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INFN
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PRIN
Progetti di Ricerca di Rilevante Interesse Nazionale

Seasons of Dark Matter Freeze-In Shaped by the Weather of the Early Universe

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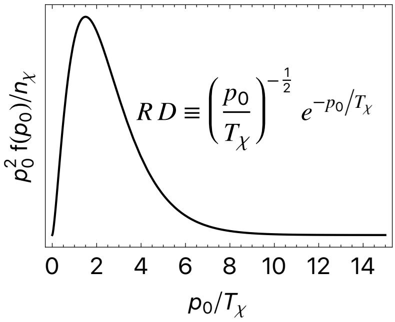
Identification of Dark Matter, Zaragoza, June 1 2026

F.D’Eramo, A. Lenoci, TS: *Phys.Rev.D 113 (2026) 8, 083502*

Framework: *Freeze-in* production of FIMPs

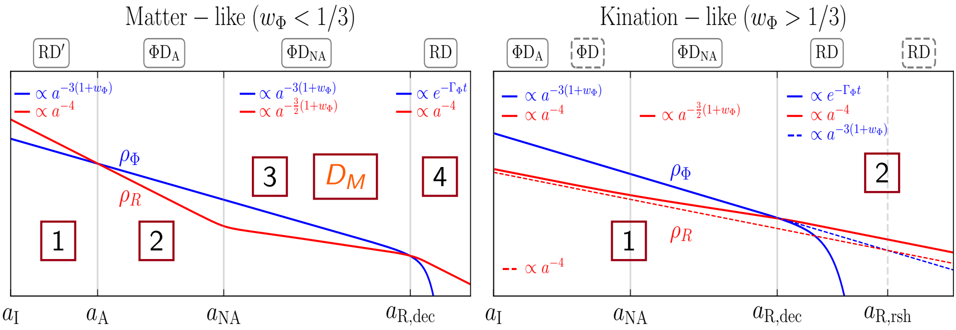
- Viable production mechanism @ small masses and tiny couplings
- Overdensities washed out by free streaming warm light particles
- Necessary to go beyond $n_\chi \to$ **PSD** [D'Eramo, Lenoci: 2012.01446]
- DM PSD affects the matter power spectrum

$B_1 \to B_2 \chi$



Explore beyond RD

$$p_\Phi = w_\Phi \, \rho_\Phi$$



1. Radiation domination
2. Φ-dom. (adiabatic)
3. Φ-dom (*NON*-adiabatic)
4. Standard RD

1. Φ-dominates until
   - Decays
   - Resdhifts away
2. Standard RD

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Focus on DM FI via $\mathcal{B}_1 \to \mathcal{B}_2\chi$: the Boltzmann Equation reads

$$\frac{df_\chi(A,p)}{d\log A} = \frac{\mathcal{C}\left(T(A),p(A)\right)}{H(A)}$$

— $\mathcal{C}\left(T(A),p(A)\right) \to$ microscopic physics input: decay
— Modify the cosmological background $\to H(A)$

$$H(A) \propto H_M A^m \qquad T(A) \propto M A^\ell \qquad q \equiv A \times p(A)/M$$

e.g. $(m,\ell) = (-3/2,-3/8)$ for Inflation, $(m,\ell) = (-2,-1)$ in RD.
If DM FI production occurs within *one* epoch

$$\boxed{\mathrm{f}_\chi(q,A_\mathrm{I},A_\mathrm{F}) \propto \int_{A_\mathrm{I}}^{A_\mathrm{F}} \frac{dA}{A^{m+1}} \frac{A^{\ell+2}}{q^2}\,\mathrm{Exp}\left[-qA^{-\ell-1} - \frac{1}{4qA^{\ell-1}}\right]}$$

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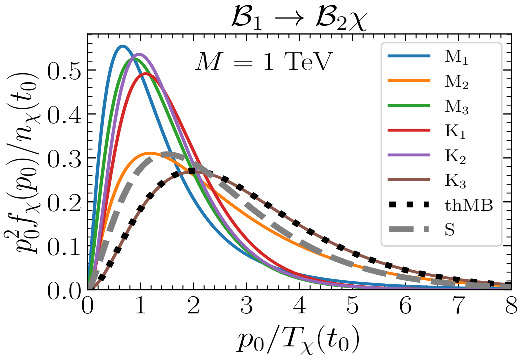

The PSD shape is fitted by

$$f_{\text{fit}}(P) \propto P^{\alpha}\ \mathrm{Exp}\left[-\beta\, P^{\gamma}\right]$$
$$P = p(t_0)/T_\chi$$

	S	M_1	M_2	M_3	K_1	K_2	K_3
α	-0.5	-0.4	-0.1	-0.5	0	0.5	0
β	1	3.1	2.7	1.7	1.9	3.1	1
γ	1	0.7	0.6	1	1	0.8	1

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We compute its second moment

$$\sigma_q = \sqrt{\frac{\int_0^\infty dq\ q^4 f_\chi(q)}{\int_0^\infty dq\ q^2 f_\chi(q)}} = \sqrt{\frac{\int_{A_\mathrm{I}}^{A_\mathrm{F}} dA\ A^{\ell-m+4} K_3(A^{-\ell})}{4\int_{A_\mathrm{I}}^{A_\mathrm{F}} dA\ A^{\ell-m+2} K_1(A^{-\ell})}}$$

This relates to the r.m.s velocity

$$W_\chi(t_0) \propto \Sigma \times \frac{T_\chi}{m_\chi} \qquad\qquad T_\chi(t_0) = T_0 \left(\frac{g_{\star s}(T_0)}{g_{\star s}(M)}\right)^{1/3}$$

The physical quantity is

$$\boxed{\Sigma = \sigma_q \times D_M^{-1/3}}$$

We impose WDM upper bounds from LSS $W_\chi(t_0) < W_{\mathrm{WDM}}(t_0)$
[D'Eramo et al:2506.13864]

$$m_\chi^{\min} = \frac{\sqrt{\langle p_0^2 \rangle}}{W_{\mathrm{WDM}}(t_0)} = \frac{\Sigma\, T_\chi(t_0)}{W_{\mathrm{WDM}}(t_0)}$$
$$= 20\ \mathrm{keV} \left(\frac{m_{\mathrm{WDM}}}{6\ \mathrm{keV}}\right)^{4/3} \left(\frac{\Sigma}{3}\right) \left(\frac{104.5}{g_\star(M)}\right)^{1/3}$$

The Dilution factor accounts for entropy injection after production

$$\boxed{\mathrm{D}_M = \frac{S(A_{\mathrm{R}})}{S(A_M)}}$$

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Exemplary cases: $M = 1\,\mathrm{TeV}$

	Φ_I	T_I	$\Gamma_\Phi[\mathrm{GeV}]$	phase	c_relic	σ_q	D_M	Σ	Σ_th
S	0	10^{12}	0	RD	1.4×10^{-8}	3	1	3	3
M_1	0.2×10^{-4}	2×10^9	2×10^{-14}	$\Phi\mathrm{D}_\mathrm{A}$	5.6×10^{-8}	4.3	11	1.9	1.2
M_2	10^{-3}	10^{15}	1.2×10^{-14}	$\Phi\mathrm{D}_\mathrm{NA}$	3.1×10^{-7}	300	10^6	3.0	3.8
M_3	2.0×10^{-7}	1.5×10^9	5.5×10^{-18}	RD'	3.2×10^{-8}	3.0	5.0	1.7	1.6
K_1	10^{15}	10^5	0.2×10^{-19}	$\Phi\mathrm{D}_\mathrm{A}$	4.4×10^{-6}	3.5	6.1	1.9	1.9
K_2	10^{15}	10^5	10^{-16}	$\Phi\mathrm{D}_\mathrm{NA}$	1.8×10^{-6}	8.7	110	1.8	1.8
K_3	10^{15}	5×10^6	0	$\Phi\mathrm{D}$	2.5×10^{-7}	3.5	1	3.5	3.5

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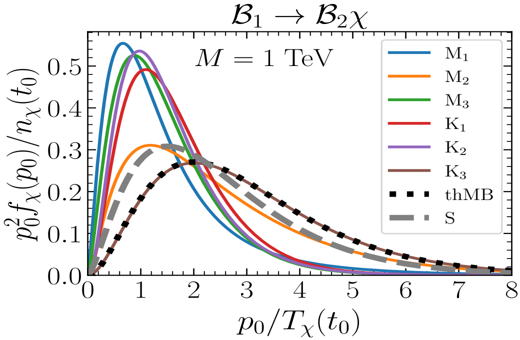

	S	M_1	M_2	M_3	K_1	K_2	K_3
numerical $m_\chi^{\min}$ [keV]	20	13	20	12	13	12	23
analytical $m_\chi^{\min}$ [keV]	20	8	25	12	13	12	23

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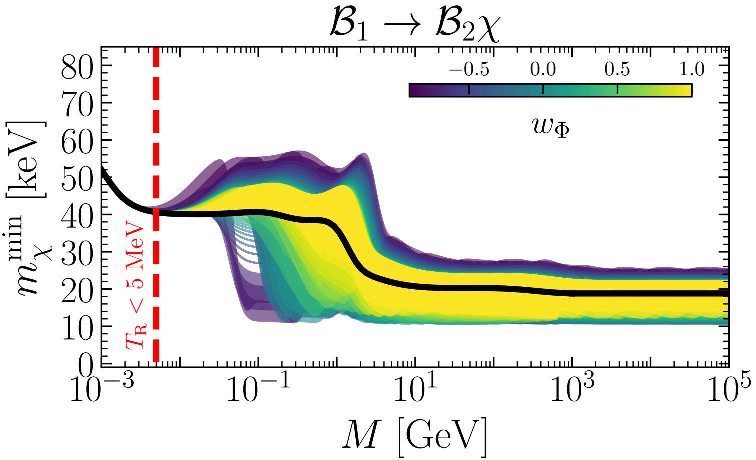

Conclusions and Outlook

Summary

- Parametrized general *modified cosmological backgrounds* (w_Φ)
- Solved BE in *phase space* $\rightarrow$ **PSD**
- Extracted *lower mass bounds* from warmness/structures
- Novel results: bounds can be relaxed/tightened by $\approx \times 2$
- Parameters space available for other experimental searches

Future developments

- Scatterings $\mathcal{B}_1 \mathcal{B}_2 \rightarrow \mathcal{B}_3\, \chi$
- Investigate IR/UV dynamics interplay
- Microscopical models for modified cosmologies

Thank you!

BACKUP MATERIAL

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Introduce an exotic species Φ behaving *macroscopically* as a fluid

$$p_\Phi = w_\Phi \, \rho_\Phi$$

The cosmological background evolves accordingly (in suitable *dimensionless* variables

$$\begin{cases} \dfrac{d\Phi}{d\log A} = -\dfrac{\Gamma_\Phi}{H}\,\Phi \\[2ex] \dfrac{dR}{d\log A} = (1-3w_R)\,R + A^{(1-3w_\Phi)}\,\dfrac{\Gamma_\Phi}{H}\,\Phi \\[2ex] H(A) = \dfrac{\sqrt{\Phi\,A^{(1-3w_\Phi)}+R}}{\sqrt{3}\,A^2}\,\dfrac{T_\mathrm{I}^2}{M_\mathrm{Pl}} \end{cases}$$

where

$$\begin{cases} A \equiv a/a_I \\ \Phi(A) \equiv \dfrac{\rho_\Phi(A)}{T_\mathrm{I}^4}A^{3(1+w_\Phi)} \\ R(A) \equiv \dfrac{\rho_R(A)}{T_\mathrm{I}^4}A^4 \end{cases}$$

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We can analytically get some intuition and results

$$w_\Phi \leq 1/3$$

$$A_{\mathrm{A}} = \left(\frac{R_{\mathrm{I}}}{\Phi_{\mathrm{I}}}\right)^{1/(1-3w_\Phi)}$$

$$A_{\mathrm{NA}} = \left[1 + \frac{5-3w_\Phi}{2}\frac{T_{\mathrm{I}}^2}{\sqrt{3}\Gamma_\Phi M_{\mathrm{Pl}}}\frac{R_{\mathrm{I}}}{\sqrt{\Phi_{\mathrm{I}}}}\right]^{2/(5-3w_\Phi)}$$

$$A_{\mathrm{R}} = \left[\frac{1}{3(1+w_\Phi)}\frac{\sqrt{3}\,\Gamma_\Phi\, M_{\mathrm{Pl}}}{T_{\mathrm{I}}^2\sqrt{\Phi_{\mathrm{I}}}}\right]^{-2/[3(1+w_\Phi)]}$$

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$$A_{\mathrm{R}} = \left[\frac{1}{3(1+w_\Phi)}\frac{\sqrt{3}\Gamma_\Phi M_{\mathrm{Pl}}}{T_{\mathrm{I}}^2\sqrt{\Phi_{\mathrm{I}}}}\right]^{-2/[3(1+w_\Phi)]}$$

$w_\Phi > 1/3$

$$A_{\mathrm{R,dec}} = \left[\frac{1}{3(1+w_\Phi)}\frac{\sqrt{3}\Gamma_\Phi M_{\mathrm{Pl}}}{T_{\mathrm{I}}^2\sqrt{\Phi_{\mathrm{I}}}}\right]^{-2/[3(1+w_\Phi)]}$$

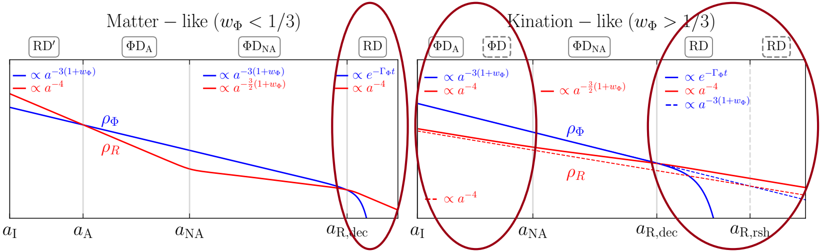
$$A_{\mathrm{R,rsh}} = \left(\frac{\Phi_{\mathrm{I}}}{R_{\mathrm{I}}}\right)^{1/(3w_\Phi-1)}$$

In general

$$H \propto H_M \widetilde{A}^m \qquad T \propto M \times \widetilde{A}^\ell \qquad \text{with:} \quad \widetilde{A} \equiv \frac{A}{A_M}$$

Phase Space Solution Analytics: Adiabatic Stages

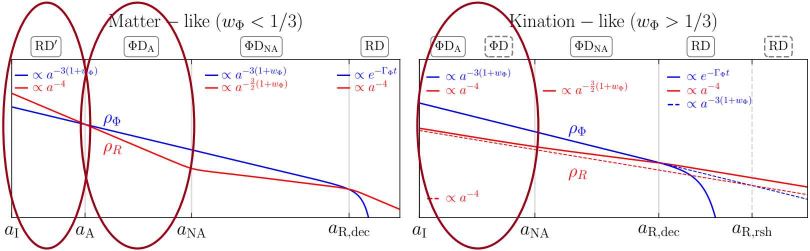
Exact analytic solutions can be found for specific cases: (RD, ΦD)



$$D_M = 1 \implies (\widetilde{A}_I \to 0,\ \widetilde{A}_F \to \infty)$$

Phase	$T \propto a^\ell$	$H \propto a^m$	$f_\chi(q)$	σ_q
RD	a^{-1}	a^{-2}	$q^{-1/2} e^{-q}$	$\sqrt{35}/2$
ΦD	a^{-4}	$a^{-\frac{3(1+w_\Phi)}{2}}$	$q^{\frac{3(w_\Phi-1)}{4}} e^{-q}$	$\sqrt{\dfrac{\Gamma\left(\frac{3w_\Phi+17}{4}\right)}{\Gamma\left(\frac{3w_\Phi+9}{4}\right)}}$

If a *NON*-adiabatic era follows DM production we need to account for dilution



and FI assumption consistency (watch out for couplings!)

$$Y_\chi(A) \ll Y_\chi^{eq}(A) \implies Y_\chi(A_0) D_M < Y_\chi^{eq}$$

$$1 \le D_M \lesssim \max\left[1, \frac{m_\chi}{2\ \mathrm{keV}}\right] \ \ldots$$

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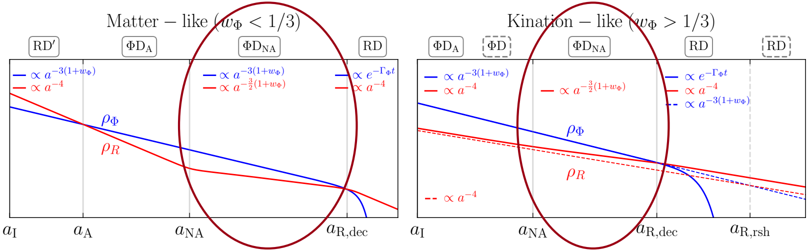
If FI happens during a *NON*-adiabatic phase physics (and calculation) gets more complicated

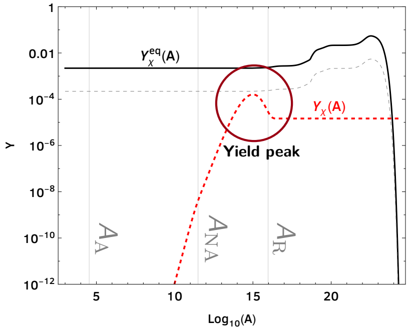


$$\widetilde{A}_F \nrightarrow \infty \qquad \text{but} \qquad \widetilde{A}_F \approx \widetilde{A}_\mathrm{R} = \left(\frac{M}{T_R}\right)^{-1/\ell}$$

$$\ell = -\frac{3}{8}(1 + w_\Phi) \qquad m = -\frac{3}{2}(1 + w_\Phi)\ .$$

Maximizing the DM yield [Calibbi et al: 2102.06221]

$$Y_\chi(\widetilde{A}) = \frac{\Gamma_{\mathcal{B}_1\to\mathcal{B}_2\chi}}{\widetilde{A}^3\, T(\widetilde{A})^3} \int_0^{\log\widetilde{A}} d\log\widetilde{A}'\, \frac{(\widetilde{A}')^3\, n^{\rm eq}_{\mathcal{B}_1}(T(\widetilde{A}'))}{H(\widetilde{A}')}\, \frac{K_1[M/T(\widetilde{A}')]}{K_2[M/T(\widetilde{A}')]}$$



$\widetilde{A}_P^{-\ell} = M/T_P = M/T_R\big|_{\min}$: how early $\mathcal{B}_1$ can decay within the NA phase – before reheating – without inducing excessive dilution and spoiling freeze-in.

$$\left(\frac{M}{T_{\rm P}}(w_\Phi)\right)^{-3(1+1/\ell(w_\Phi))} \lesssim D_M^{\rm NA}$$

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Estimating the *maximum* $D_M^{\text{max}} = \# \, Y_\chi^{eq}/(0.44\,\mathrm{eV}/m_\chi)$

$$D_M^{\text{max}} \geq \frac{\max Y_\chi(\widetilde{A}_{\mathrm{P}})}{Y_\chi(\widetilde{A}_{\mathrm{R}})} = \frac{\mathcal{X}_\chi(\widetilde{A}_{\mathrm{P}})}{\mathcal{X}_\chi(\widetilde{A}_{\mathrm{R}})} \left(\frac{T_{\mathrm{R}}}{M}\right)^{3(1+1/\ell(w_\Phi))} \left(\frac{T_{\mathrm{P}}}{M}\right)^{-3(1+1/\ell(w_\Phi))}$$

leading to

$$\frac{M}{T_{\mathrm{R}}}(w_\Phi)\bigg|_{\text{max}} = \frac{M}{T_{\mathrm{P}}}(w_\Phi) \left[\left(\frac{m_\chi}{2\ \mathrm{keV}}\right) \frac{\mathcal{X}_\chi(x^{-1/\ell(w_\Phi)})}{\mathcal{X}_\chi((M/T_{\mathrm{P}})^{-1/\ell(w_\Phi)})}\right]^{-\frac{1}{(3(1+1/\ell(w_\Phi))}}$$

$$D_M^{\mathrm{NA}} \lesssim \left(\frac{M}{T_{\mathrm{R}}}\bigg|_{\mathrm{max}}(w_\Phi)\right)^{-3(1+1/\ell(w_\Phi))}$$

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Some hidden subtleties

- Assumption of FI during a *single* phase
- Critical behavior for $w_\Phi \to -1$
- Low-T FI tail missed

Solutions

- Overestimate $T_R \to (M/T_R)/c_1$ with $c_1 \approx (1, 1.7)$ (a posteriori)
- Account for production after $T_P \to c_2 \approx 5$

$$\left(\frac{M}{T_\mathrm{R}}\right)_{\max}(c_1, c_2) = \frac{x}{c_1}$$

$$x = \frac{M}{T_\mathrm{P}}(w_\Phi)\left[\left(\frac{m_\chi}{2\ \mathrm{keV}}\right)\frac{c_2\,\mathcal{X}_\chi(x^{-1/\ell(w_\Phi)})}{\mathcal{X}_\chi((M/T_\mathrm{P})^{-1/\ell(w_\Phi)})}\right]^{-1/(3(1+1/\ell(w_\Phi)))}$$

Mass bounds analytical estimates I

$$\mathrm{RD}: \qquad m_\chi^{\mathrm{min}} \approx 2.96 \frac{T_\chi(t_0)}{W_{\mathrm{WDM}}(t_0)} = 19.7\ \mathrm{keV} \left(\frac{m_{\mathrm{WDM}}}{6\ \mathrm{keV}}\right)^{4/3} \left(\frac{104.5}{g_\star(M)}\right)^{1/3}$$

$$\Phi\mathrm{D}: \qquad m_\chi^{\mathrm{min}} \approx (1.94 - 3.46) \frac{T_\chi(t_0)}{W_{\mathrm{WDM}}(t_0)} = (12.9 - 23.1)\ \mathrm{keV} \left(\frac{m_{\mathrm{WDM}}}{6\ \mathrm{keV}}\right)^{4/3} \left(\frac{104.5}{g_\star(M)}\right)^{1/3}$$

$$\mathrm{RD}': \qquad m_\chi^{\mathrm{min}} \approx 11.1\ \mathrm{keV} \left(\frac{m_{\mathrm{WDM}}}{6\ \mathrm{keV}}\right) \left(\frac{104.5}{g_\star(M)}\right)^{1/4}$$

$$\Phi\mathrm{D}_A: \qquad m_\chi^{\mathrm{min}} \approx (8.1 - 12.5)\ \mathrm{keV} \left(\frac{m_{\mathrm{WDM}}}{6\ \mathrm{keV}}\right) \left(\frac{104.5}{g_\star(M)}\right)^{1/4}$$

Mass bounds analytical estimates II

$$w_\Phi \gtrsim -0.4$$

$$\tilde{\sigma}^{\mathrm{NA}}_{D\mathrm{max,c_1=1}}(w_\Phi) = \sqrt{\frac{\int_0^{\tilde{A}^{\mathrm{max}}_{\mathrm{R}}} d\tilde{A}\ \tilde{A}^{\ell-m+4} K_3(\tilde{A}^{-\ell})}{4\int_0^{\tilde{A}^{\mathrm{max,1}}_{\mathrm{R}}} d\tilde{A}\ \tilde{A}^{\ell-m+2} K_1(\tilde{A}^{-\ell})}}\ (\tilde{A}^{\mathrm{max,1}}_{\mathrm{R}})^{-(1+\ell)}$$

$$\tilde{A}^{\mathrm{max,1}}_{\mathrm{R}} = \left[\left(\frac{M}{T_{\mathrm{R}}}\right)_{\mathrm{max}}(1,5)\right]^{-1/\ell(w_\Phi)}$$

$$w_\Phi \lesssim -0.4$$

$$\tilde{\sigma}^{\mathrm{NA}}_{D\mathrm{max,c_1=1.7}}(w_\Phi) = \sqrt{\frac{\int_0^{\tilde{A}^{\mathrm{max,2}}_{\mathrm{R}}} d\tilde{A}\ \tilde{A}^{\ell-m+4} K_3(\tilde{A}^{-\ell})}{4\int_0^{\tilde{A}^{\mathrm{max,2}}_{\mathrm{R}}} d\tilde{A}\ \tilde{A}^{\ell-m+2} K_1(\tilde{A}^{-\ell})}}\ (\tilde{A}^{\mathrm{max,2}}_{\mathrm{R}})^{-(1+\ell)}$$

$$\tilde{A}^{\mathrm{max,2}}_{\mathrm{R}} = \left[\left(\frac{M}{T_{\mathrm{R}}}\right)_{\mathrm{max}}(1.7,5)\right]^{-1/\ell(w_\Phi)}$$

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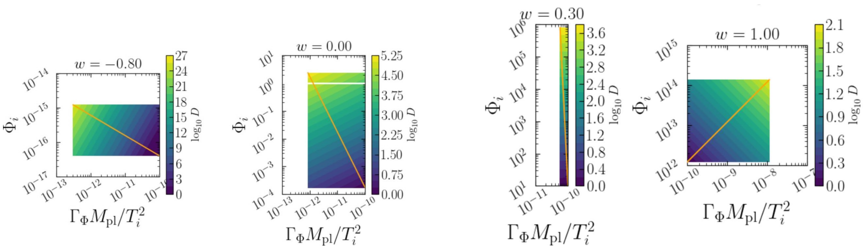
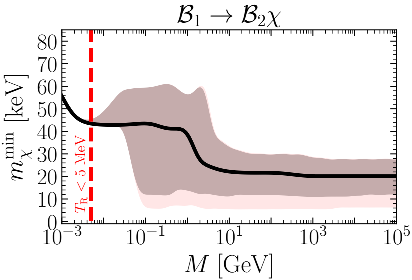
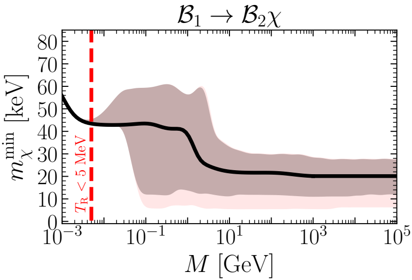
We vary $w_\Phi \in [-0.9, 1]$ and $T_I \in [10^{-2}, 10^{12}]\,\mathrm{GeV}$ and change other conditions to let the dilution factor vary over a few orders of magnitude

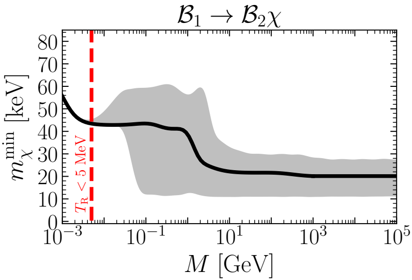
PRELIMINARY



Marginalizing cosmologies







Imposing Freeze-In consistency *a posteriori* is necessary!