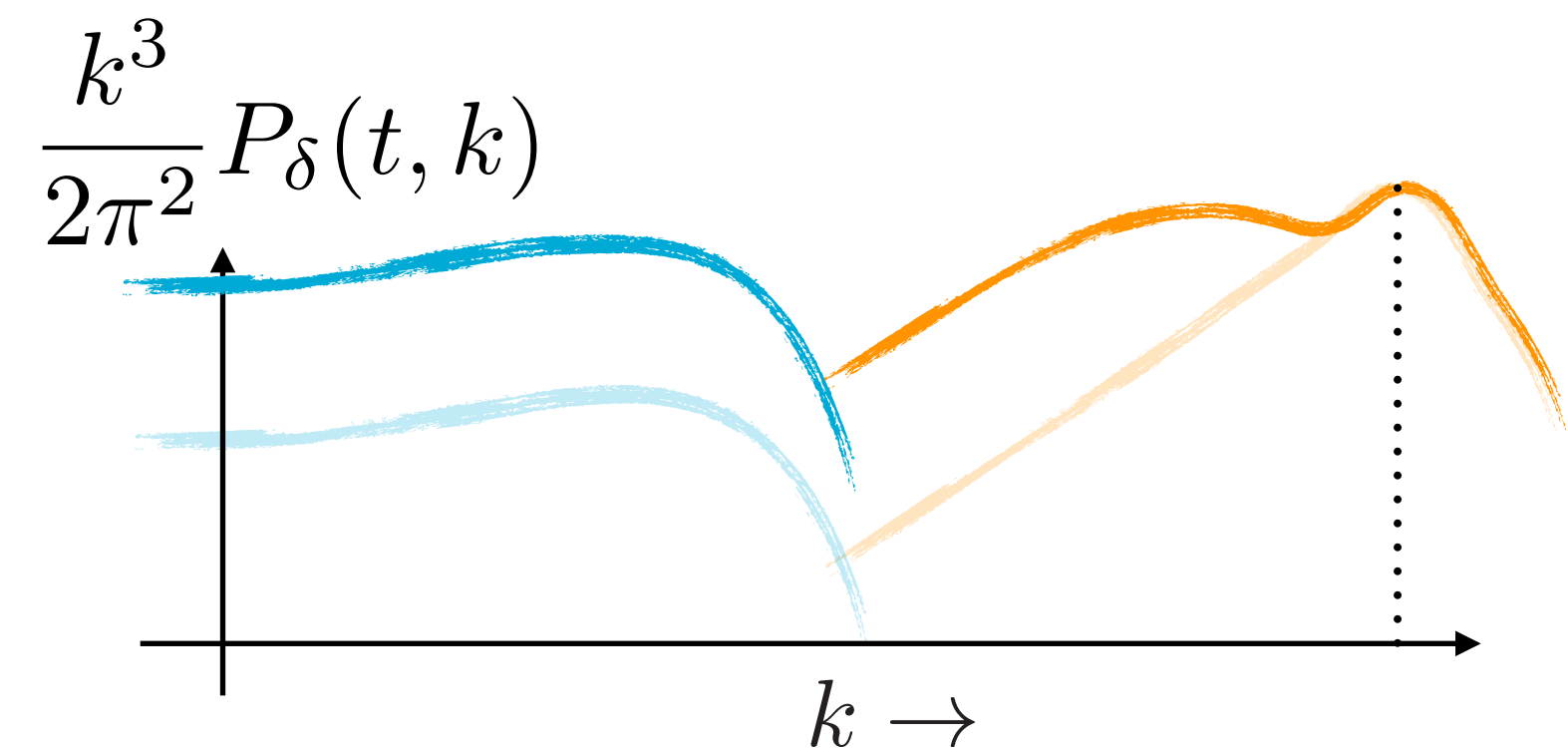
$$i\partial_\eta\psi = -\frac{\nabla^2}{2m}\psi + ma^2\Phi\psi$$

$$\nabla^2\Phi = \frac{4\pi G}{a}(\rho - \bar{\rho}), \quad \rho = m|\psi|^2$$

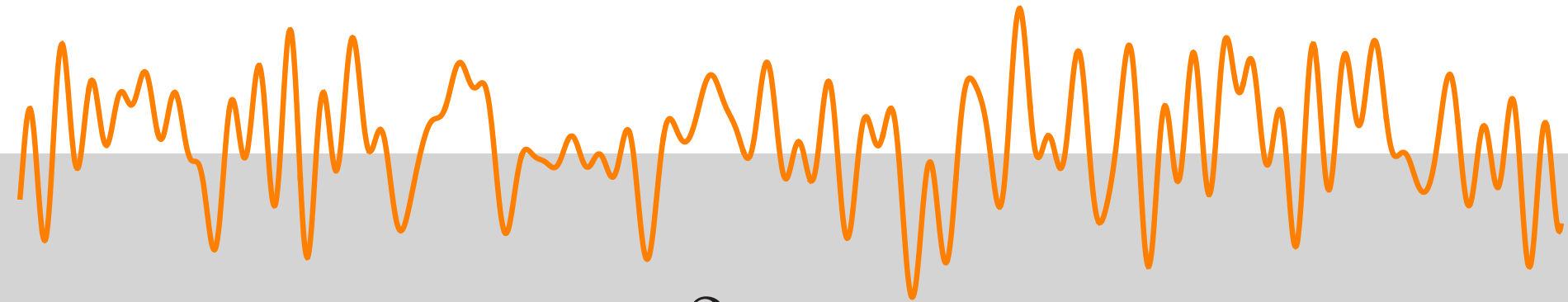
$$d\eta = a^{-2}dt$$



$$\langle \delta_{\mathbf{k}} \delta_{\mathbf{k}'} \rangle \propto P_\delta(k)$$



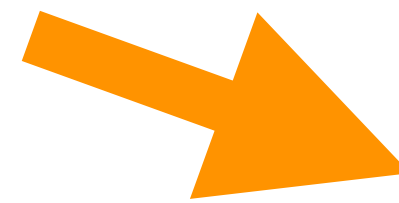
2-pt density correlation function = 4 pt field corr. func.



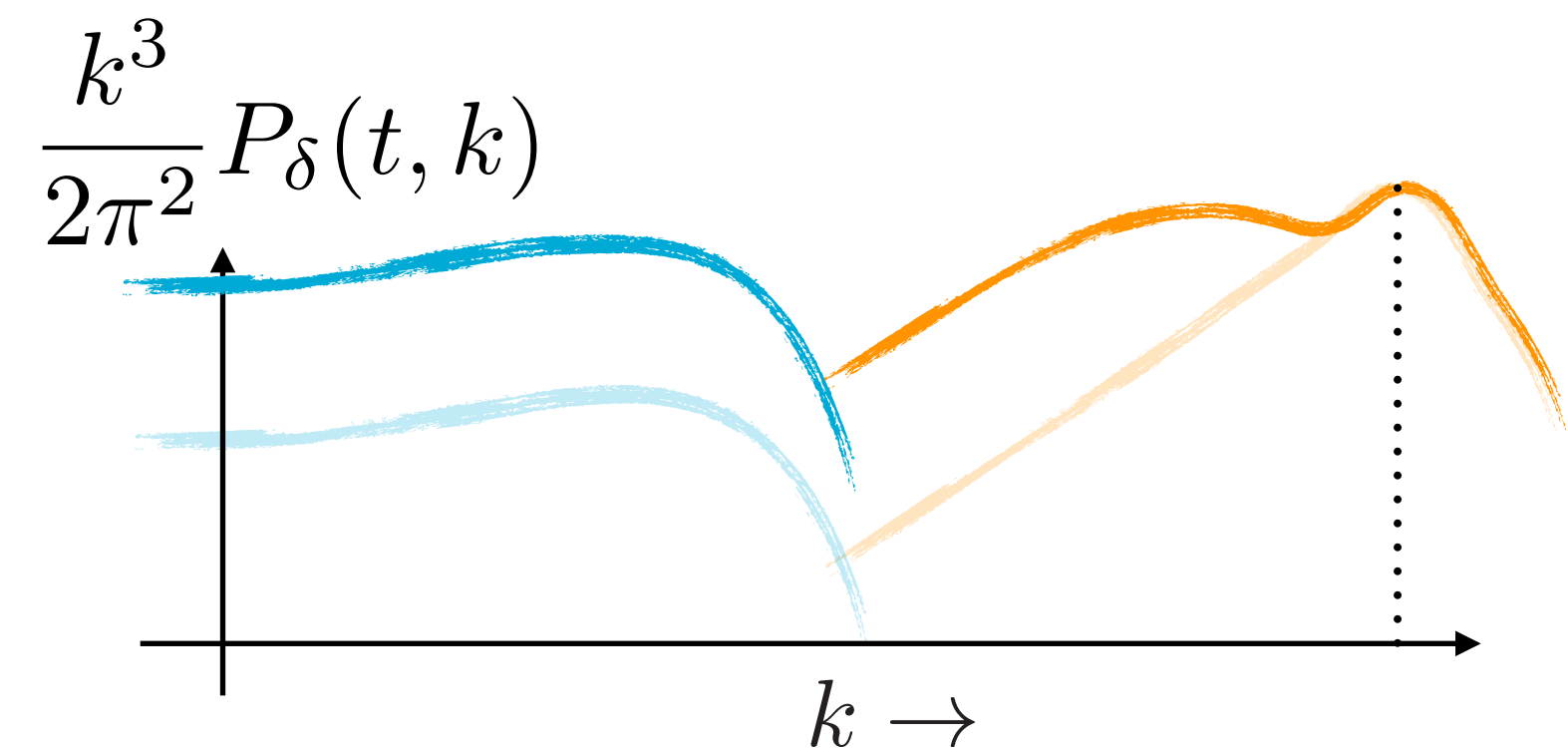
$$i\partial_\eta\psi = -\frac{\nabla^2}{2m}\psi + ma^2\Phi\psi$$

$$\nabla^2\Phi = \frac{4\pi G}{a}(\rho - \bar{\rho}), \quad \rho = m|\psi|^2$$

$$d\eta = a^{-2}dt$$



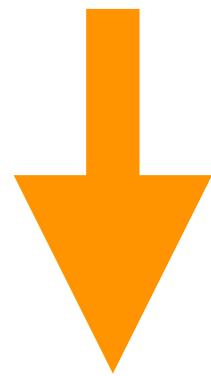
$$\langle\psi\psi\psi\psi\rangle \sim \langle\delta_{\mathbf{k}}\delta_{\mathbf{k}'}\rangle \propto P_\delta(k)$$



field eq. to eqns. for distribution (correlation) functions

$$i\partial_\eta \psi_{\mathbf{q}} = \frac{q^2}{2m} \psi_{\mathbf{q}} + ma^2 \int_{\mathbf{p}} \Phi_{\mathbf{p}} \psi_{\mathbf{q}-\mathbf{p}}$$

$$\Phi_{\mathbf{q}} = -\frac{4\pi G}{aq^2} \rho_{\mathbf{q}}$$



$$(2\pi)^3 \delta_D(\mathbf{k} + \mathbf{k}') \partial_\eta f(p) = \frac{3\bar{H}_0^2}{2} a \int_{\mathbf{k}, \mathbf{q}} \Delta_{\mathbf{k}} g_{\mathbf{k}\mathbf{k}'}(\mathbf{q}, \mathbf{p}) \approx 0$$

$$\begin{aligned} \partial_\eta g_{\mathbf{k}\mathbf{k}'}(\mathbf{p}, \mathbf{p}') + i \left(\frac{\mathbf{p} \cdot \mathbf{k}}{m} + \frac{\mathbf{p}' \cdot \mathbf{k}'}{m} \right) g_{\mathbf{k}\mathbf{k}'}(\mathbf{p}, \mathbf{p}') - \frac{3}{2} \bar{H}_0^2 a \left[\Delta_{\mathbf{k}} f(p) \int_{\mathbf{l}} g_{\mathbf{k}\mathbf{k}'}(\mathbf{l}, \mathbf{p}') + \Delta_{\mathbf{k}'} f(p') \int_{\mathbf{l}} g_{\mathbf{k}'\mathbf{k}}(\mathbf{l}, \mathbf{p}) \right] \\ \approx \frac{3}{2} \bar{H}_0^2 a [\Delta_{\mathbf{k}} f(p) f(|\mathbf{p}' + \mathbf{k}/2|) f(|\mathbf{p}' - \mathbf{k}'/2|) + (1 \leftrightarrow 2)] (2\pi)^3 \delta_D(\mathbf{k} + \mathbf{k}'). \end{aligned}$$

$$\mathcal{L}\psi = 0 \Rightarrow \begin{aligned} \mathcal{L}_1 f &= S[g] \\ \mathcal{L}_2 g &\approx S[f] \end{aligned}$$

similar to truncated BBGKY

ignore connected 3~pt

power spectrum evolution

$$P_\delta(\eta, k) = P_\delta(\eta_0, k) + 3\bar{H}_0^2 \int_{\eta_0}^{\eta} d\eta' a(\eta') T_k^{(b)}(\eta, \eta') T_k^{(c)}(\eta, \eta')$$

$$T_k^{(i)}(\eta, \eta_0) = T_k^{\text{fs}(i)}(\eta, \eta_0) + \frac{3\bar{H}_0^2}{2} \int_{\eta_0}^{\eta} d\eta' a(\eta') T_k^{\text{fs}(b)}(\eta, \eta') T_k^{(i)}(\eta', \eta_0) \quad i = \text{b, c}$$

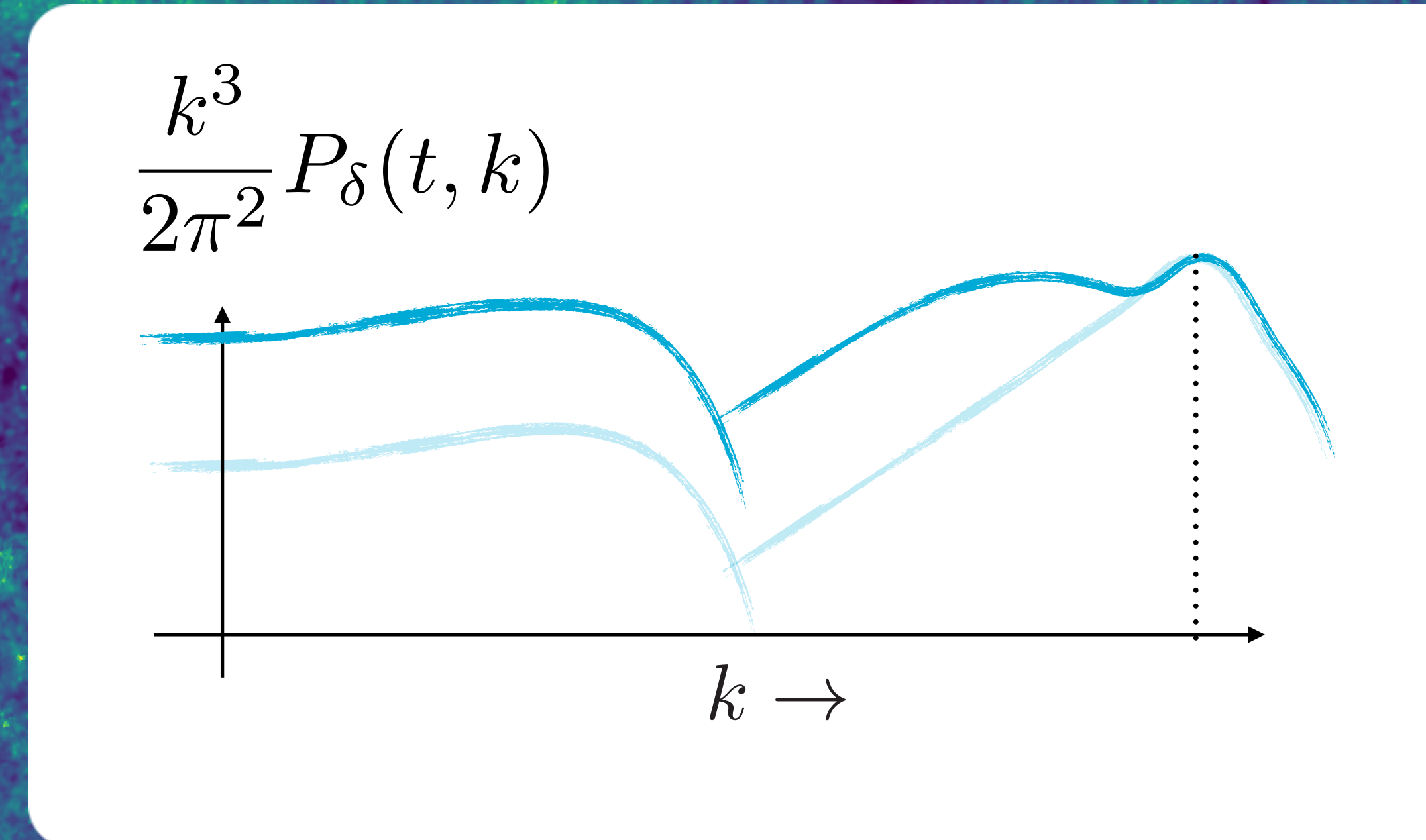
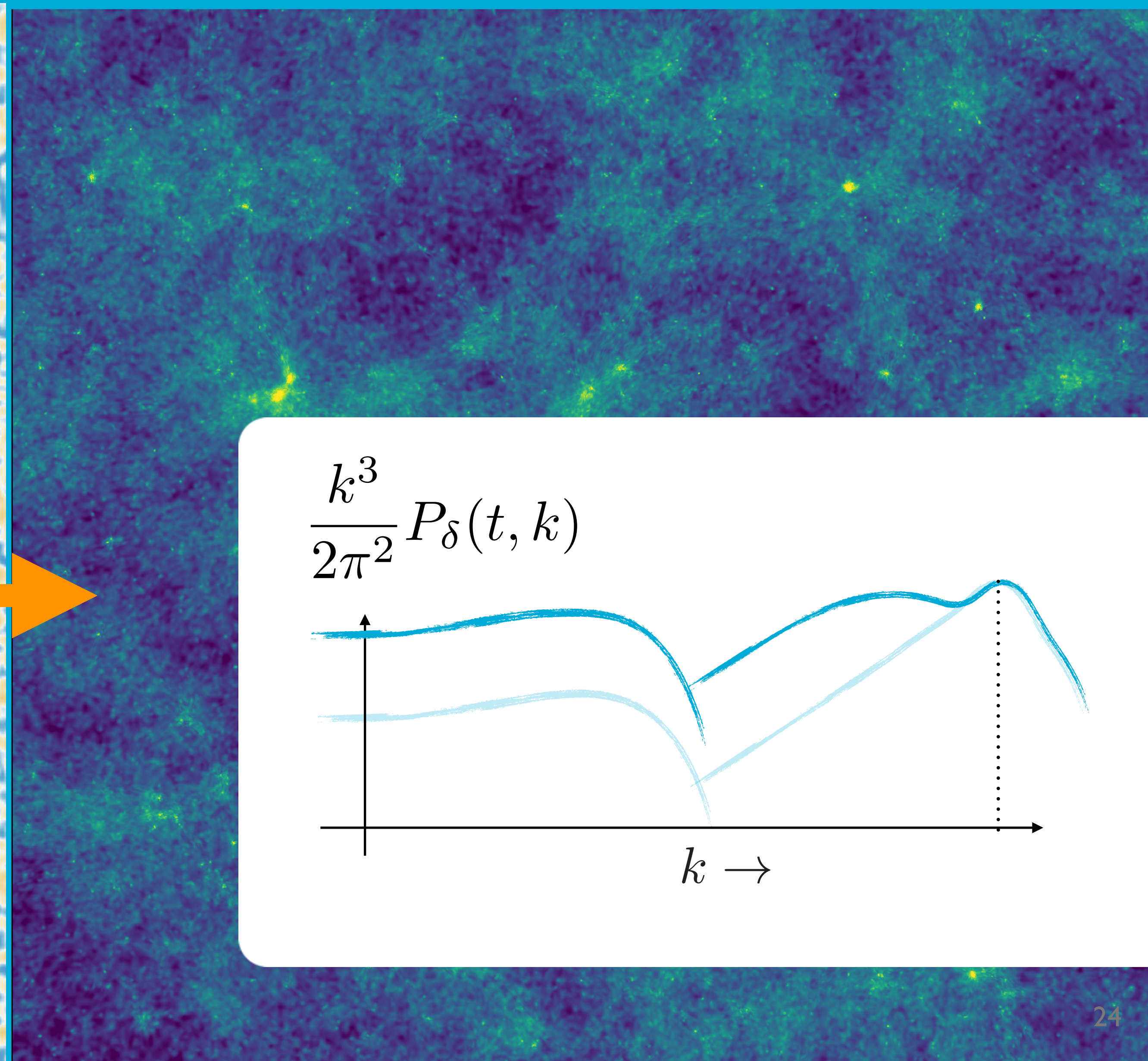
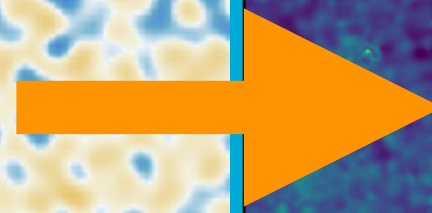
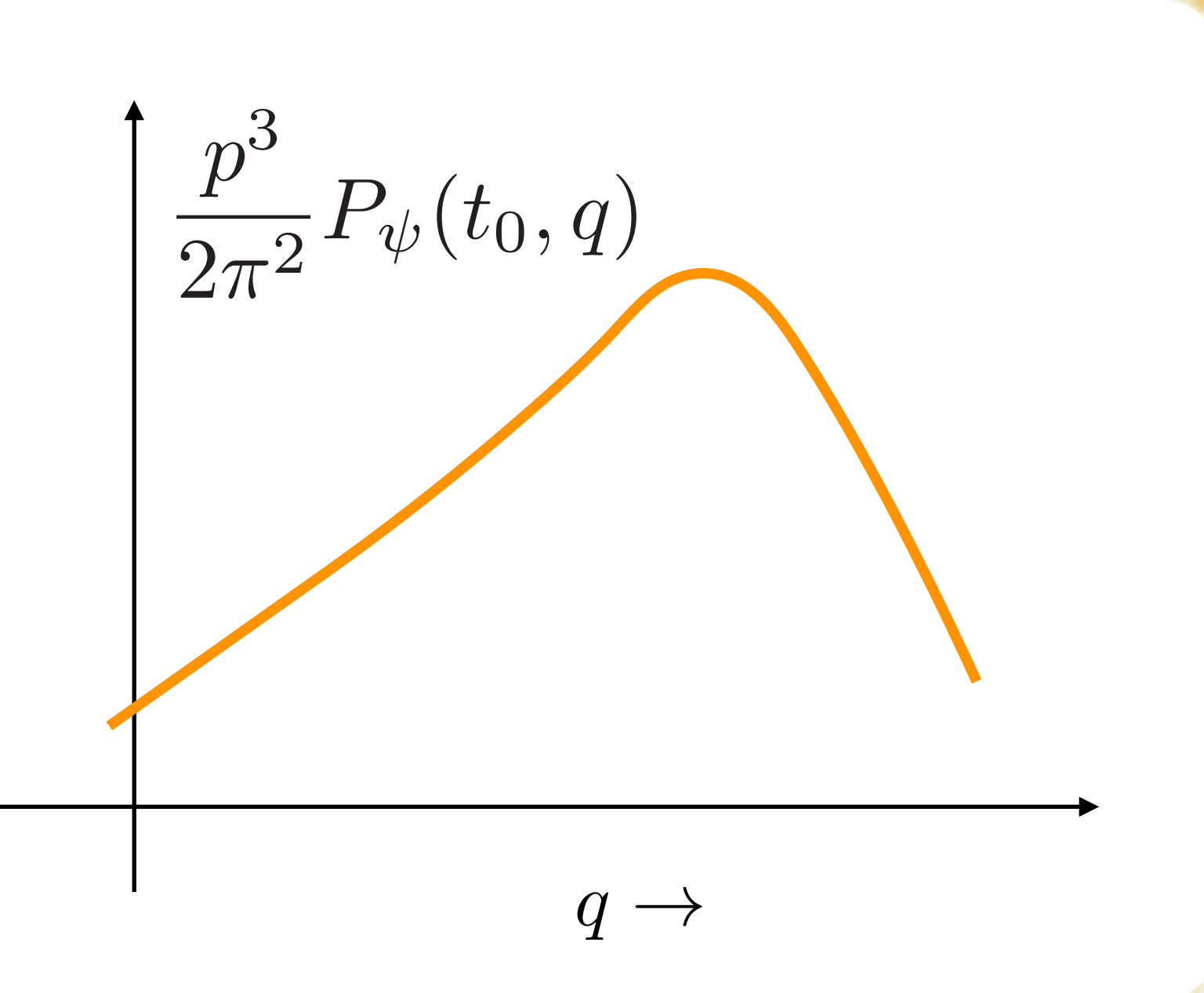
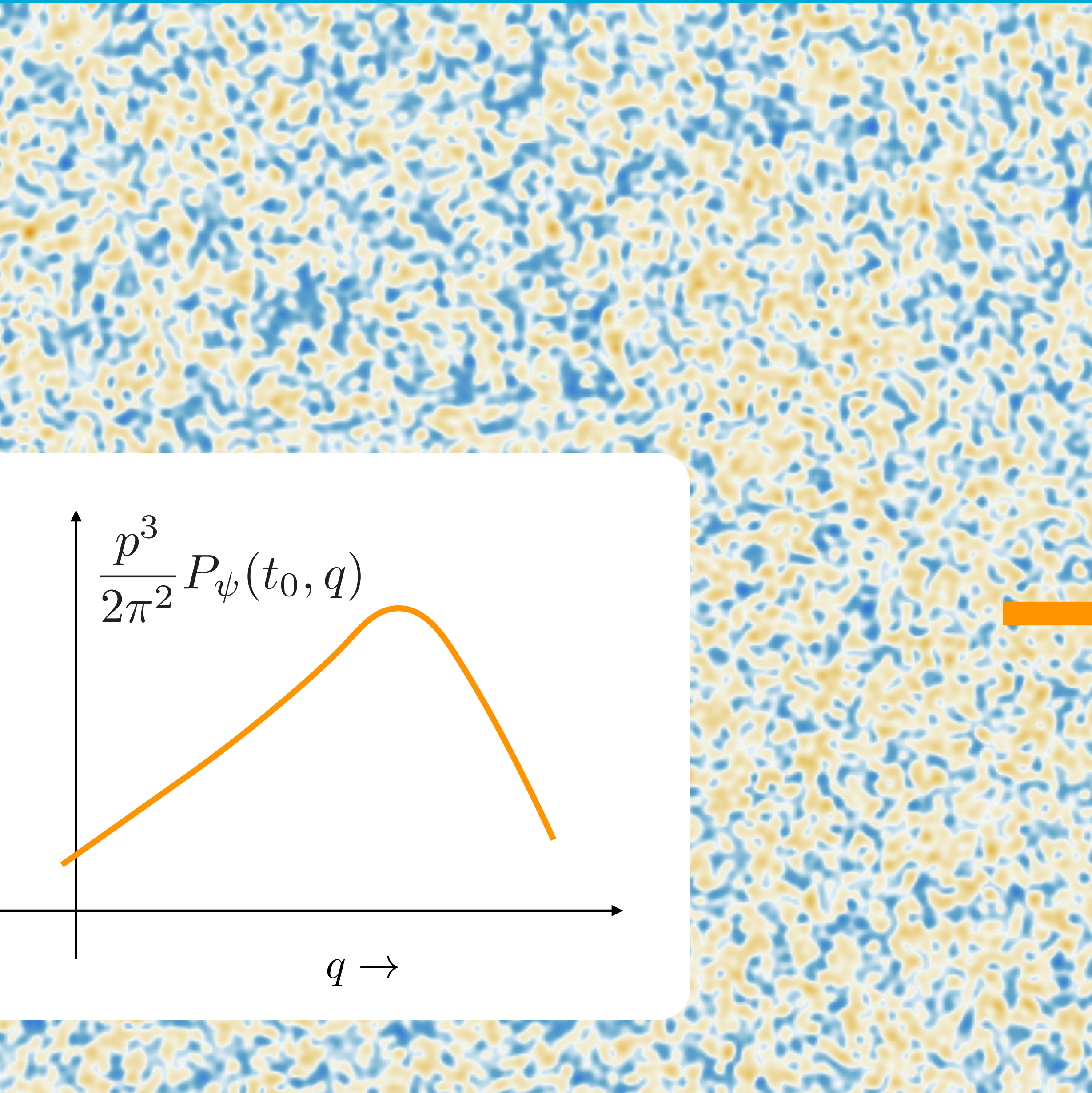
pure free-streaming gravitational clustering (with free-streaming)

$$T_k^{\text{fs}(b)}(\eta, \eta') = \frac{2m}{k^2} \sin \left[\frac{k^2}{2m} (\eta - \eta') \right] \int_{\mathbf{p}} f(p) e^{-i \frac{\mathbf{p} \cdot \mathbf{k}}{m} (\eta - \eta')},$$

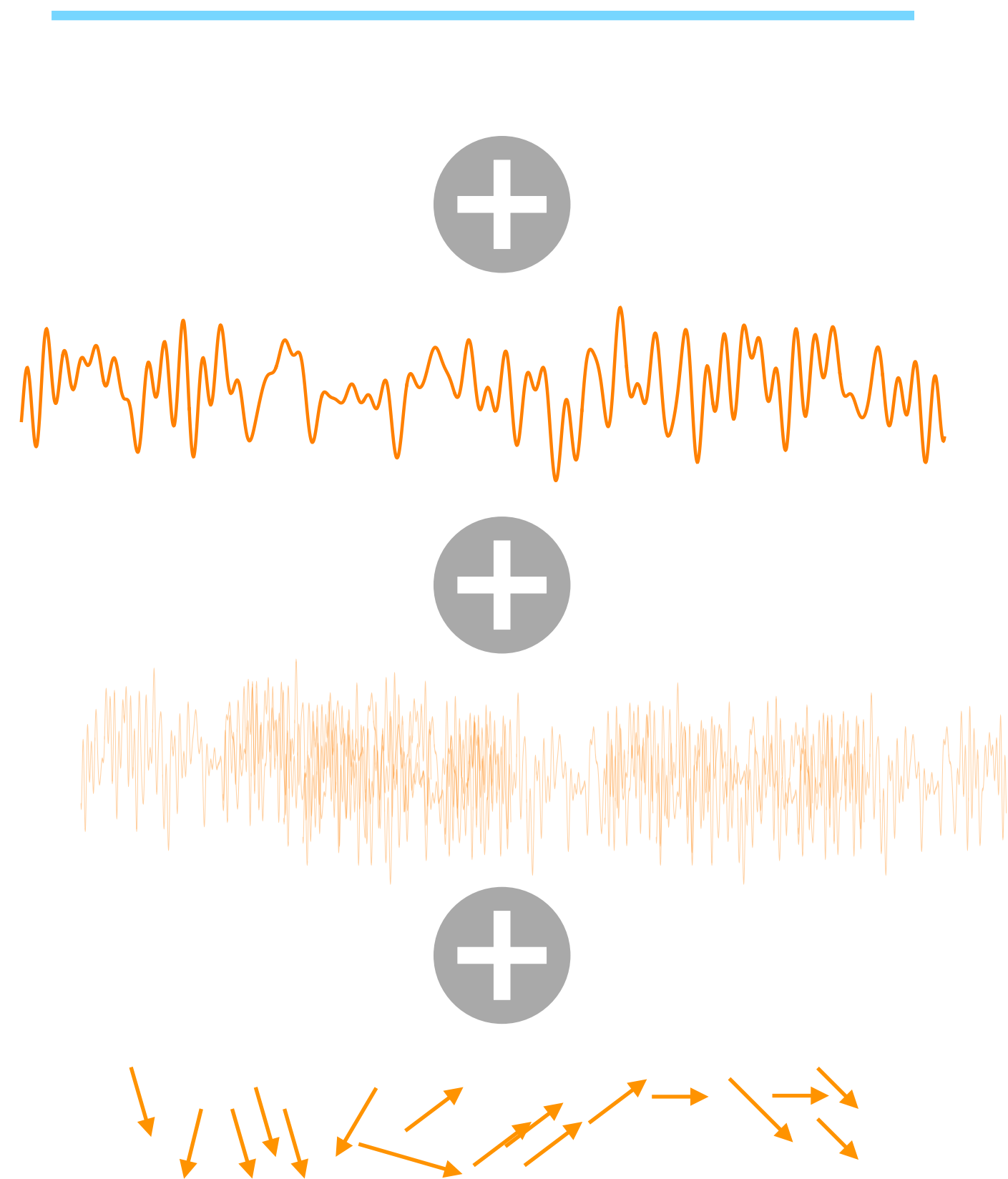
$$T_k^{\text{fs}(c)}(\eta, \eta') = \int_{\mathbf{p}} f(|\mathbf{p} + \mathbf{k}/2|) f(|\mathbf{p} - \mathbf{k}/2|) e^{-i \frac{\mathbf{p} \cdot \mathbf{k}}{m} (\eta - \eta')}$$

$$\bar{H}_0^2 = (8\pi G/3)\bar{\rho} \quad \Delta_{\mathbf{k}} f(p) \equiv (-im/k^2) [f(|\mathbf{p} + \mathbf{k}/2|) - f(|\mathbf{p} - \mathbf{k}/2|)] \quad 23$$

key input and output



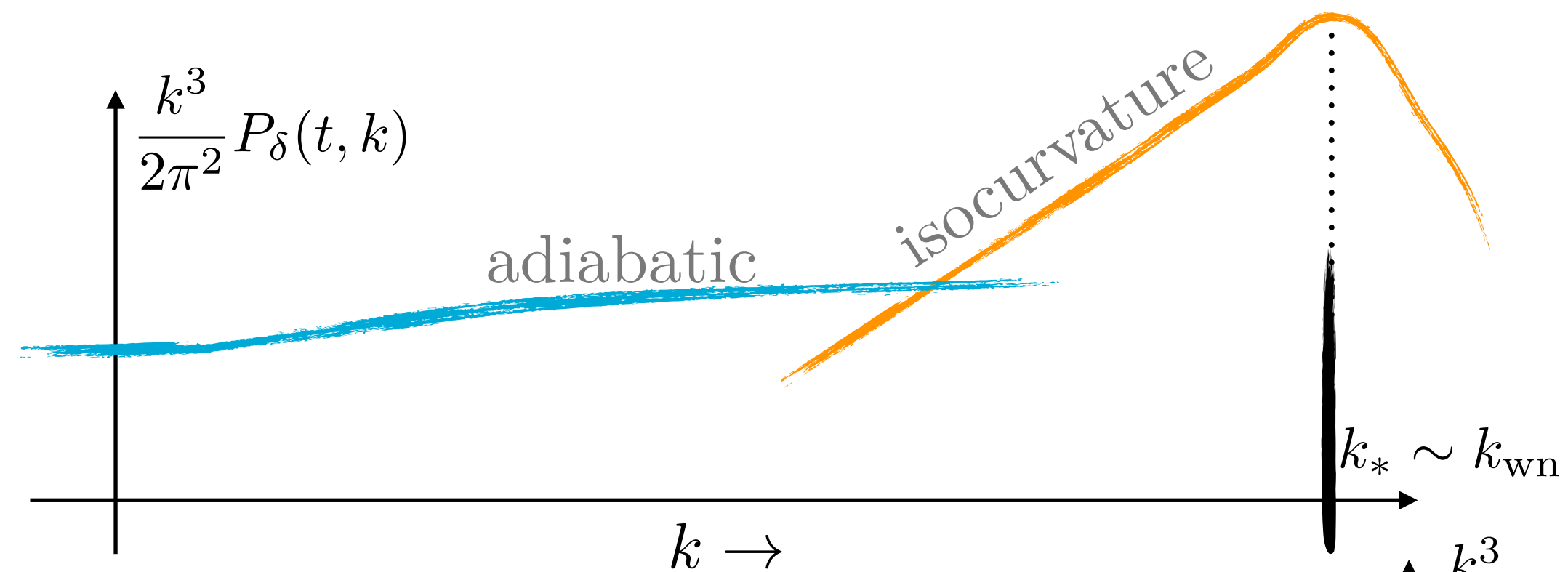
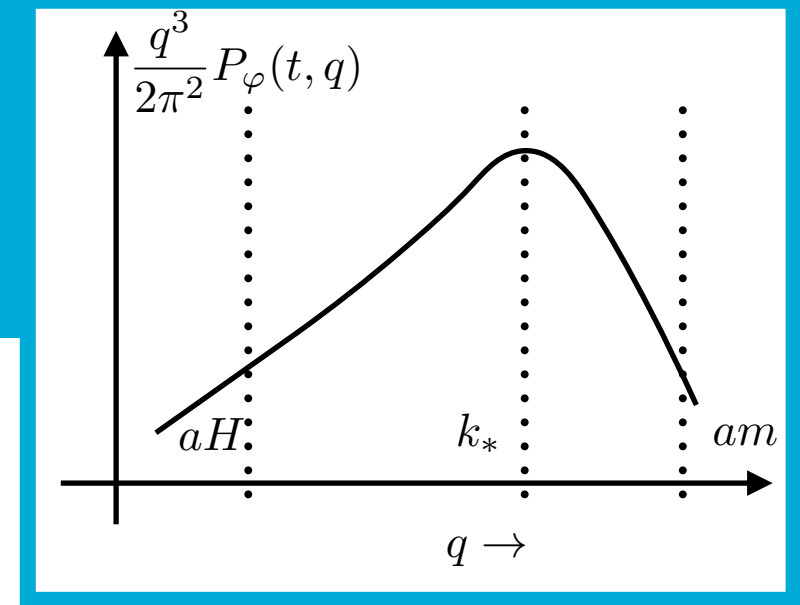
generality and computational efficiency



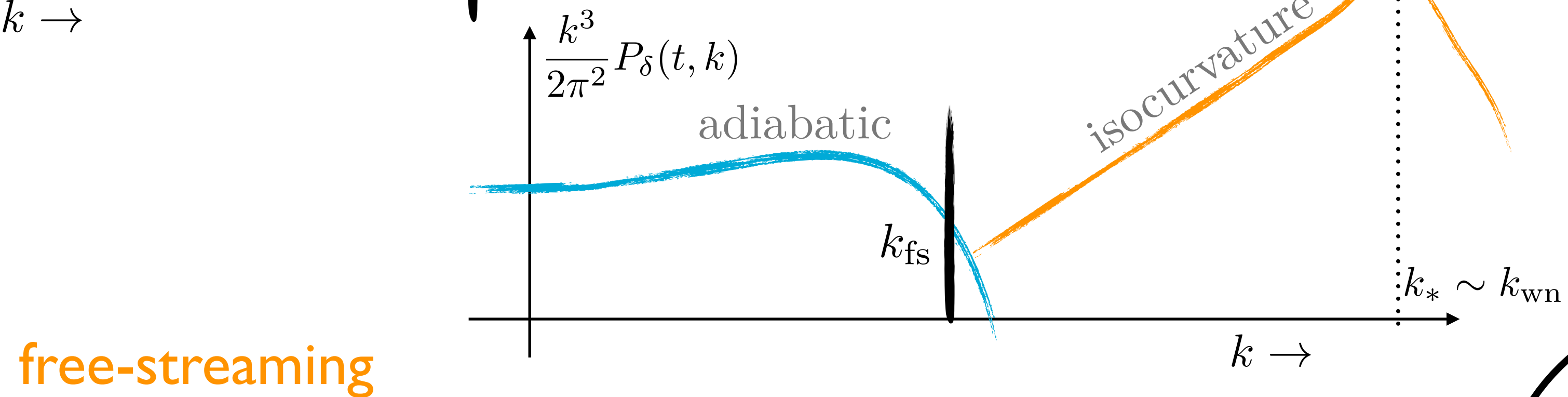
- works for fields with **arbitrary initial Field spectra**
- includes effects of **warmth and noisiness**
- works for **arbitrary # of components**
- requires solving **Volterra equations** (we have a **fast, efficient** code < few sec for PS)

<https://github.com/mustafaamin/warm-and-random>

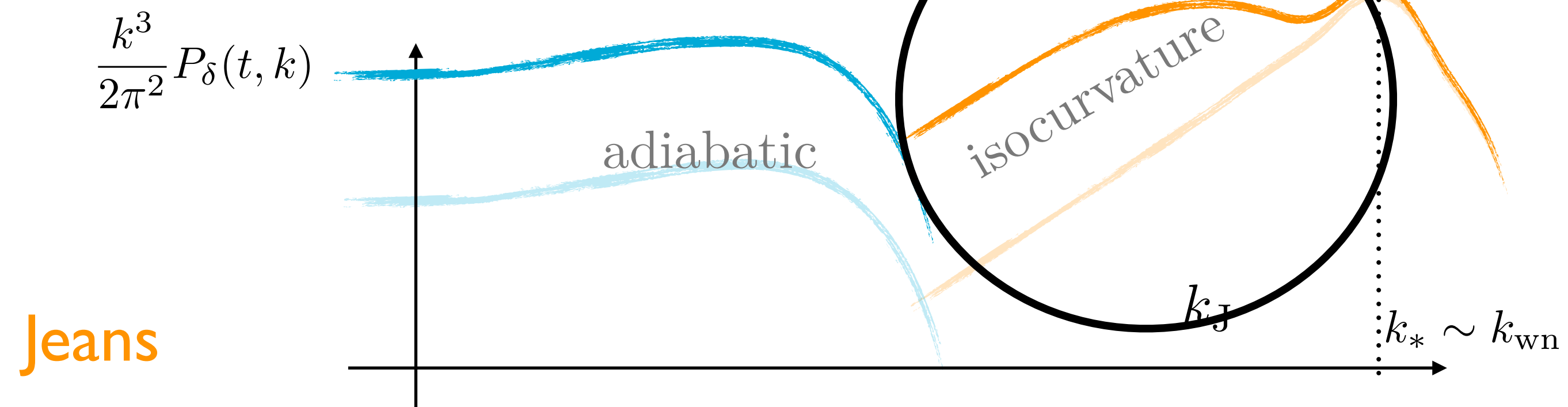
evolution of the matter power spectrum



white noise

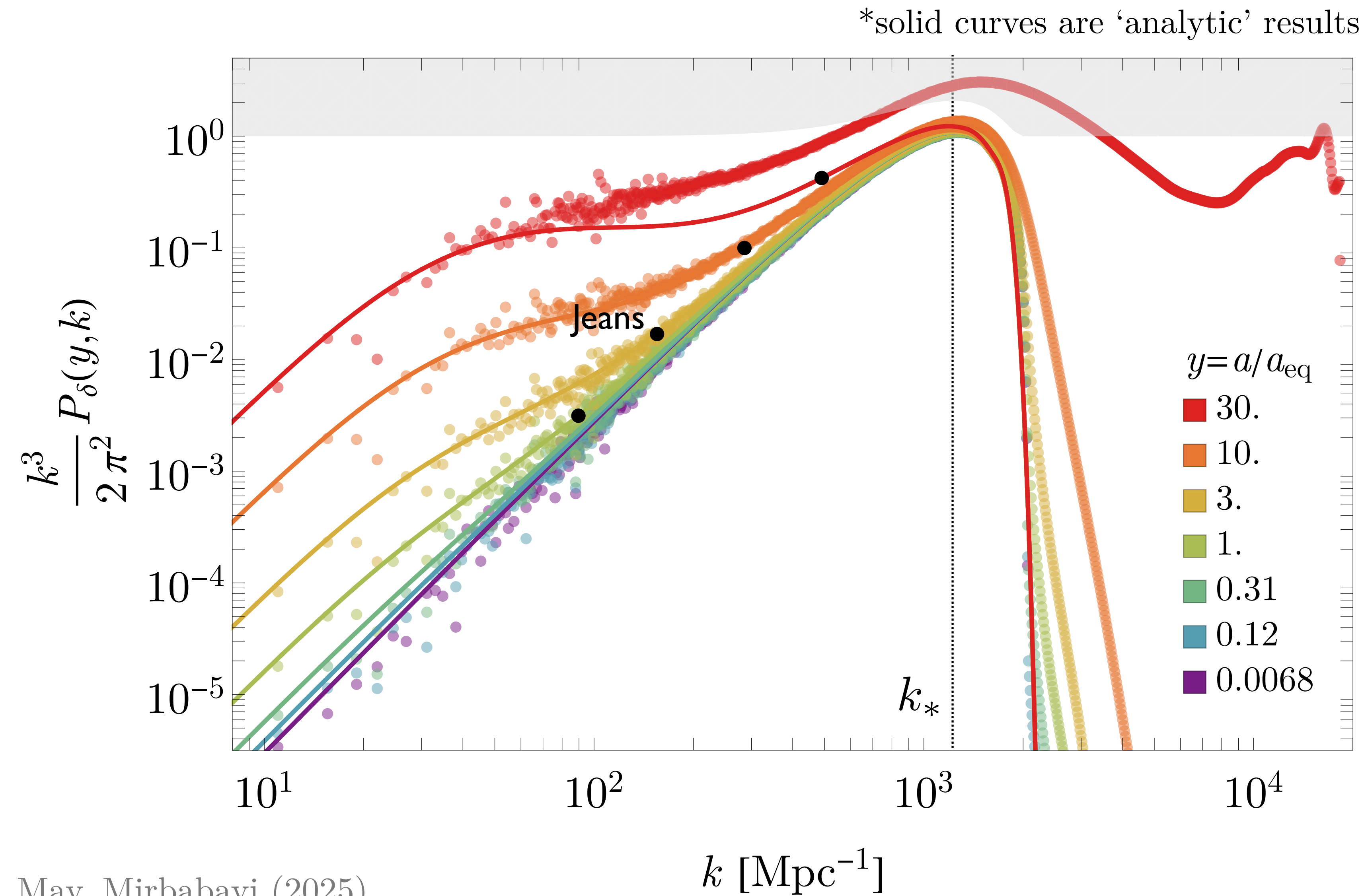


free-streaming



Jeans

comparison with simulations



$a/a_{\text{eq}} = 0.85$

$100 h^{-1} \text{ kpc}$

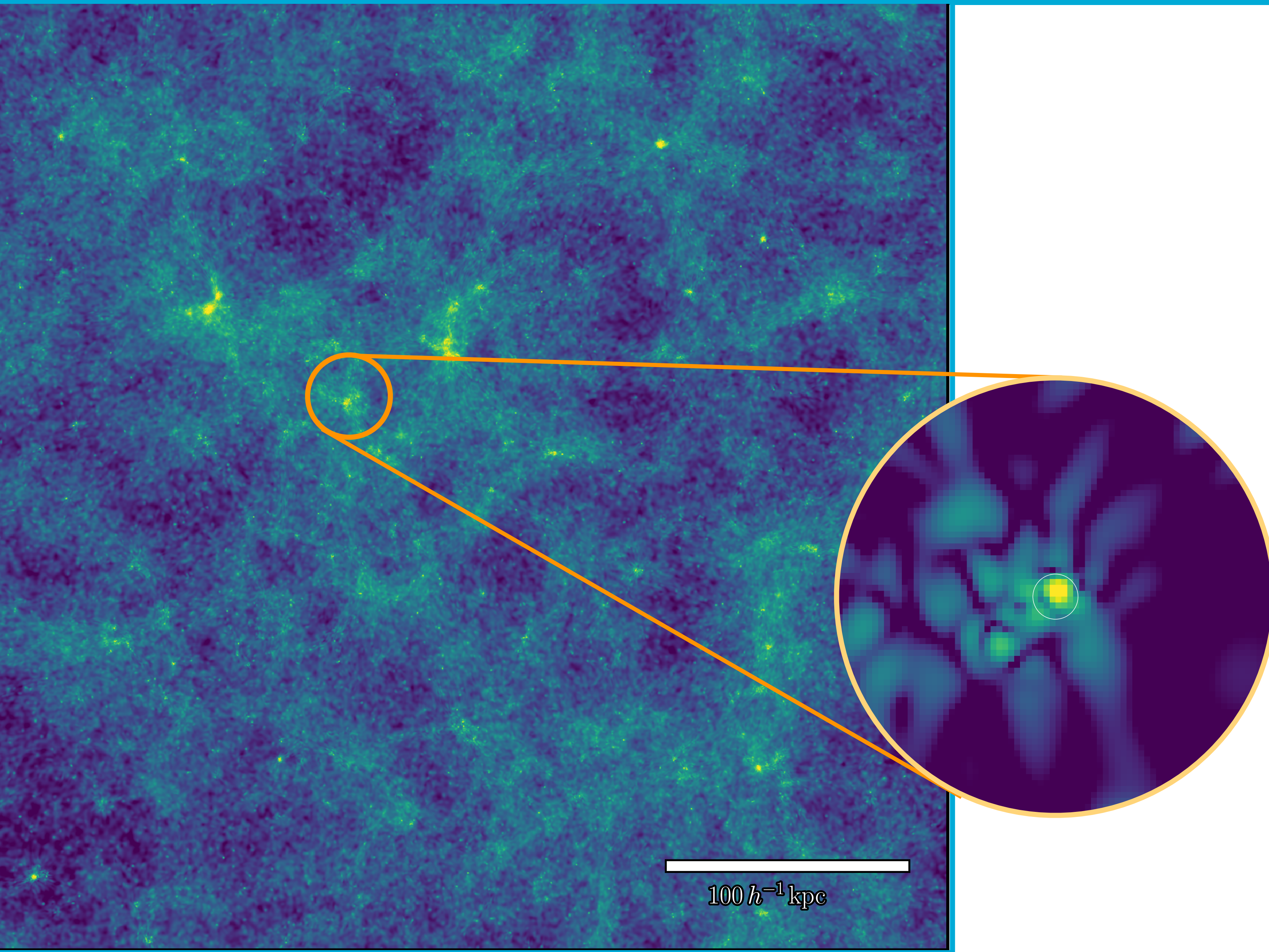
$a/a_{\text{eq}} = 5.6$

$100 h^{-1} \text{ kpc}$

$a/a_{\text{eq}} = 33.8$

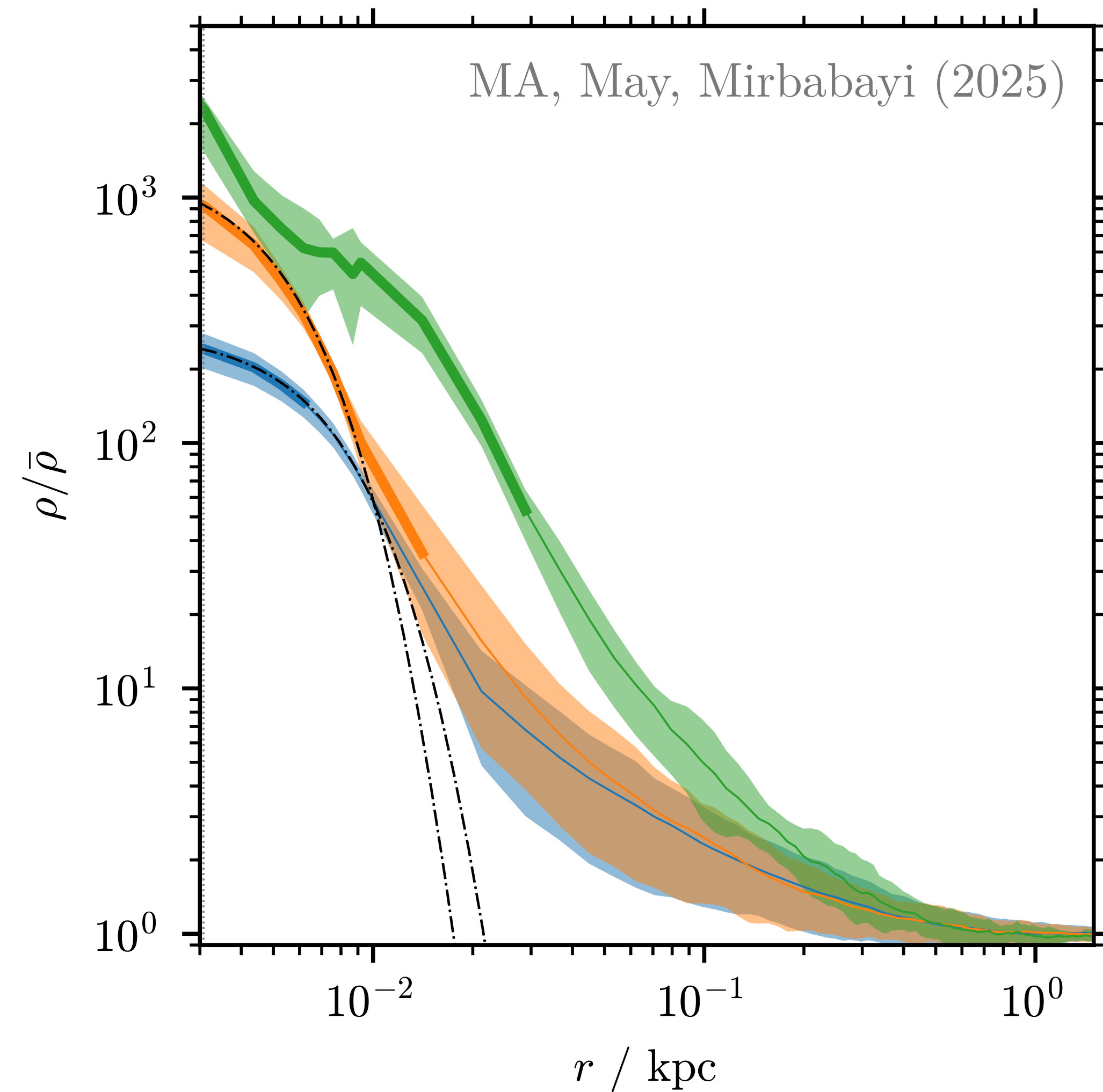
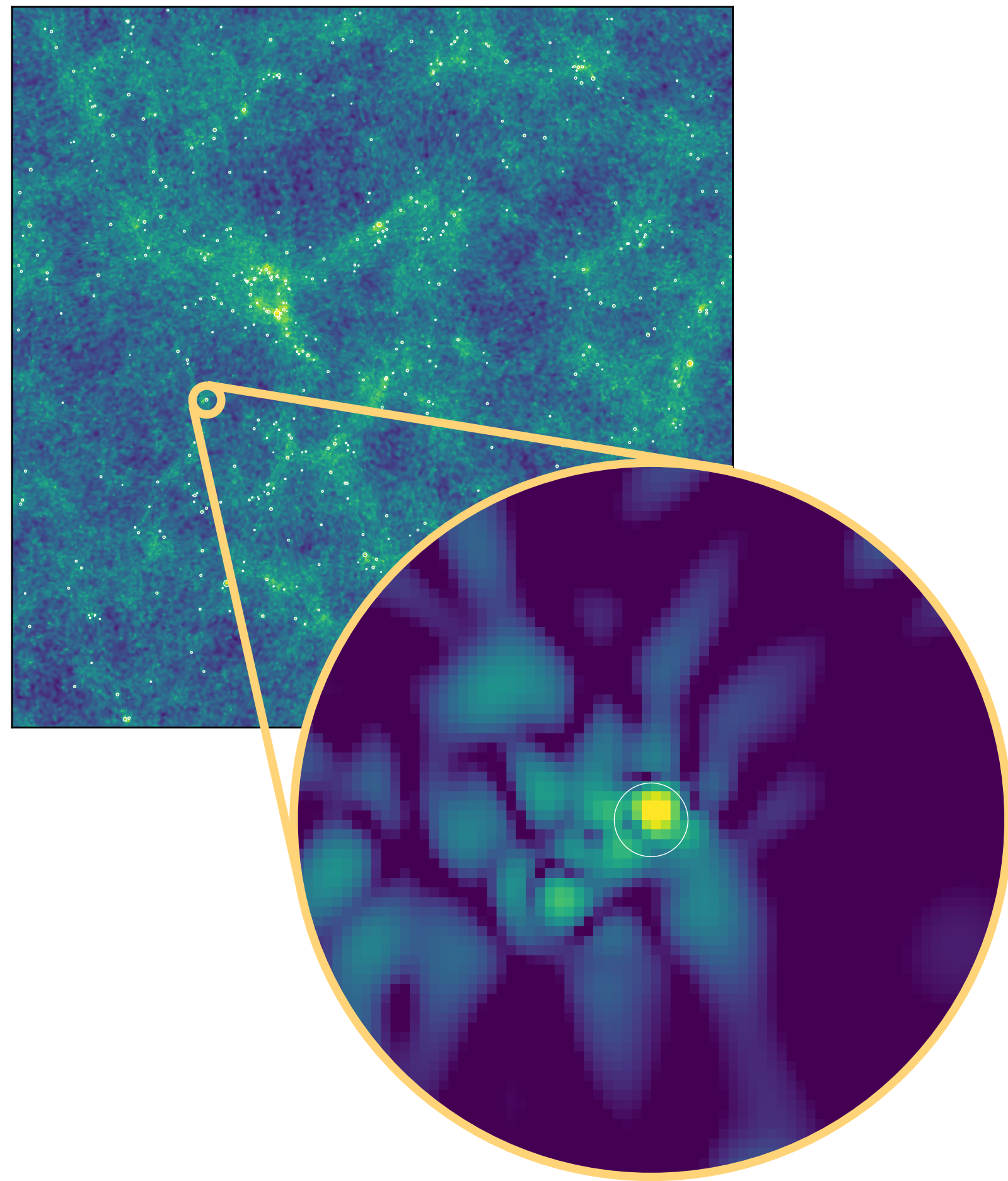
$100 h^{-1} \text{ kpc}$

early nonlinear structure formation



halos+solitons at $z > 100$

solitons — when do they form, in what fraction ?



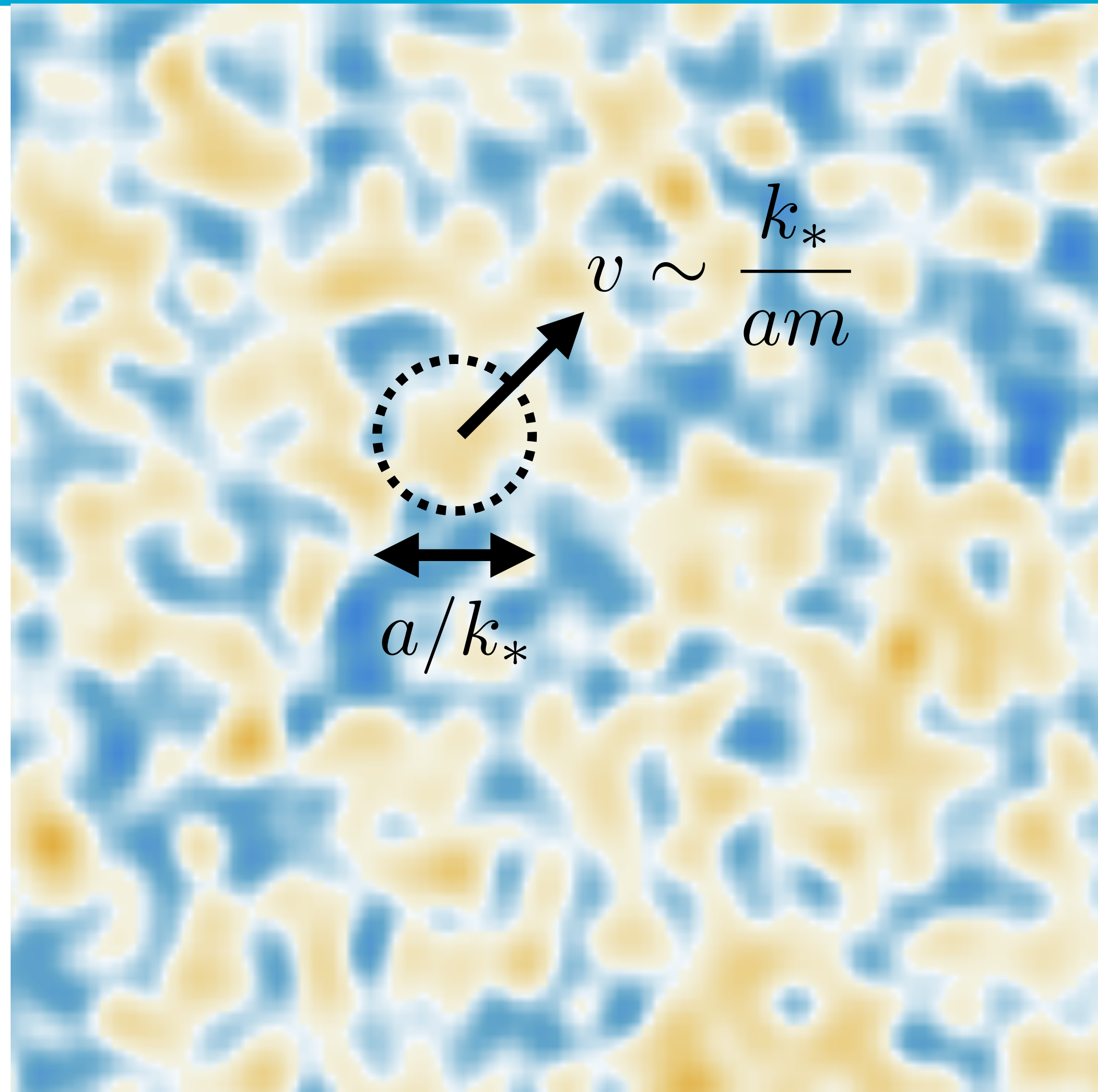
* can be order unity fraction of mass in collapsed objects, at least at early times. Also see Gorghetto, Hardy and Villadoro (2024)

generalizations — noise and warmth everywhere

quasi-particle picture — noise and warmth

$$M \sim \bar{\rho} \left(\frac{a}{k_*} \right)^3$$

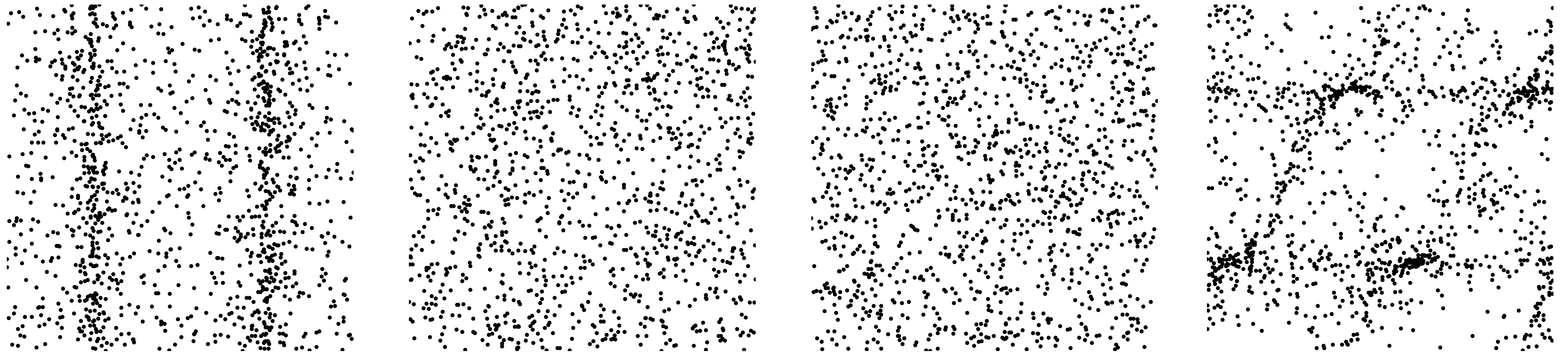
$$v \sim \frac{d\omega}{dk} \sim \frac{k_*}{am}$$



structure formation with finite number density and warmth

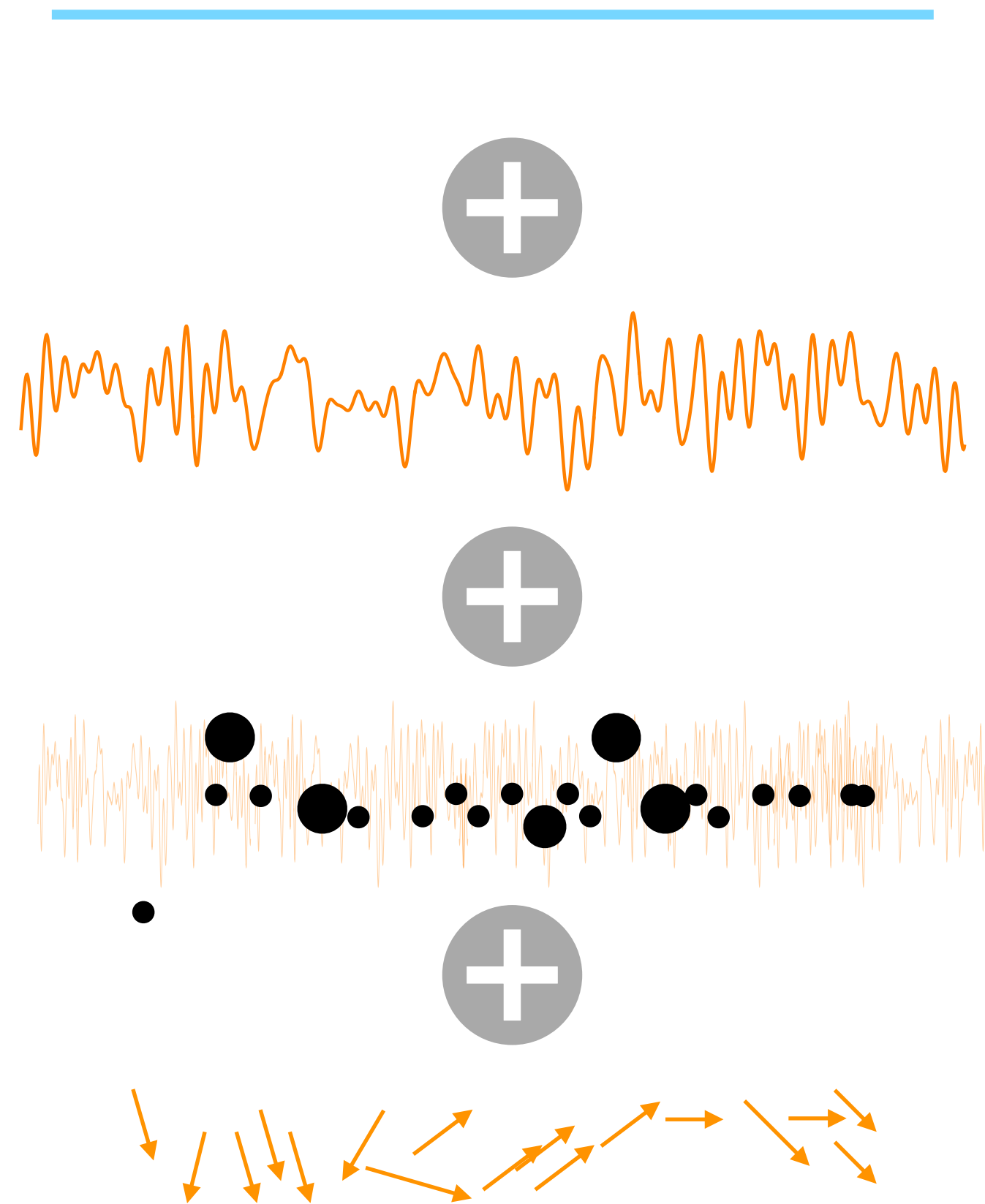
radiation domination

matter domination



(a) —free streaming→ (b) —free streaming→ (c) — grav. clustering → (d)

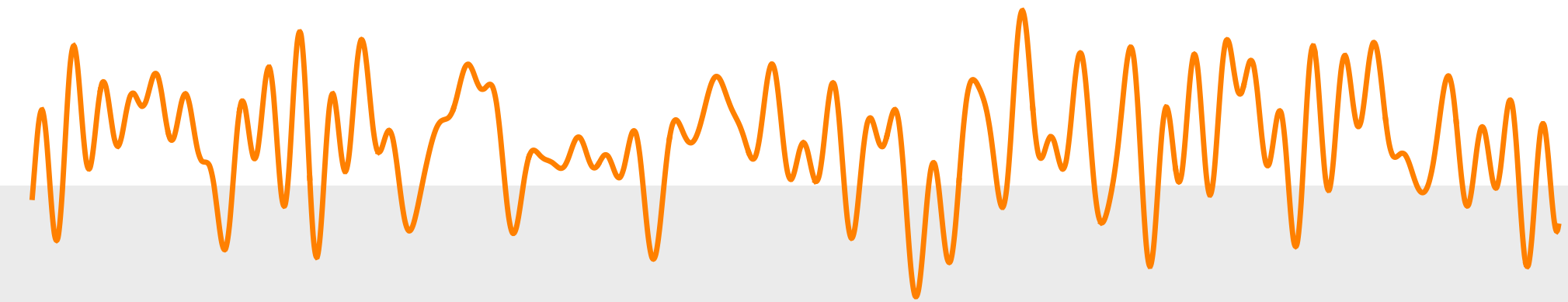
generality of the framework



- works for point particles, solitons and fields with **arbitrary initial spectra**
- includes effects of **noise and warmth** (not captured by CLASS, AxiCAMB etc.)
- works for **arbitrary # of components**
- can include non-gravitational interactions*

already being applied to MW Satellites populations (Nadler et. al (2026), subdominant ALPs (Gorghetto et. al, 2026)

summary



- **Warmth** related suppression features & **Poisson fluctuations** related enhancements are generic features in many cosmological fields
- Many **observational. probes** of these features in power spectra and nonlinear structures (eg. solitons): Ly α , high-redshift observables, galaxy clustering and early universe observables (g-waves?)
- A new **fast, theoretical and numerical framework** for computation exists (linear regime). Exciting signatures in the **nonlinear regime**.