

Picolensing as a probe of massive compact DM states

IDM 2026

16th International Workshop on the Identification of Dark Matter
Zaragoza, Spain

June 4, 2026

Phys. Rev. D 111 (2025) 063060 [2411.12947]

w/ Sergey Sibiryakov

+ [260X.YYYYYY]

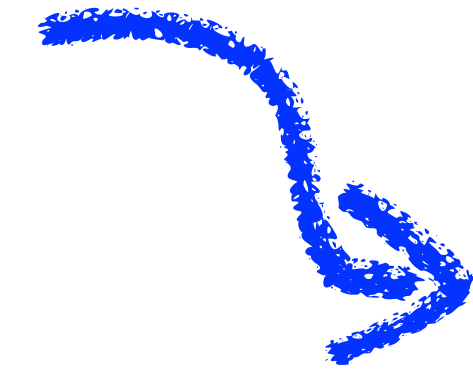
w/ Sergey Sibiryakov + Eleni Anna Kolonia

Michael A. Fedderke

mfedderke@perimeterinstitute.ca



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PBH + extended lenses

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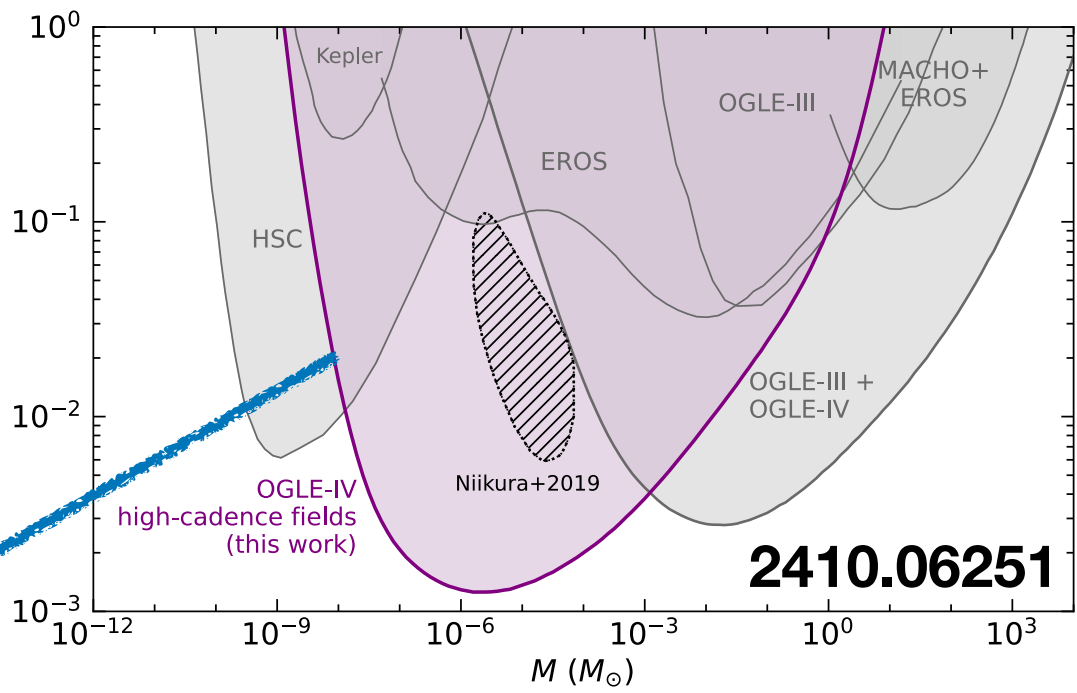
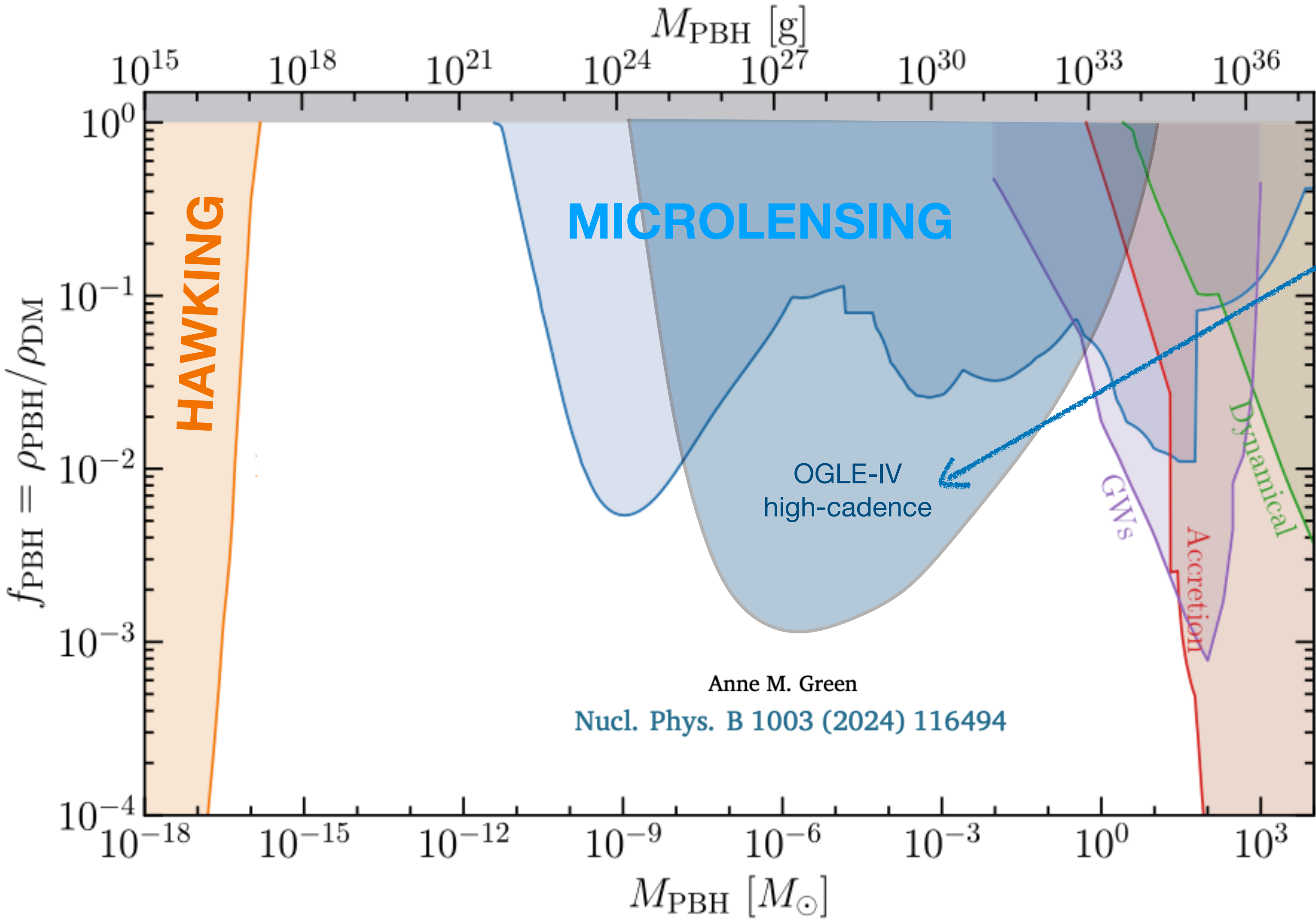


PBH dark matter

$M_{\text{PBH}} \ll M_{\odot}$. Formed in the early-universe.

GRAVITATIONALLY COLLAPSED OBJECTS OF VERY LOW MASS

Stephen Hawking



This talk

**Agnostic to the
production
mechanism**

Annu. Rev. Nucl. Part. Sci. 2020. 70:355–94

Bernard Carr¹ and Florian Kühnel²

A. M. Green and B. J. Kavanagh, *Primordial Black Holes as a dark matter candidate*, *J. Phys. G* **48** (2021) 043001 [[arXiv:2007.10722](#)].

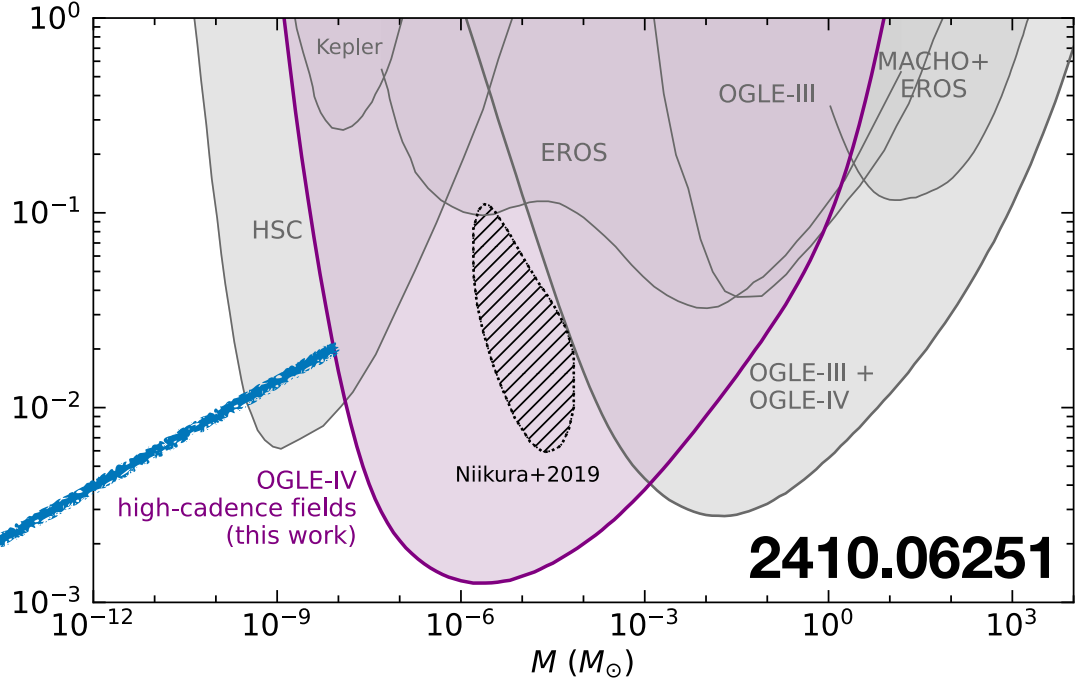
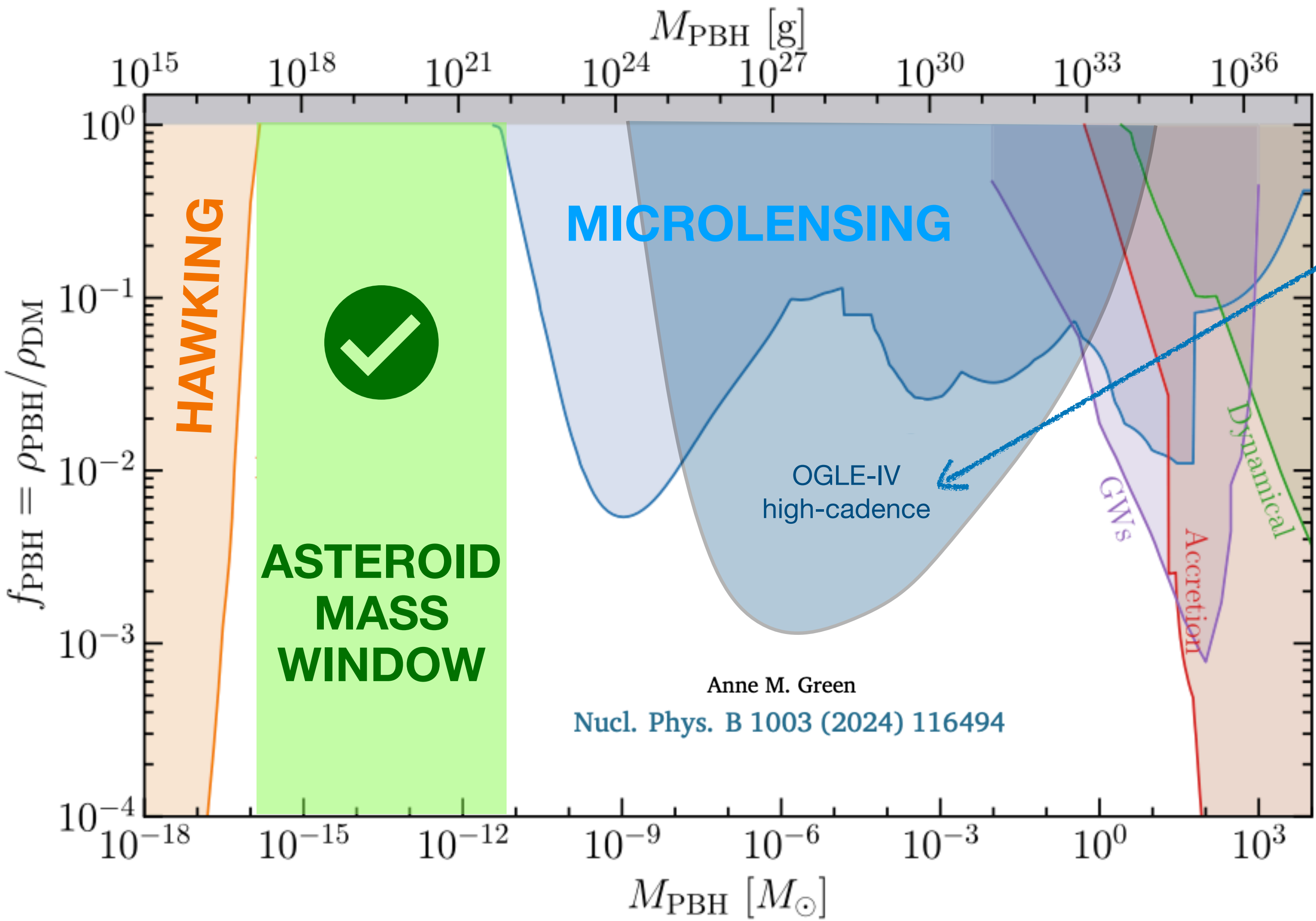
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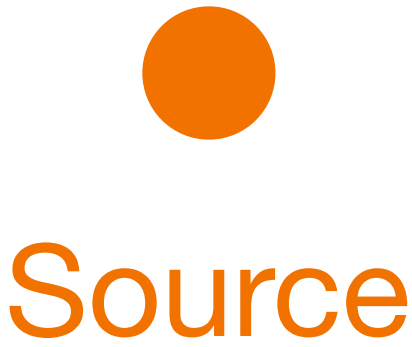
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Anne M. Green
Nucl. Phys. B 1003 (2024) 116494

Picolensing

Picolensing

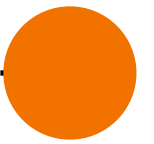


Picolensing

Observer 1



Observer 2

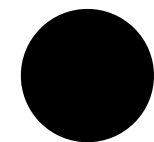


Source

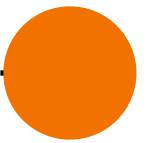
NOT TO SCALE (angles similar to a basketball at the outer edge of the Oort cloud) 3

Picolensing

Observer 1



PBH = Lens



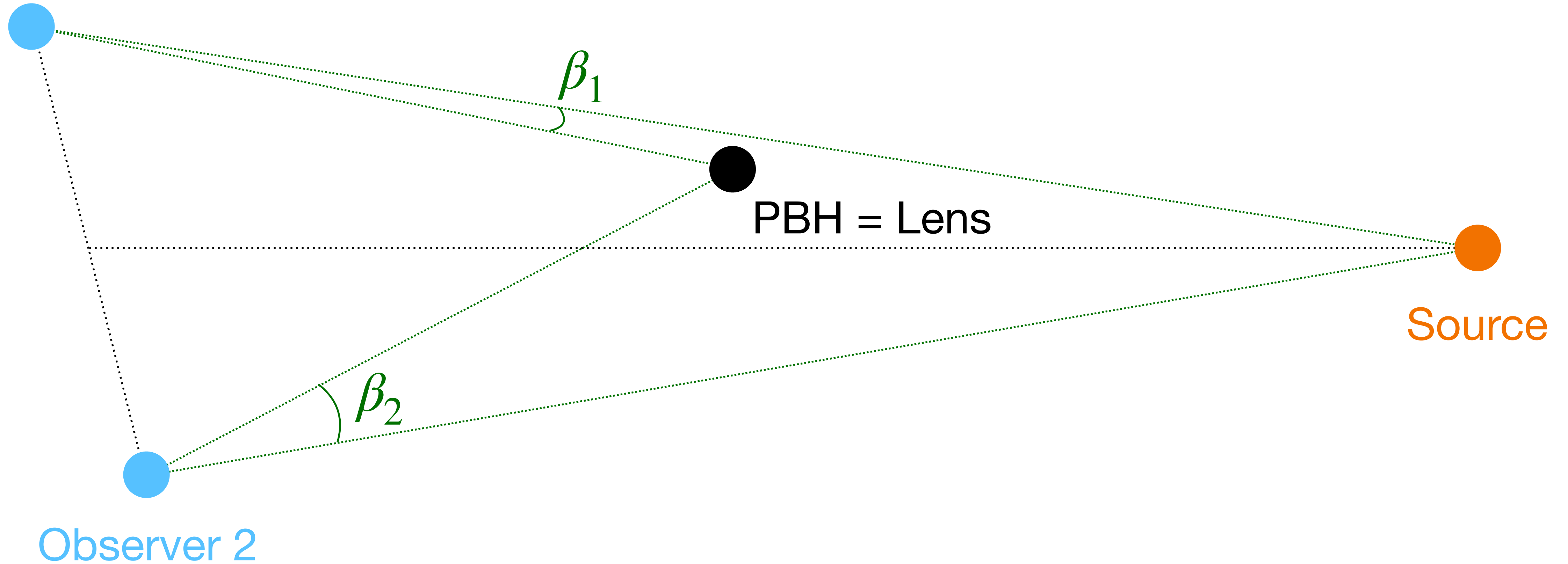
Source



Observer 2

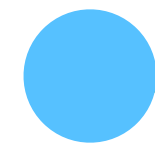
Picolensing

Observer 1

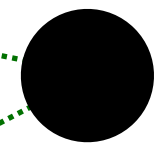


Picolensing

Observer 1

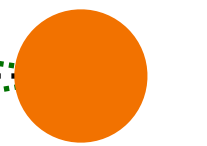


β_1



PBH = Lens

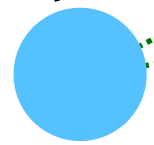
Parallax!



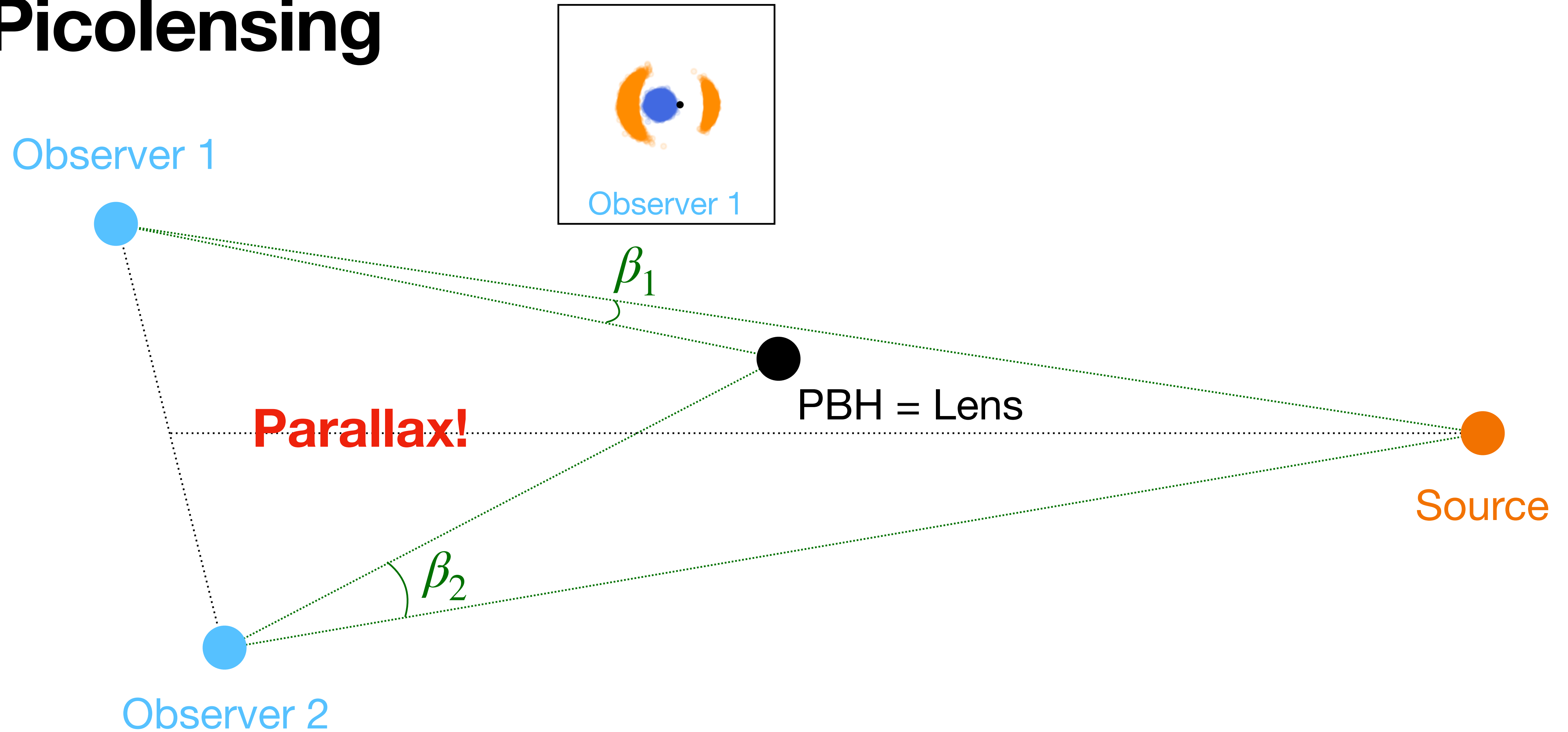
Source

β_2

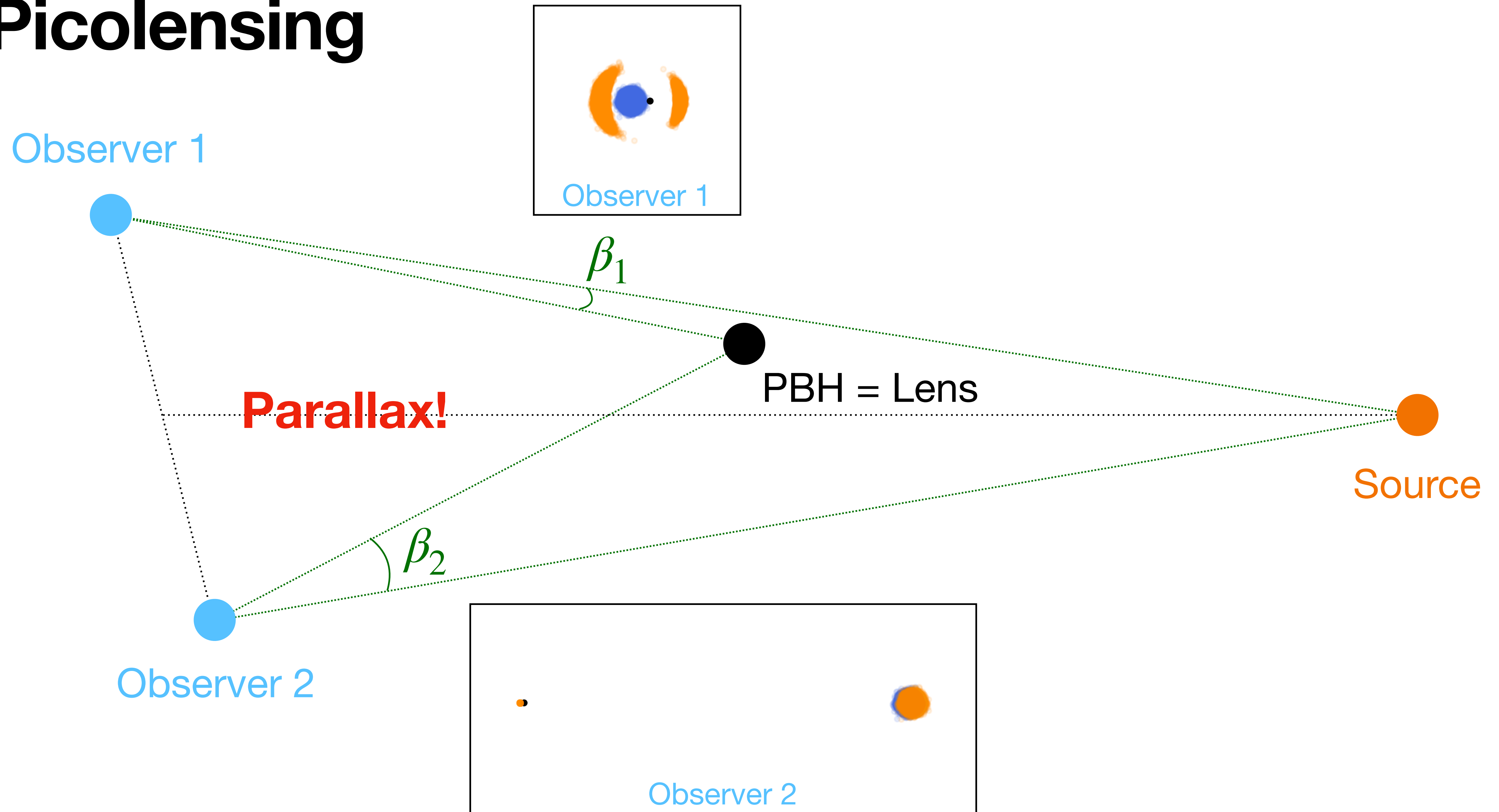
Observer 2



Picolensing



Picolensing



NOT TO SCALE (angles similar to a basketball at the outer edge of the Oort cloud)

Picolensing

Observer 1



Signal: **differential observed brightness of a single source that is observed simultaneously by spatially separated detectors**

Nemiroff and Gould [astro-ph/9505019]
Kolb and Tkachev [astro-ph/9510043]
Marani et al. [astro-ph/9810391]
Nemiroff [astro-ph/9806012]

Parallax!

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Source

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Picolensing

Observer 1



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Parallax!

PBH = Lens

$z \sim 1$

Source
GRB

Observer 2



Picolensing

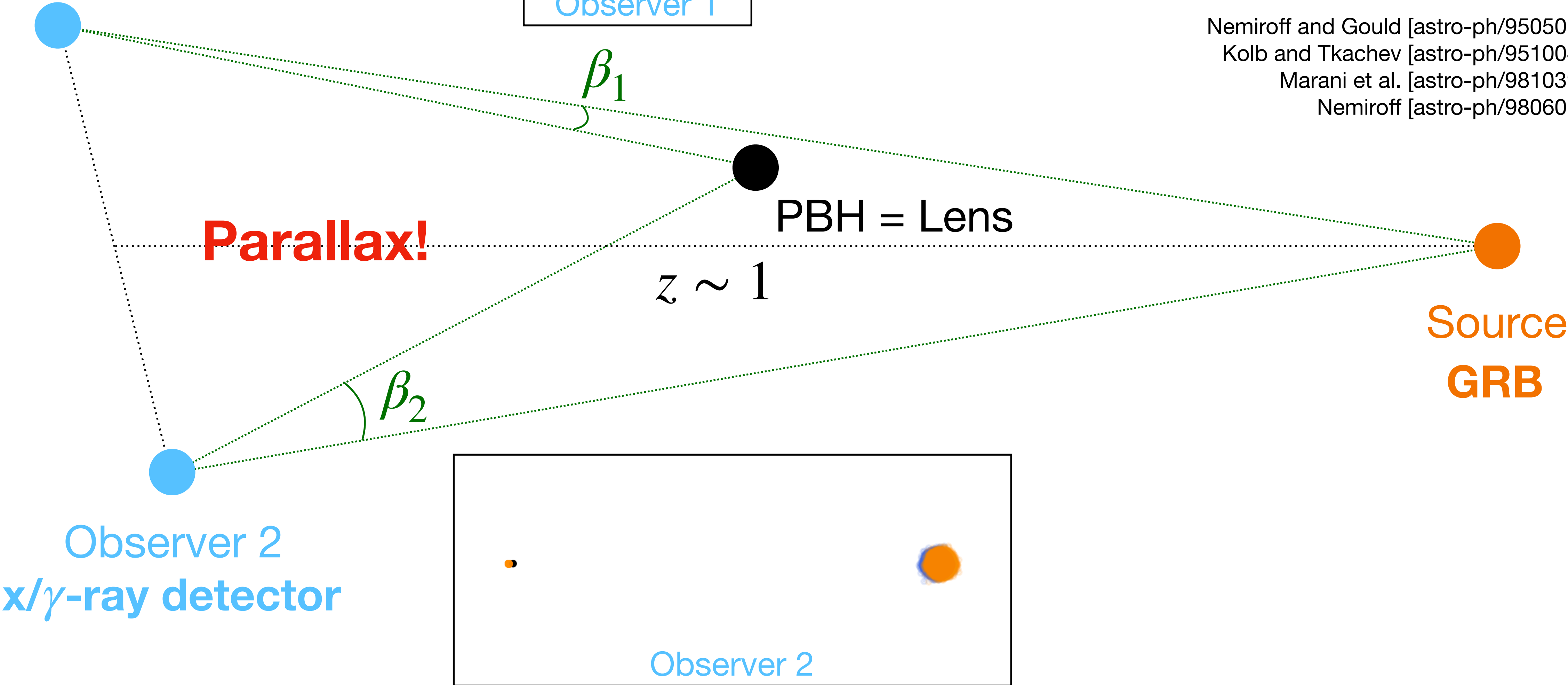
x/ γ -ray detector

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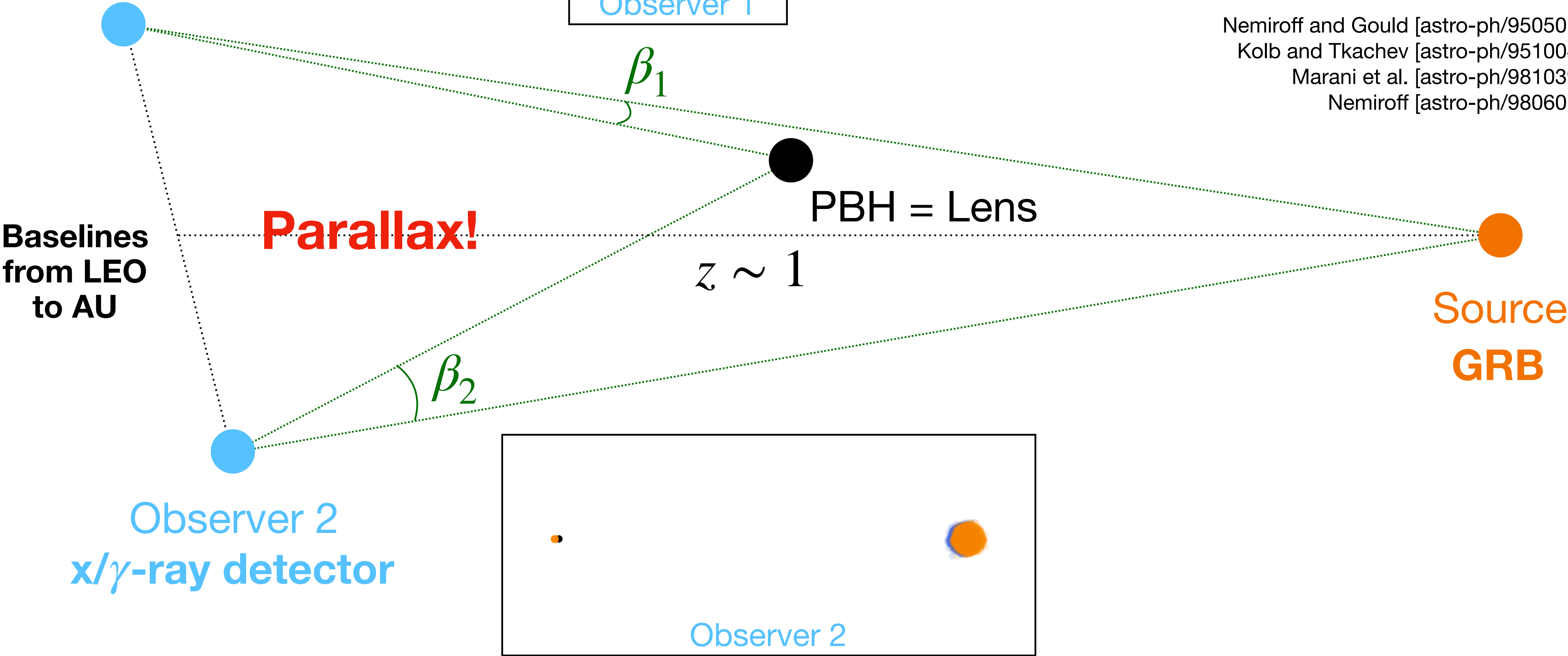
Picolensing

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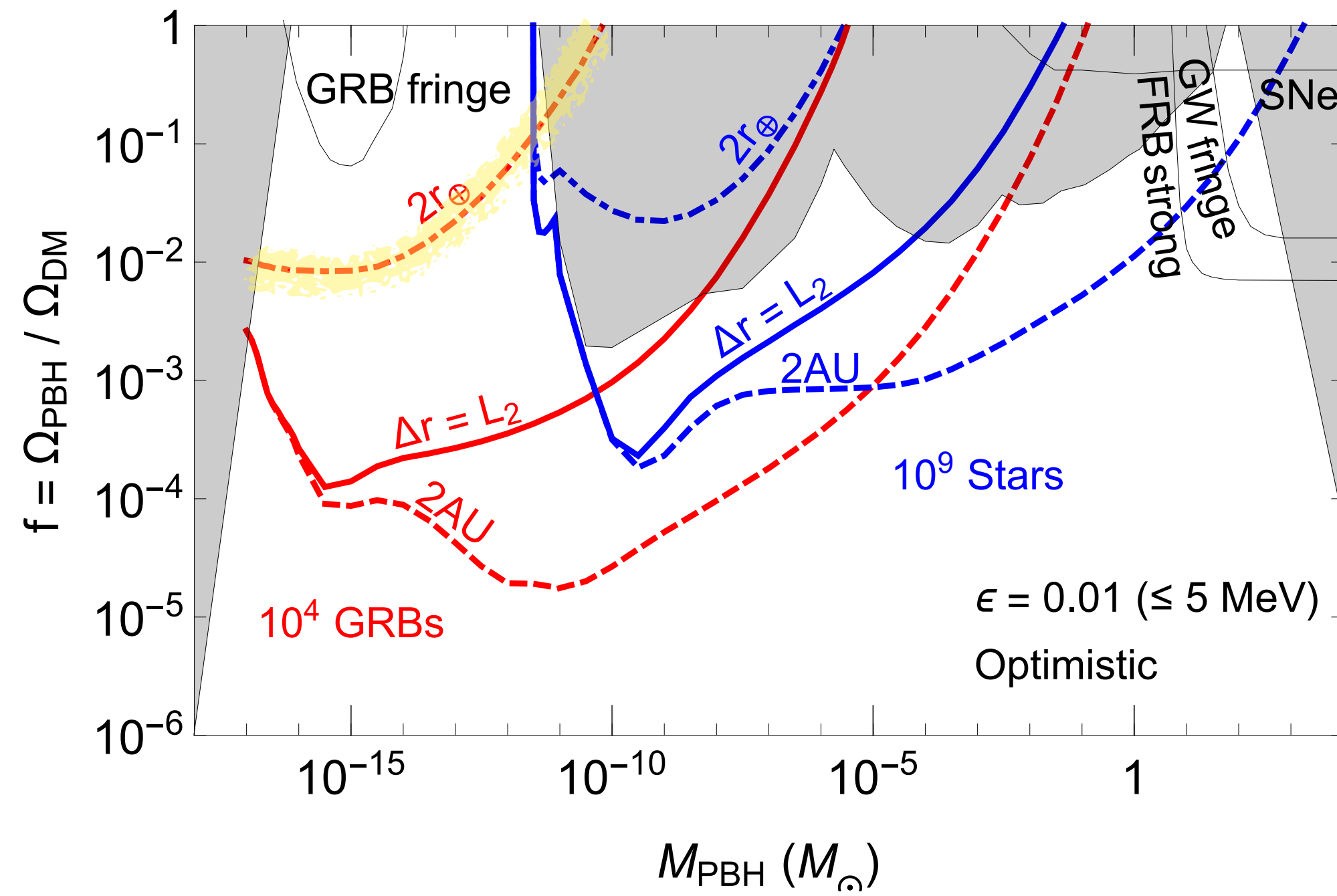
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Prior state of the literature

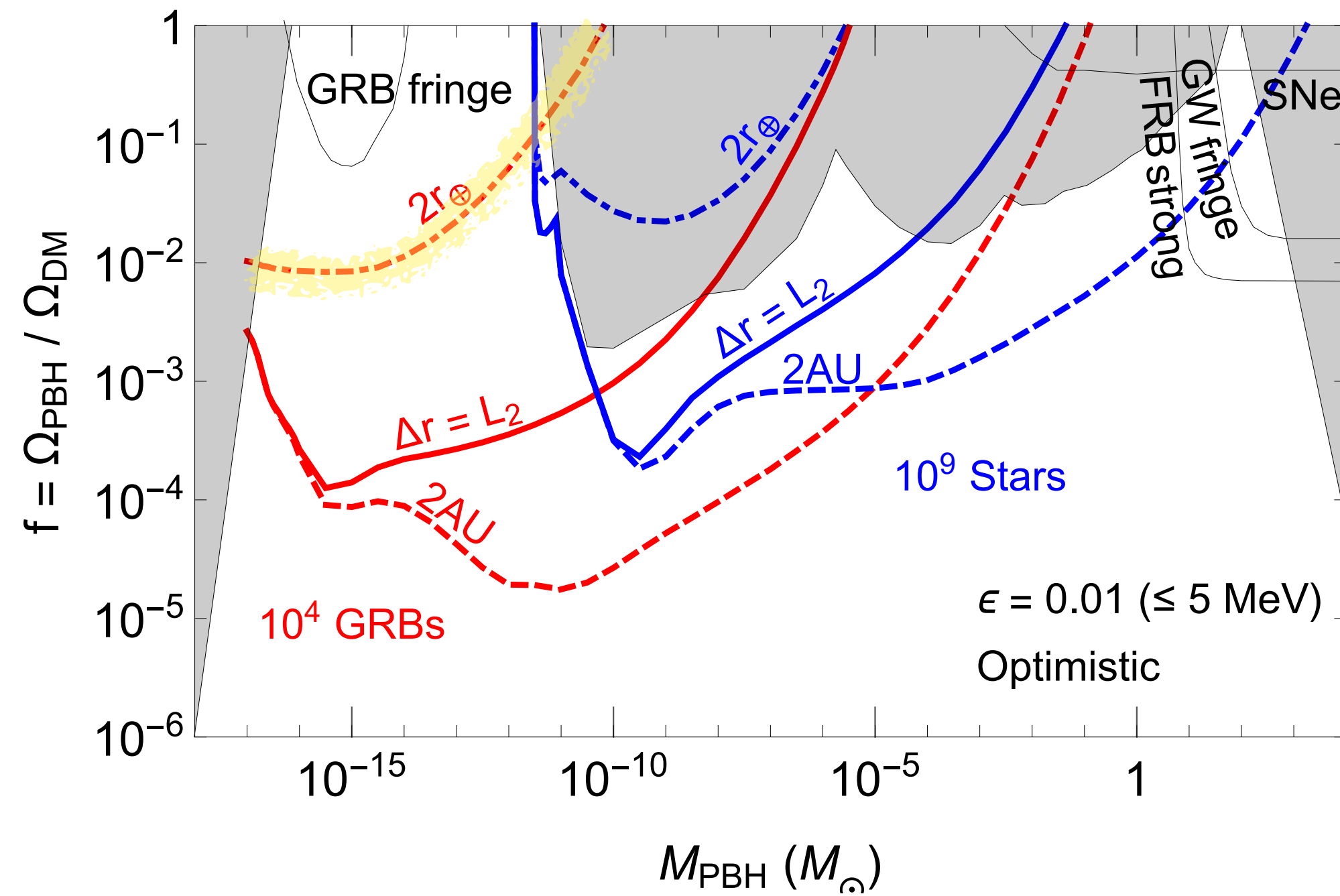
(b)



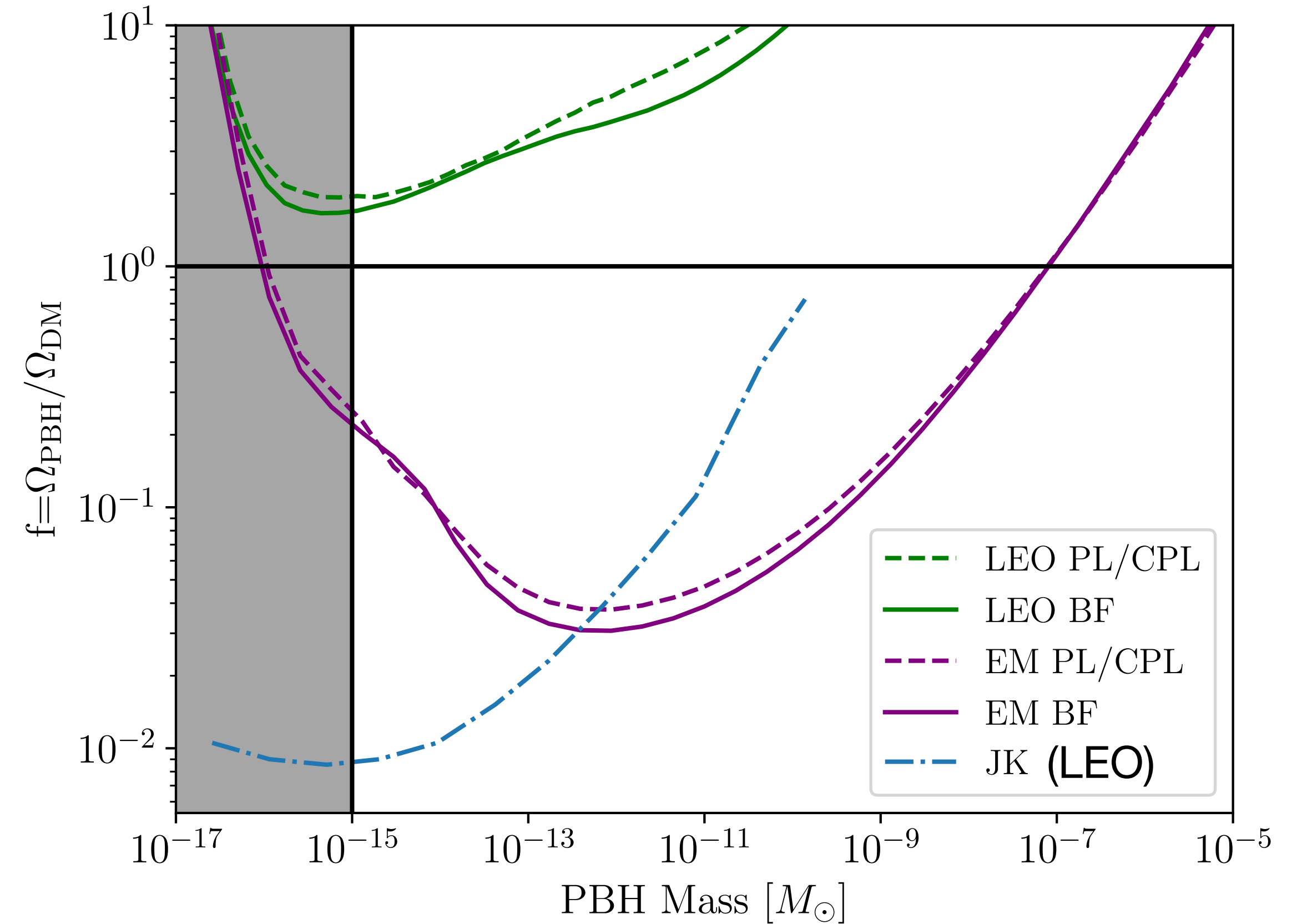
Jung and Kim [1908.00078]

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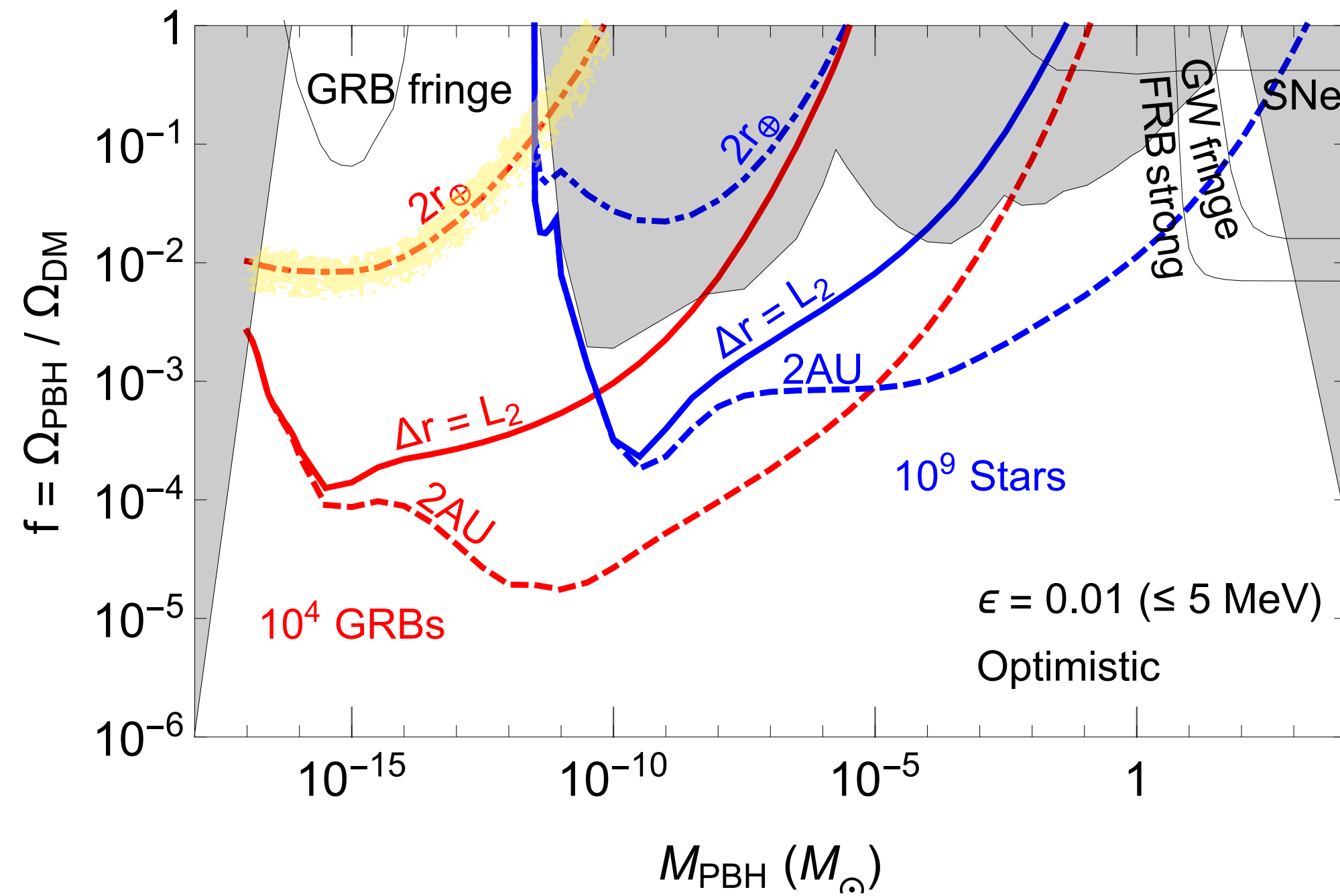
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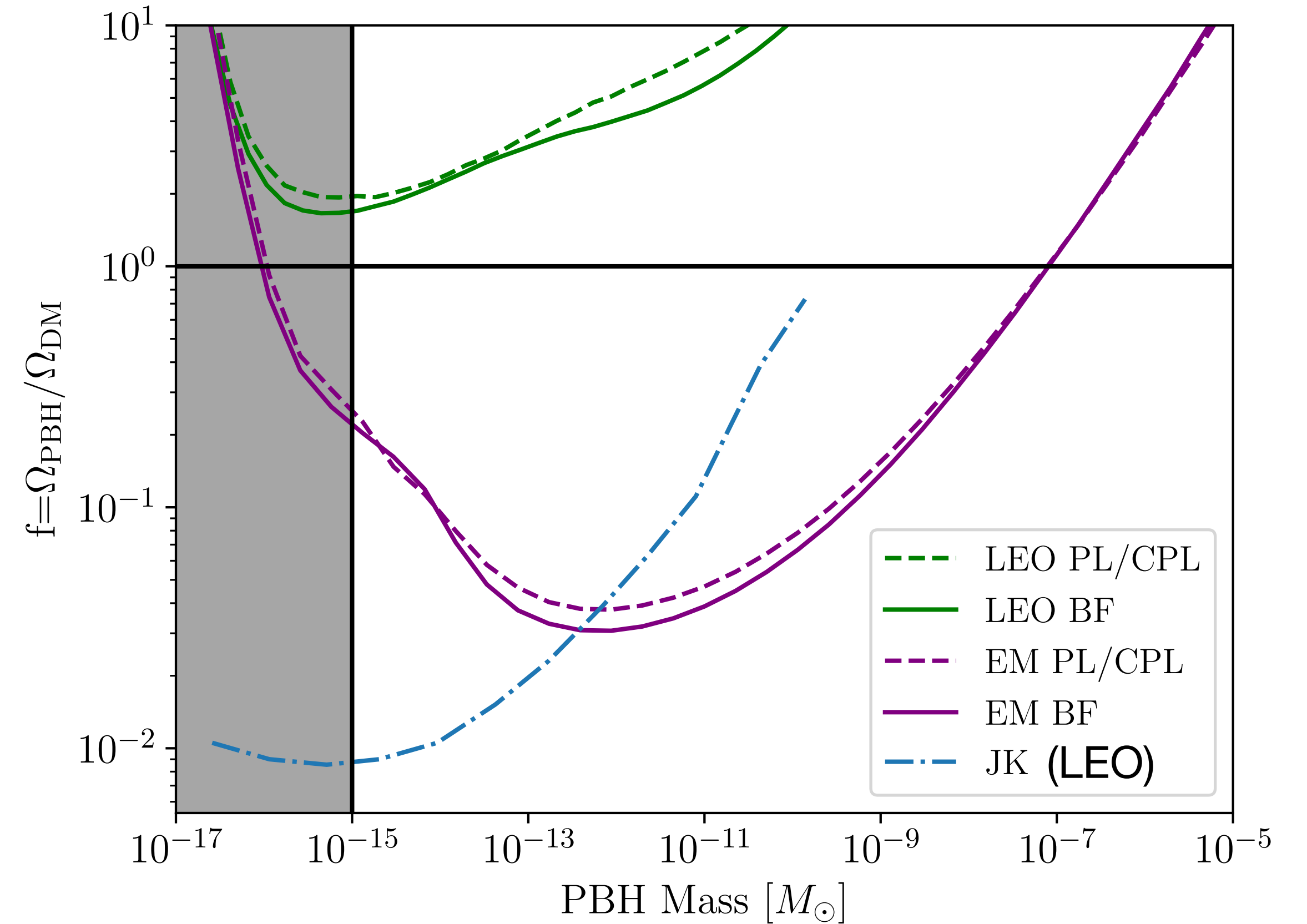
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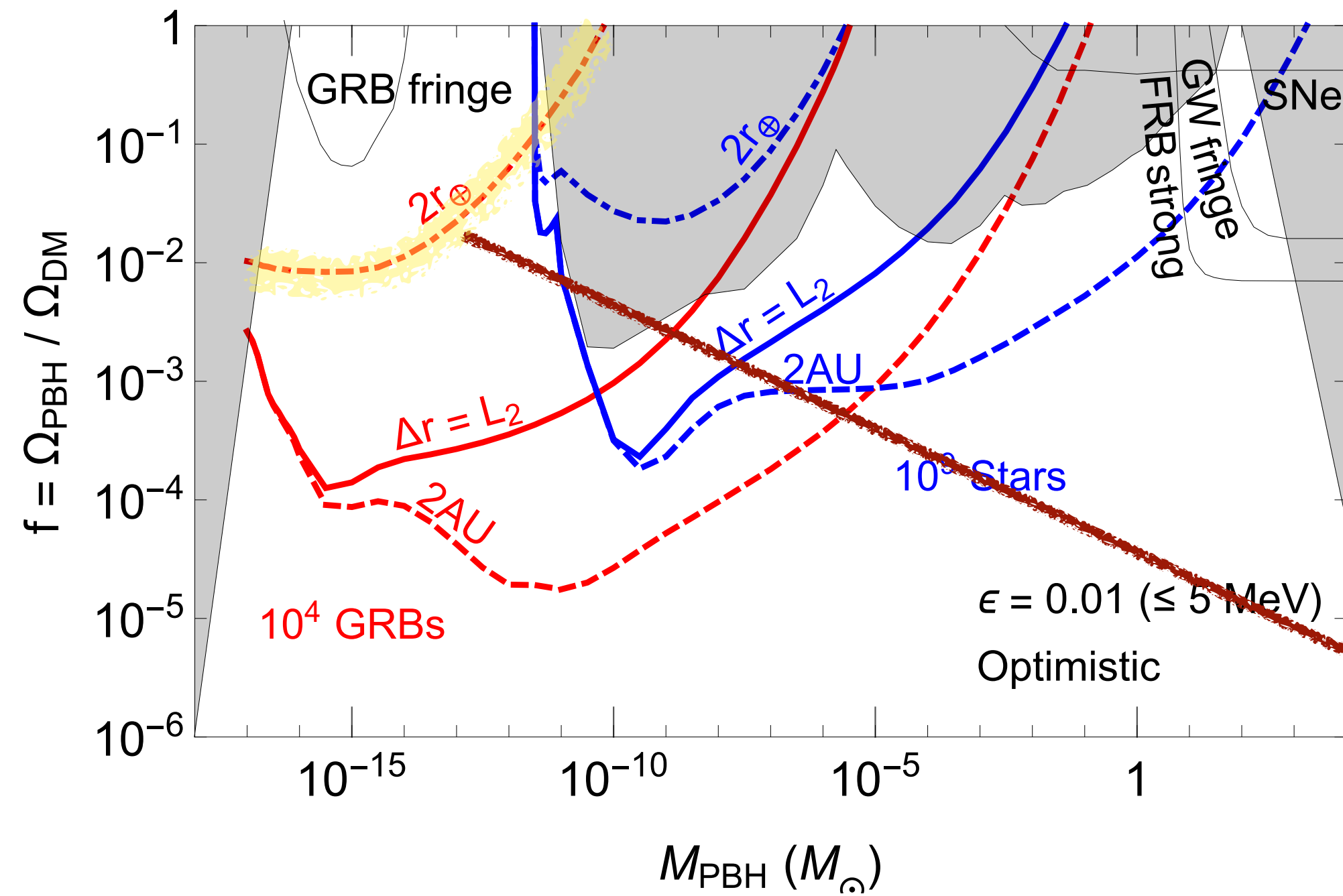


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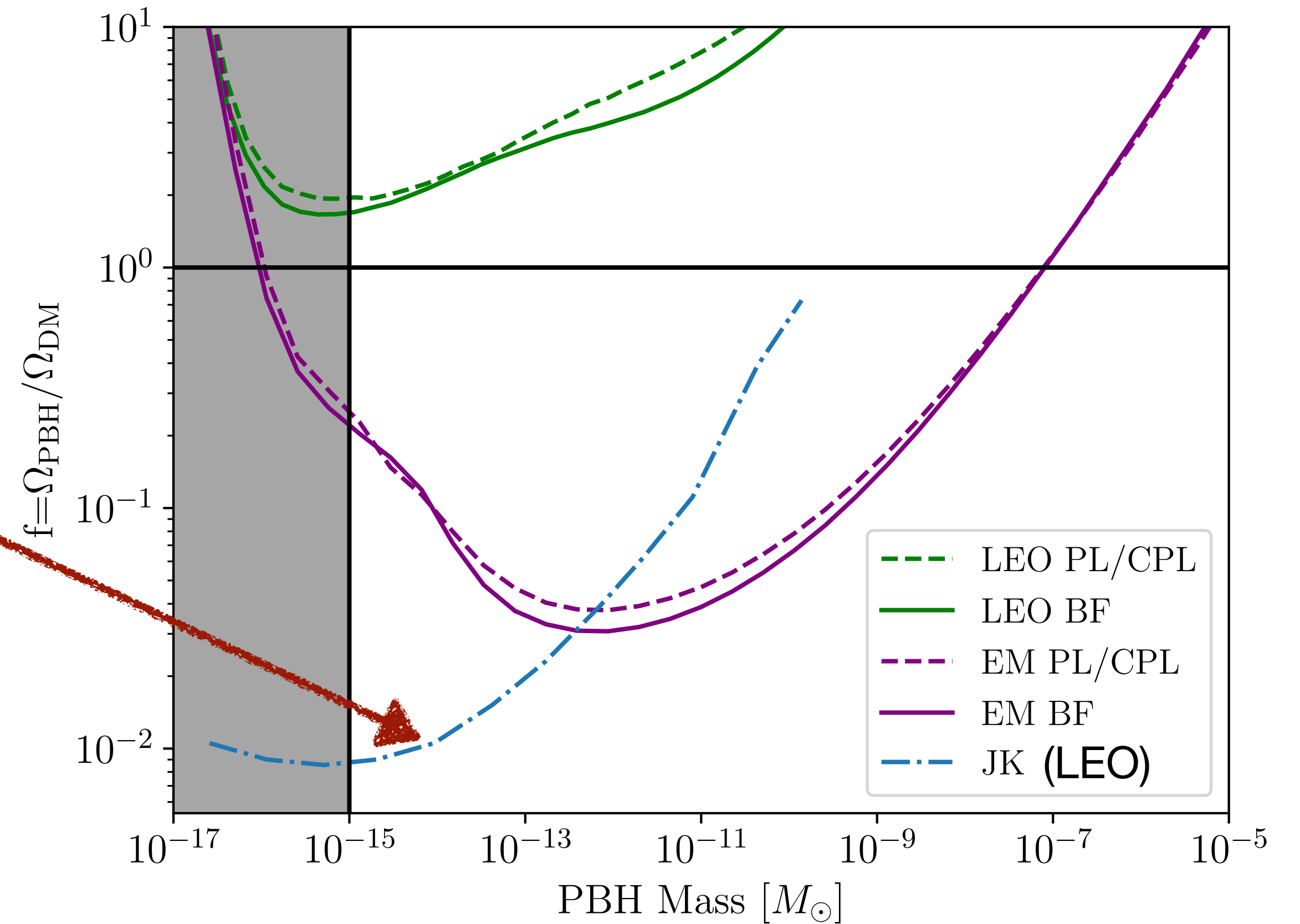
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- * more conservative detector assumptions

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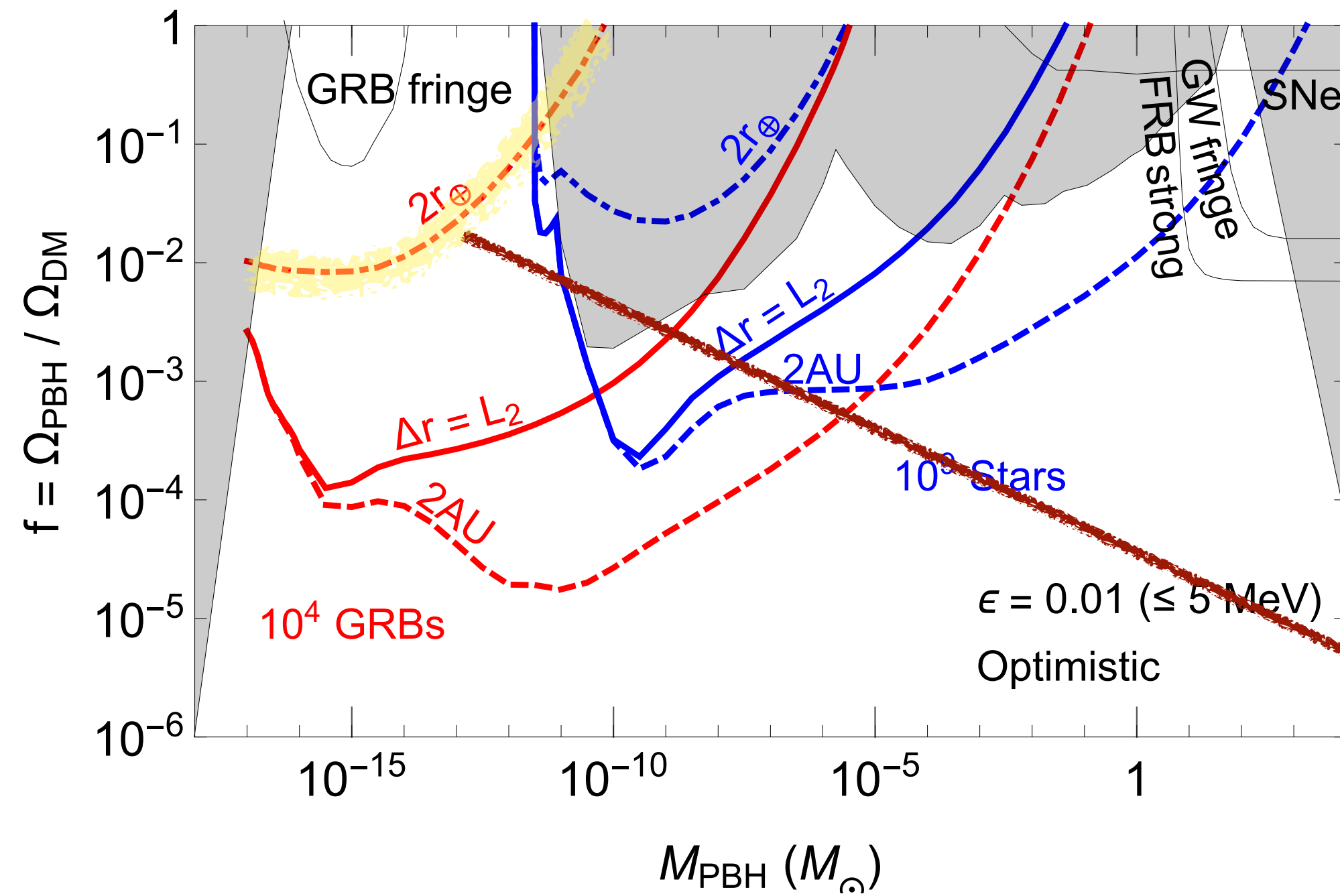


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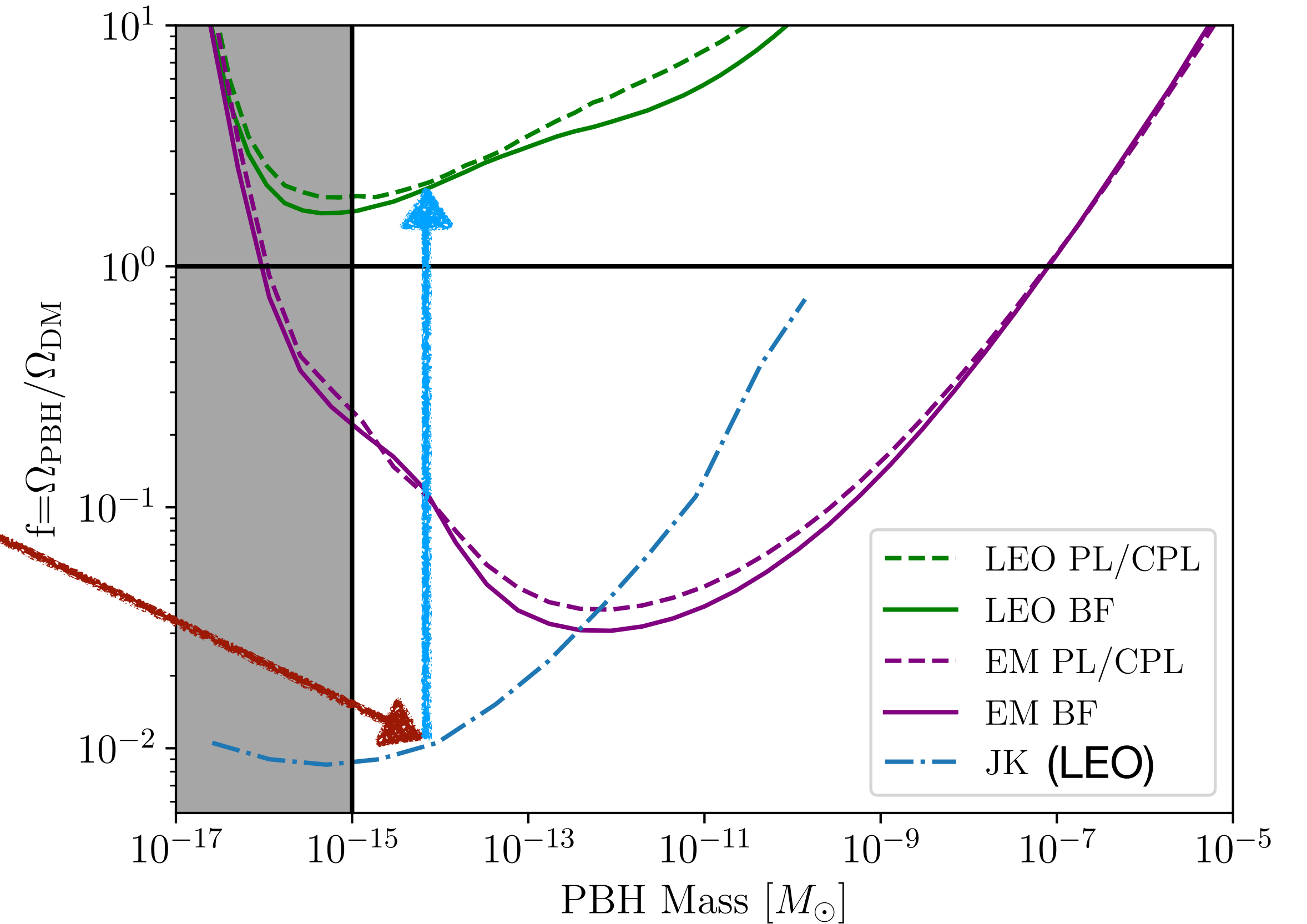
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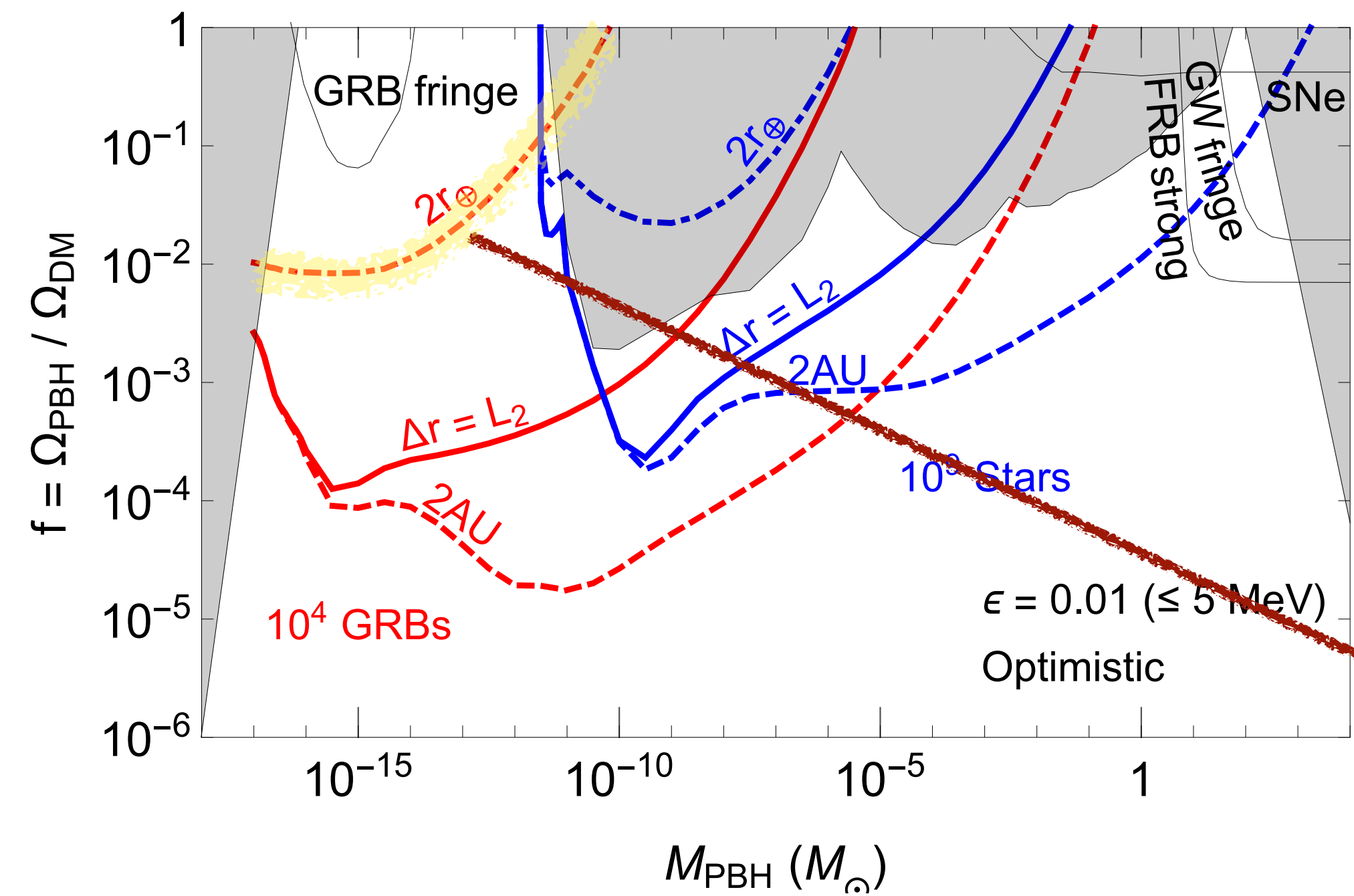


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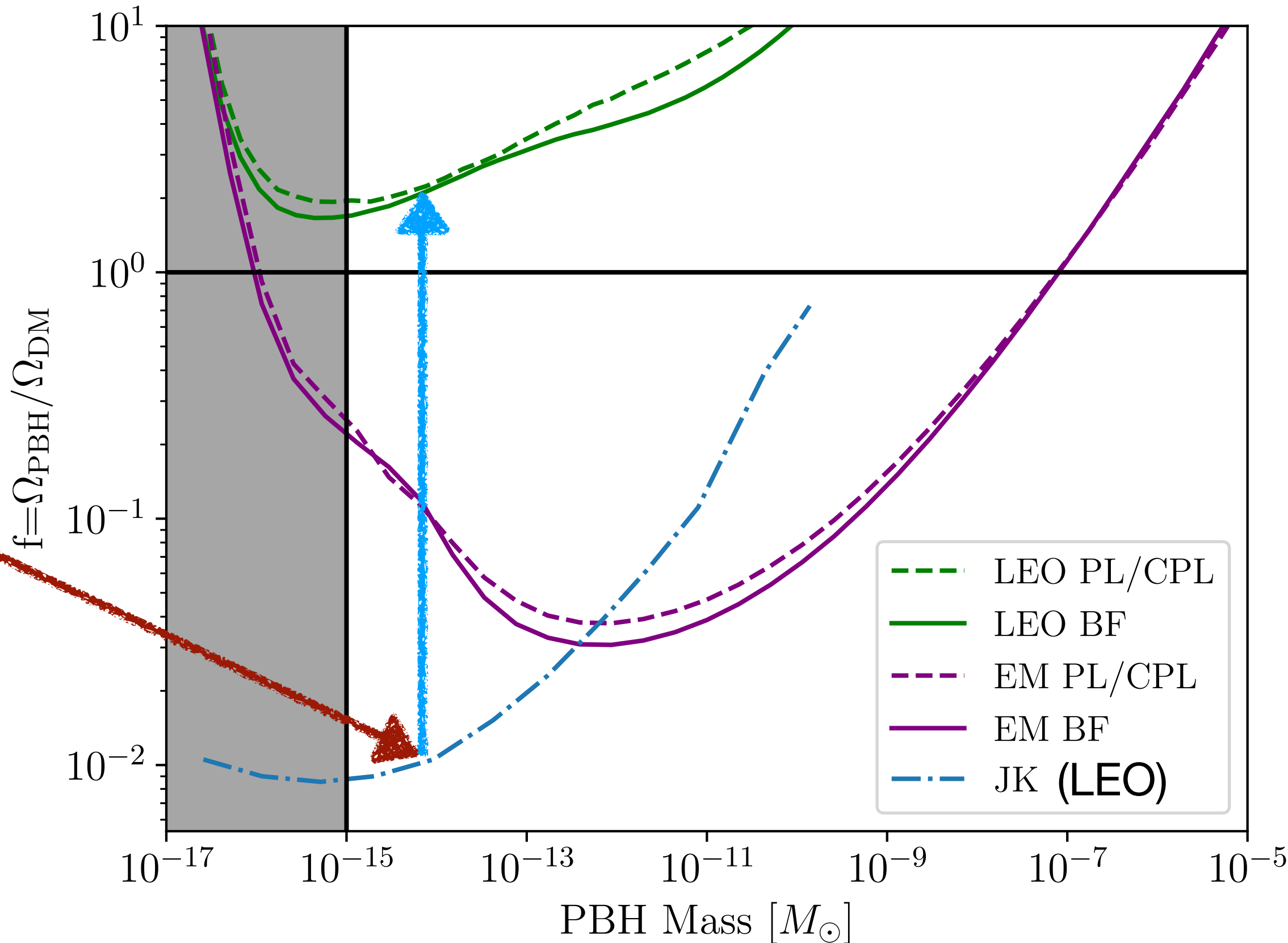


Jung and Kim [1908.00078]

ROBUSTNESS? SOURCE SIZE UNCERTAINTIES

LARGER DETECTOR SEPARATION DONE CAREFULLY?

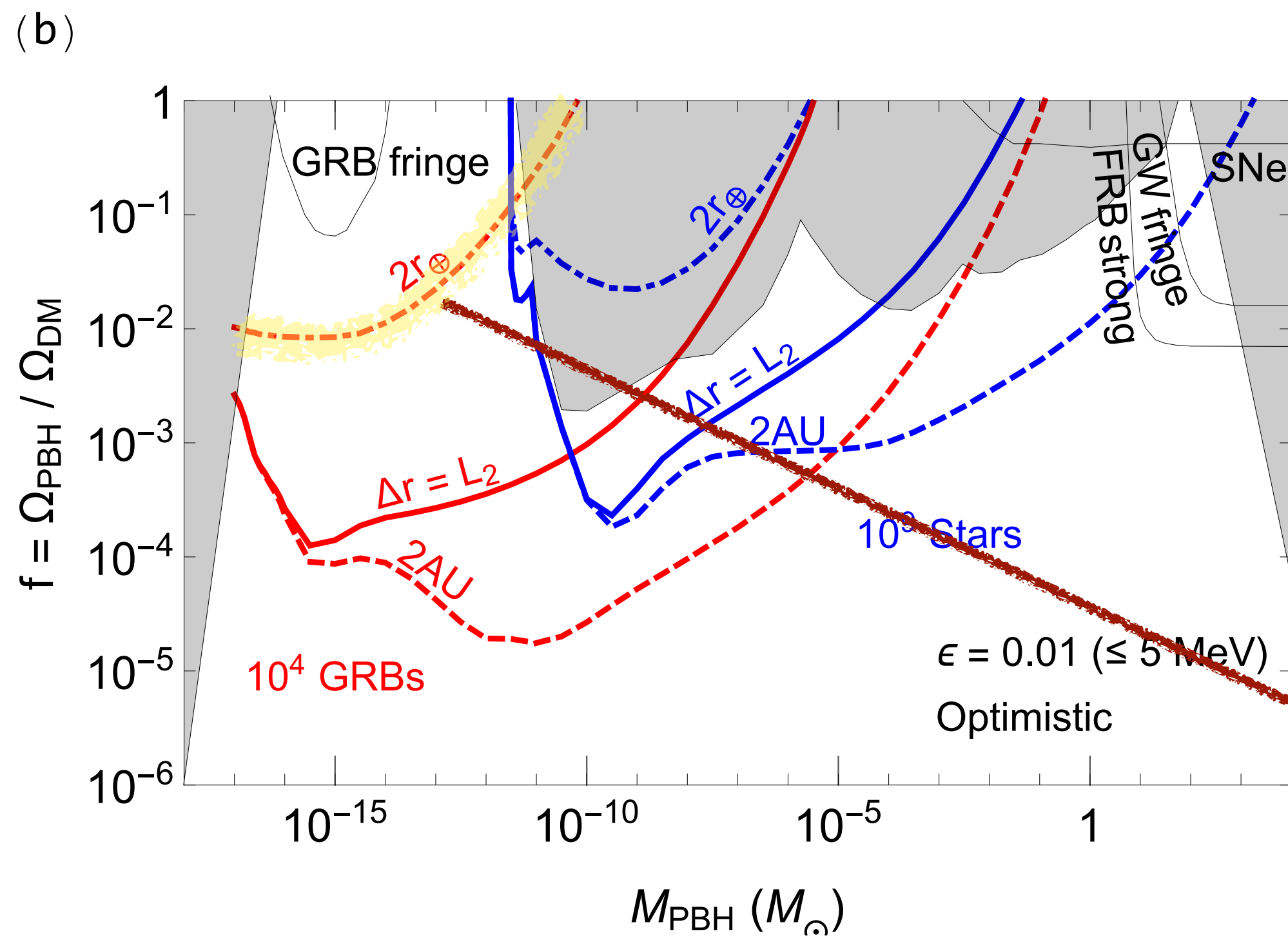
(++)



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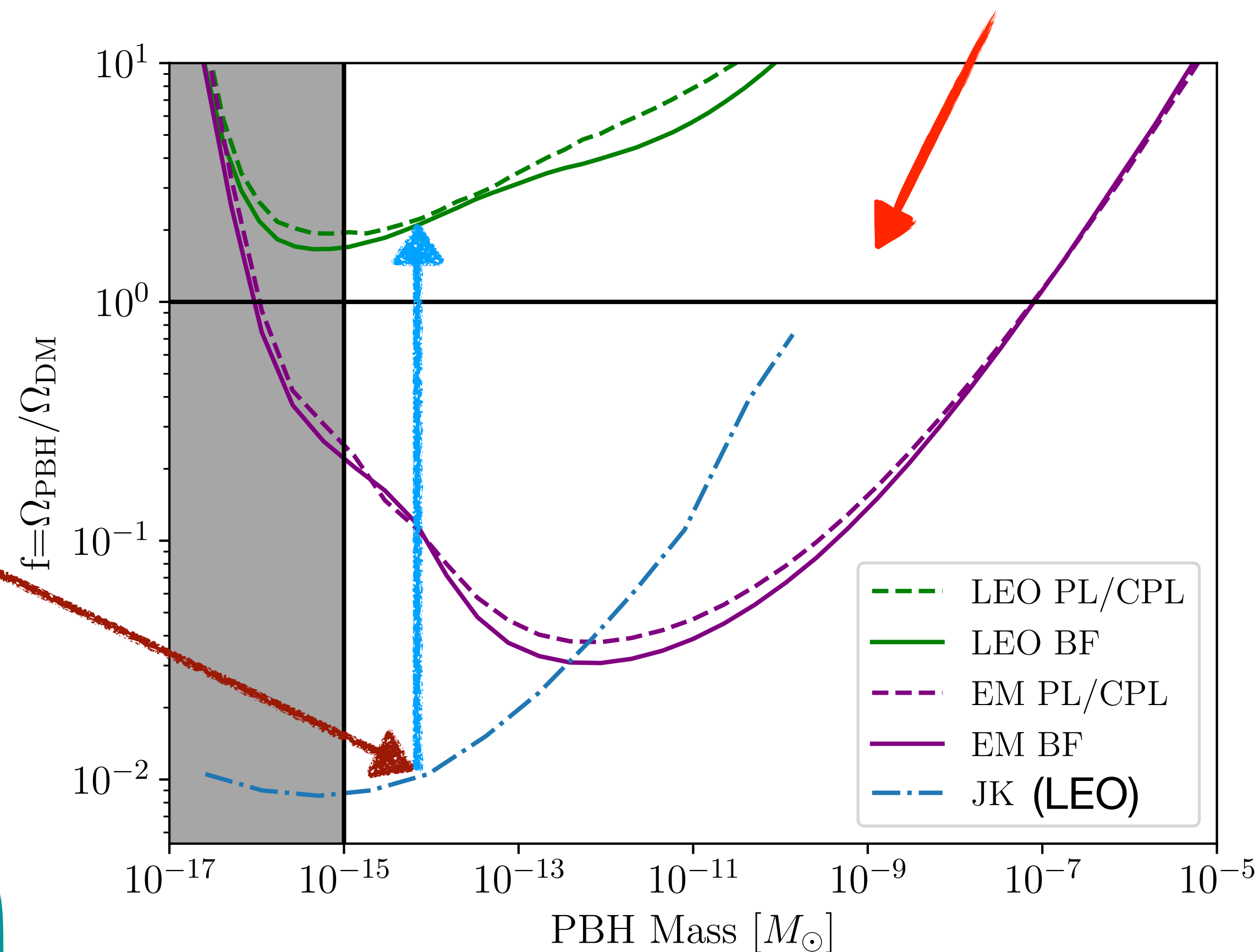
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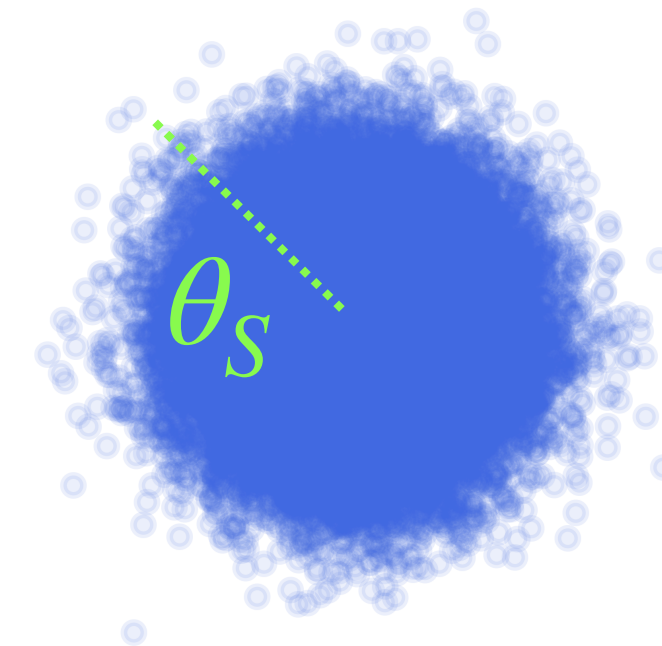
We follow the methodology of this paper



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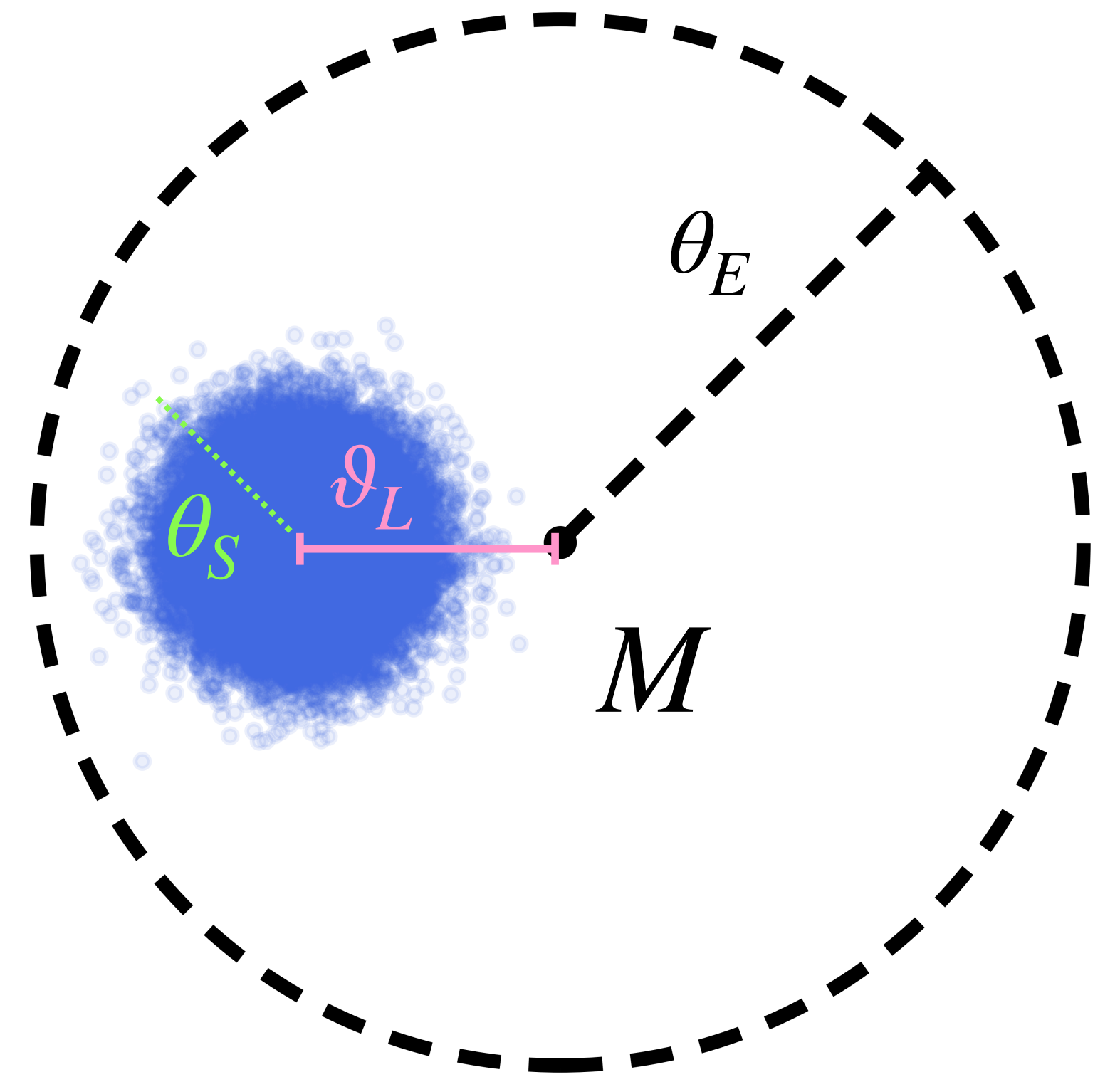
Gravitational Lensing 101



Gravitational Lensing 101

Typical angular scale associated with strong lensing: **Einstein angle**

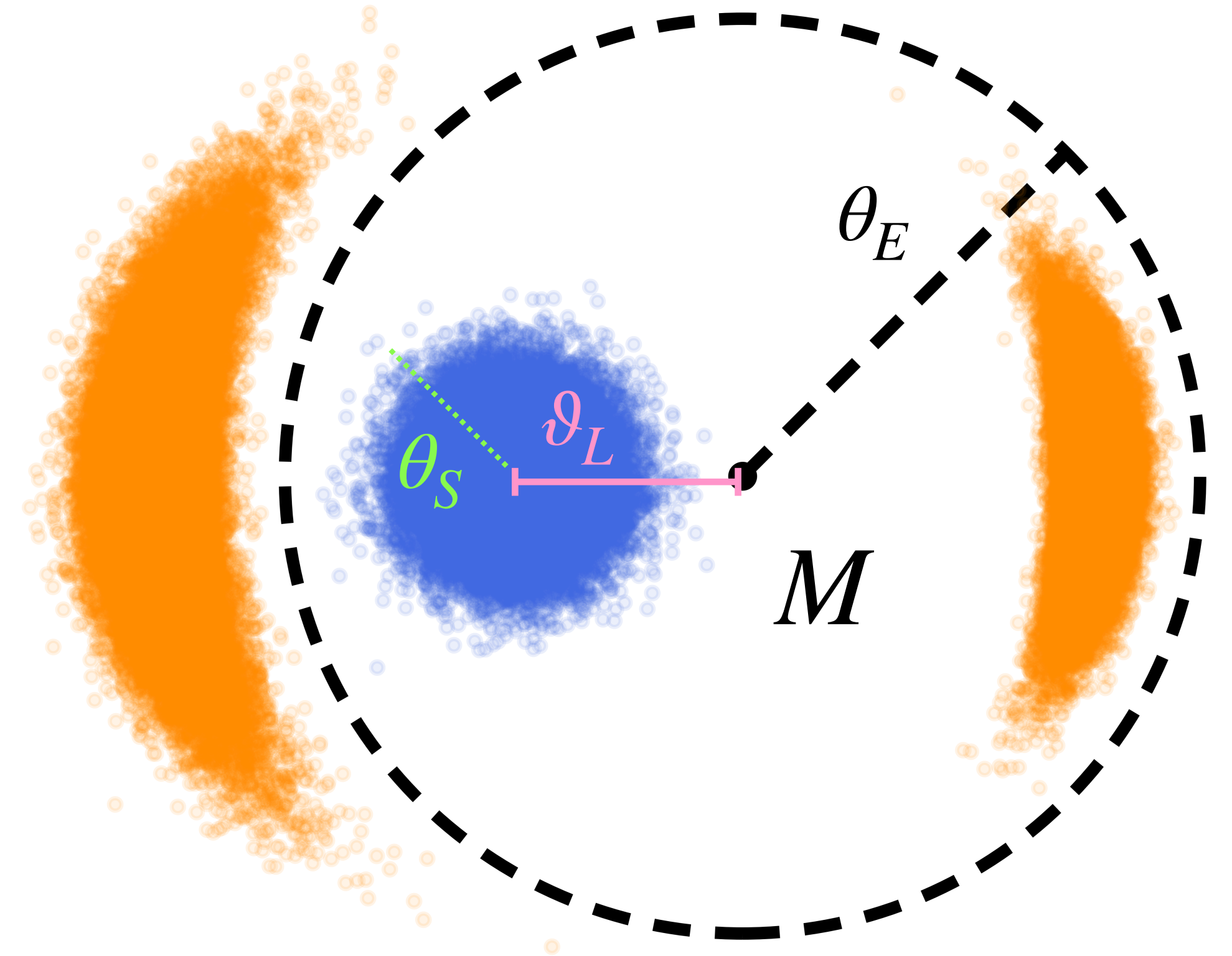
$$\theta_E \equiv \sqrt{\frac{4G_N M (1 + z_L) \chi_{LS}}{\chi_S \chi_L}}$$



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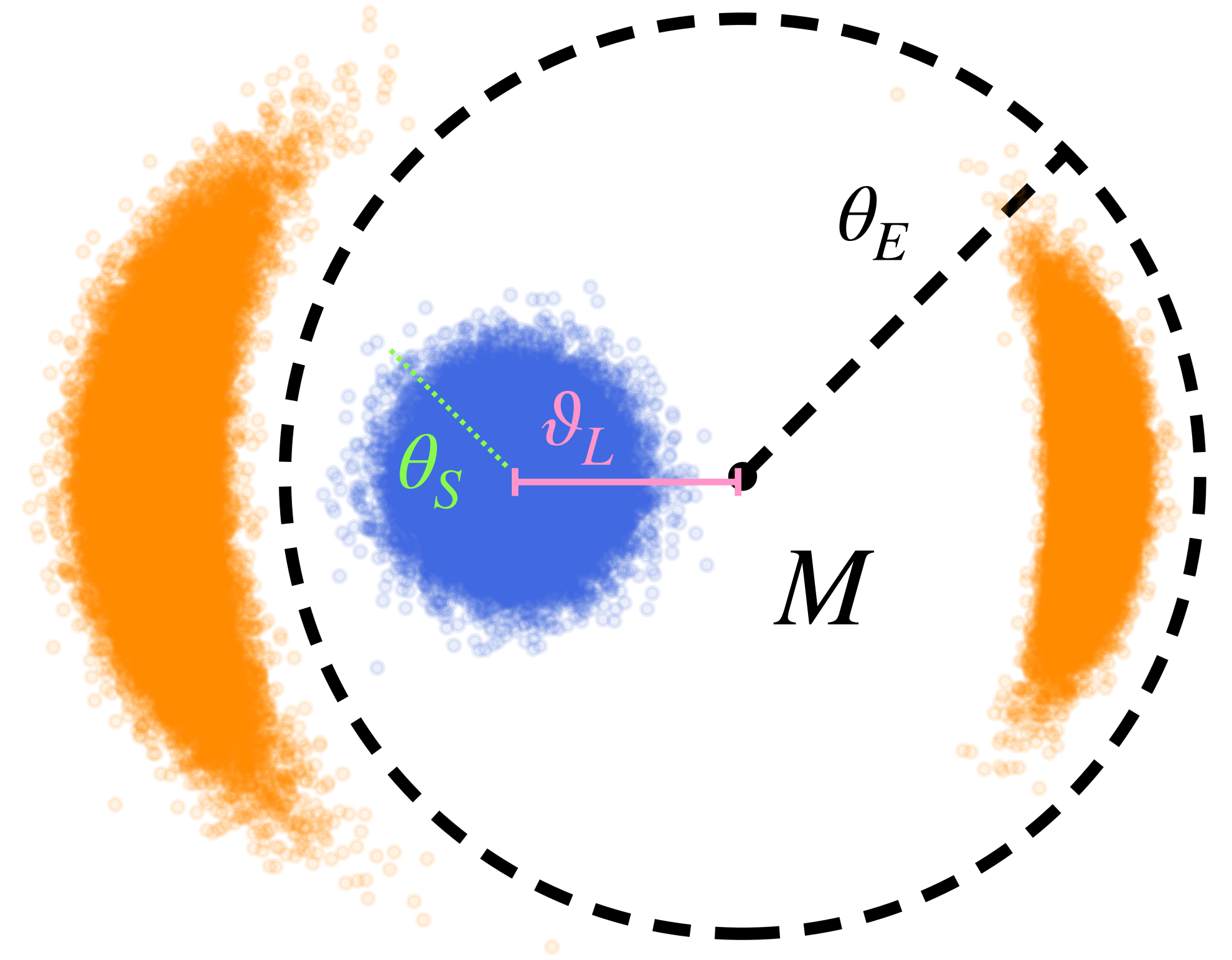


Image is magnified: $\mu \sim \frac{\theta_E}{\vartheta_L} \quad (\vartheta_L, \theta_S \ll \theta_E)$

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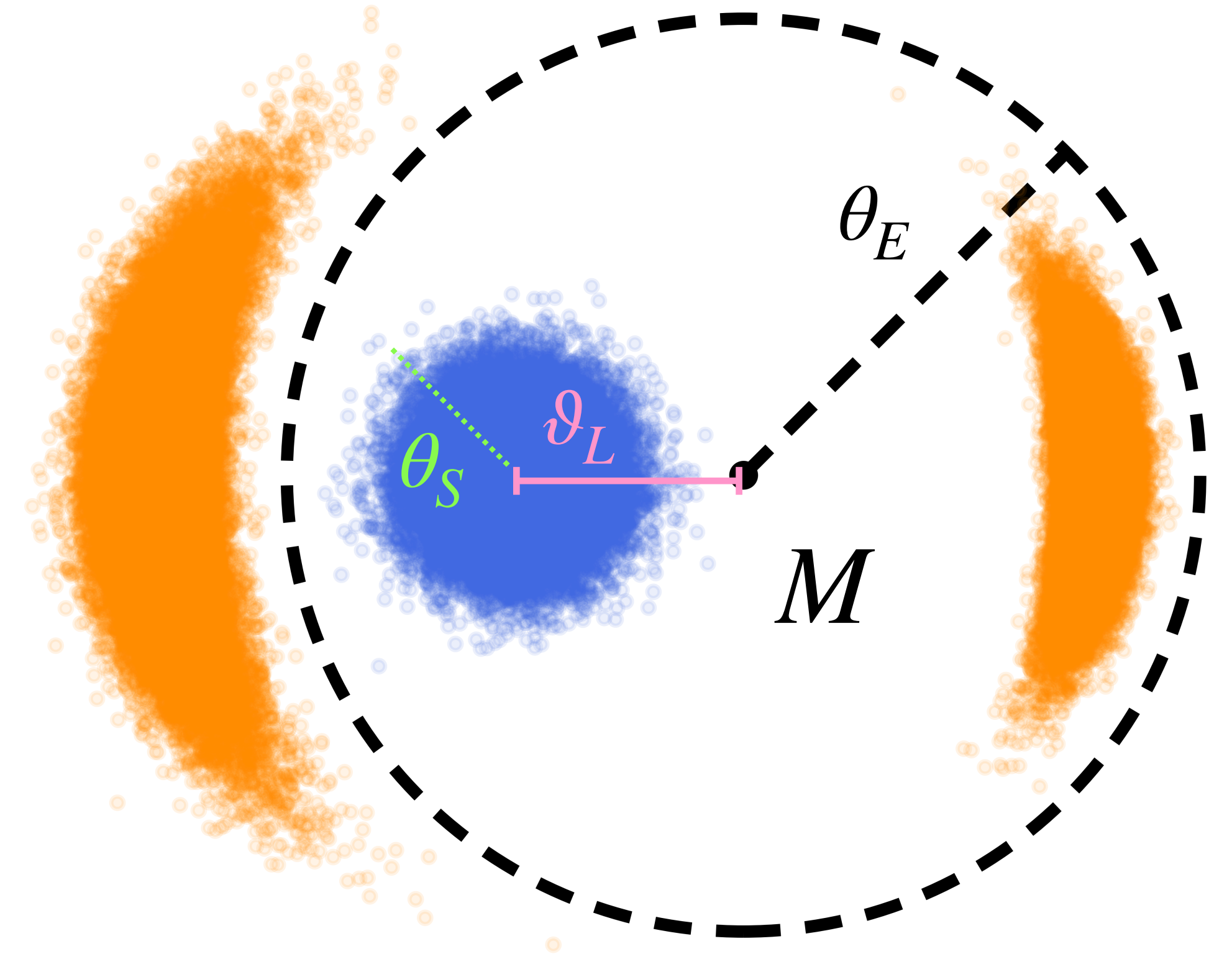


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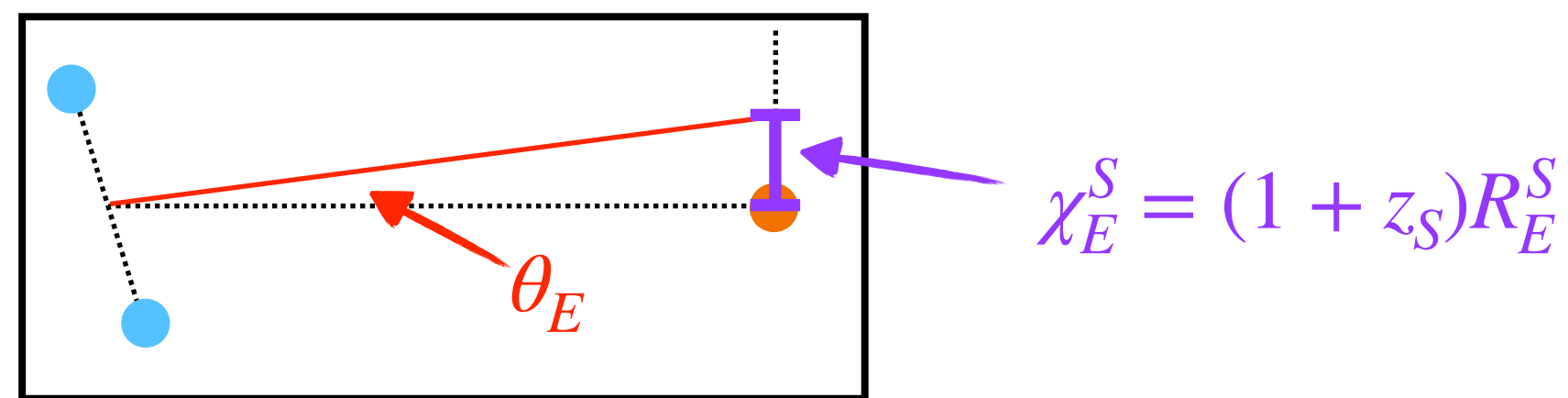
BUT: Finite sources ($\theta_S \gtrsim \theta_E$) have suppressed source-averaged magnification: $\bar{\mu} - 1 \sim \mathcal{O}(\theta_E^2/\theta_S^2)$

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$$\theta_E \sim \text{picoarcsec} \times \sqrt{\frac{M}{10^{-12} M_\odot}}$$



$$R_E^S = \mathcal{D}_S \theta_E \sim R_\odot \times \sqrt{\frac{M}{10^{-12} M_\odot}}$$

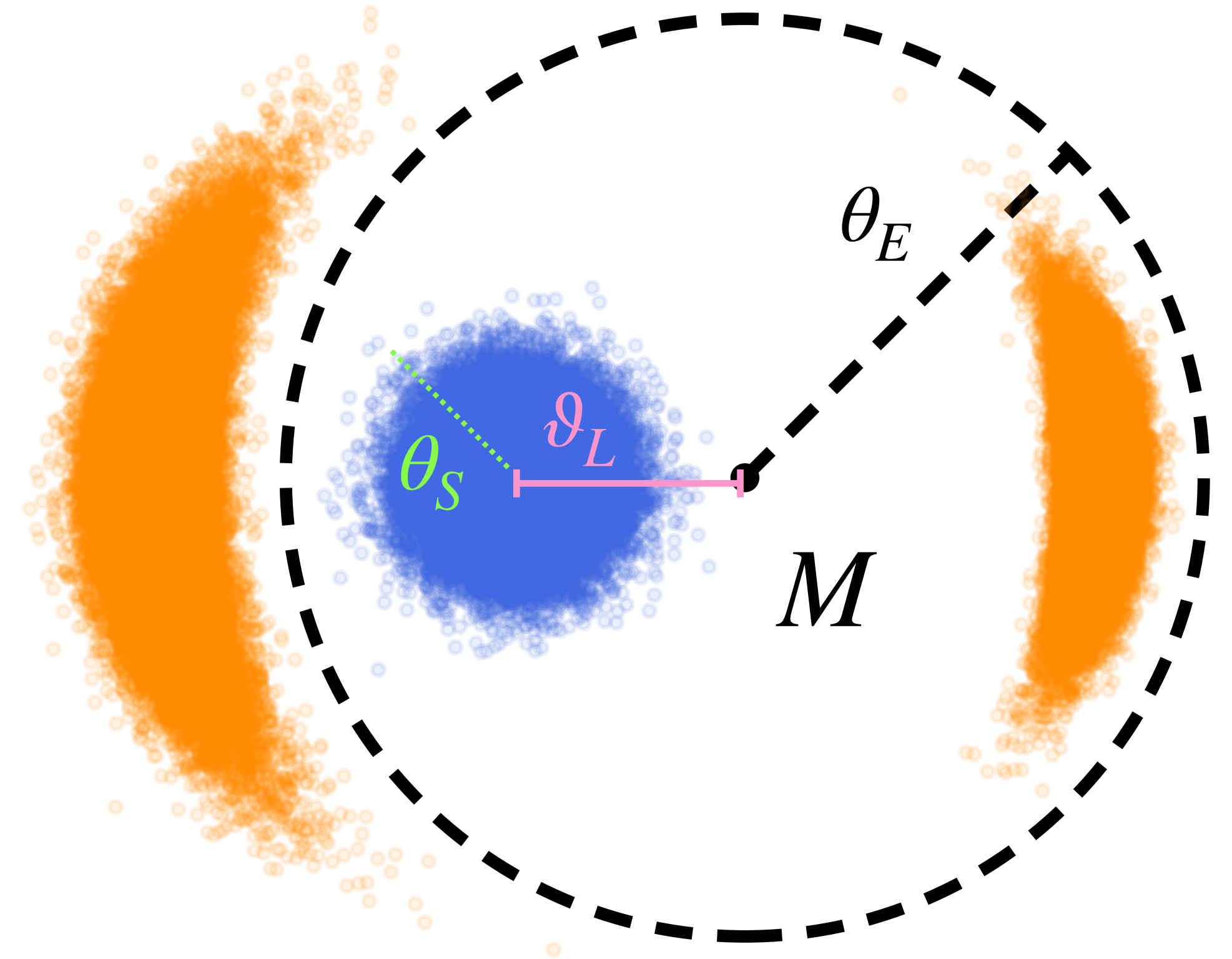


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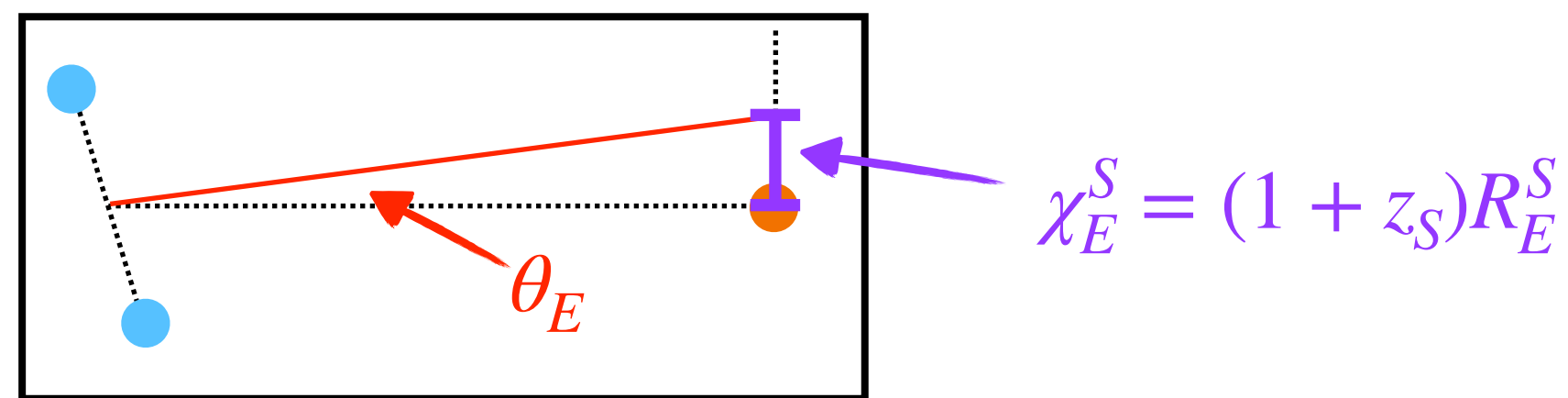
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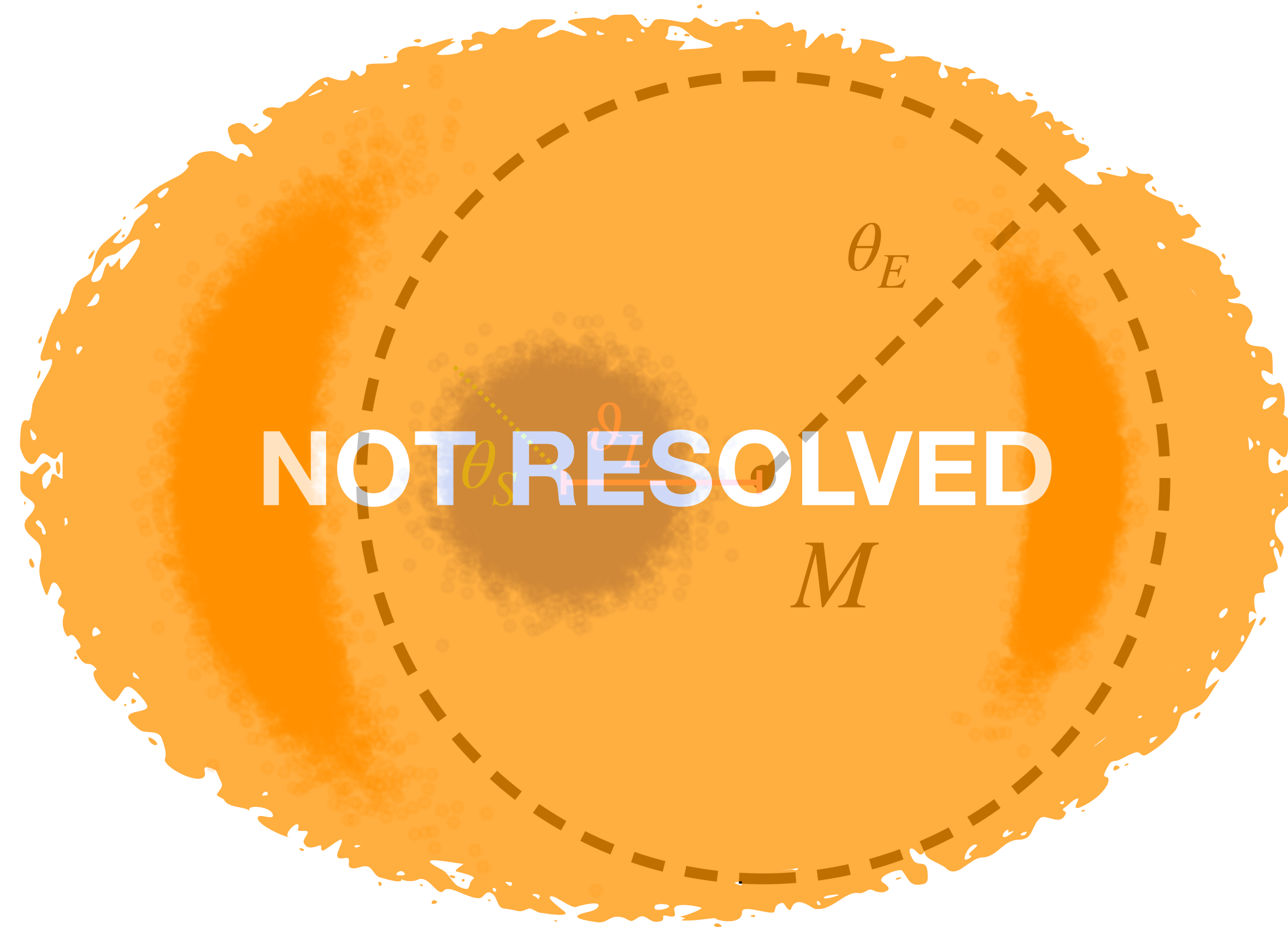
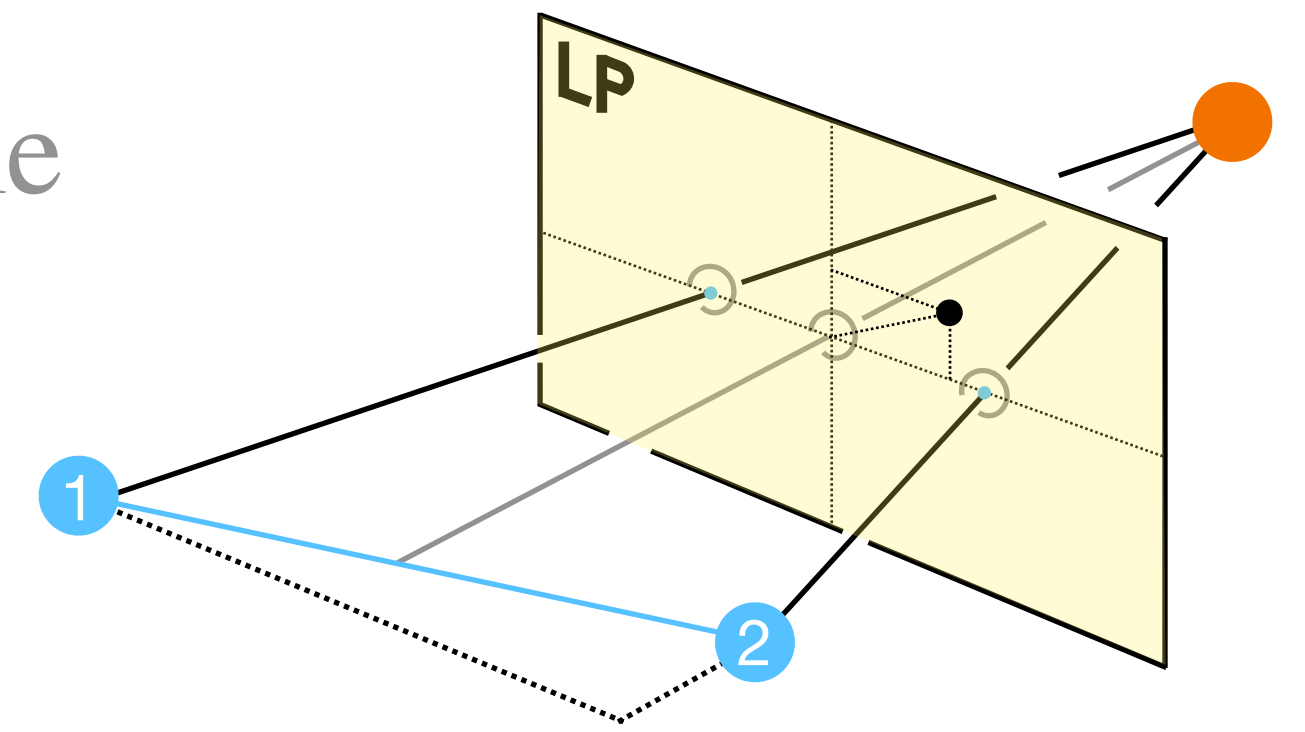


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Picolensing signal

$f = \text{flux} = \text{photons/area/time}$



Two detectors each measure photons: $N = S + B$.

Signal: $\langle S \rangle = \bar{\mu} \cdot f_S \cdot A_S \cdot T$

Background: $\langle B \rangle = f_B \cdot A_B \cdot T$

Uncertainty: $\sqrt{\langle N \rangle}$

Assumed identical
detectors, except for
magnification

Could generalise

Picolensing signal: $\Delta N = |N_2 - N_1|$.

Uncertainty: $u[\Delta N] = \sqrt{2B + S_1 + S_2}$

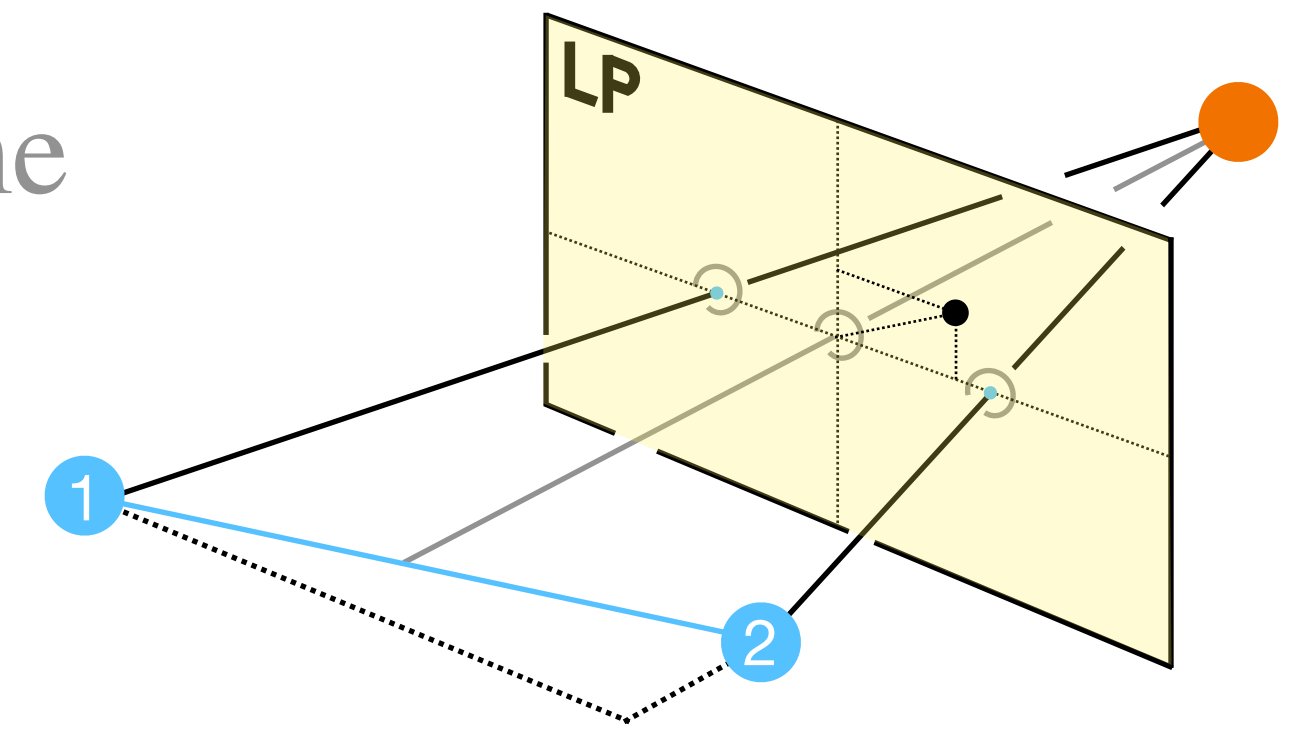
SNR: $\rho = \langle \Delta N \rangle / u[\Delta N]$

$\langle \Delta N \rangle = |\langle S_1 \rangle - \langle S_2 \rangle|$

$\propto |\bar{\mu}_1 - \bar{\mu}_2|$

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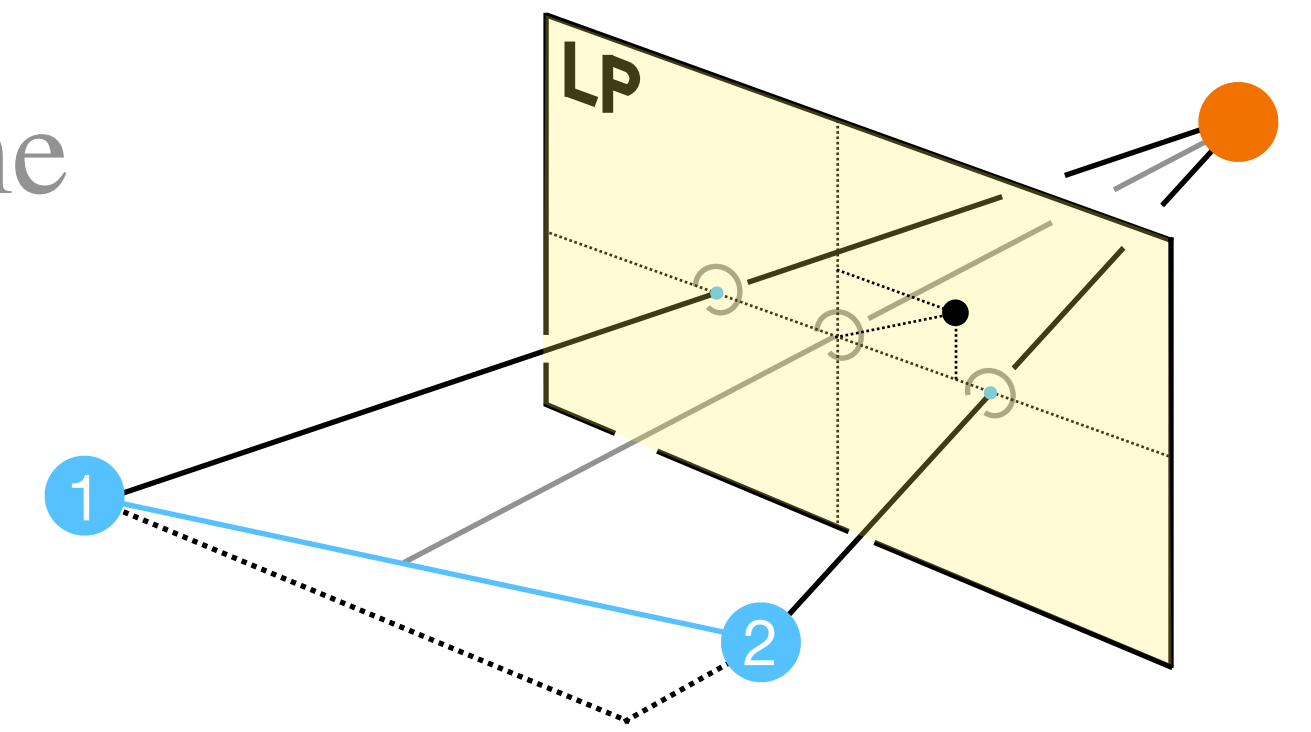
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$$\begin{aligned} \langle \Delta N \rangle &= | \langle S_1 \rangle - \langle S_2 \rangle | \\ &\propto | \bar{\mu}_1 - \bar{\mu}_2 | \end{aligned}$$

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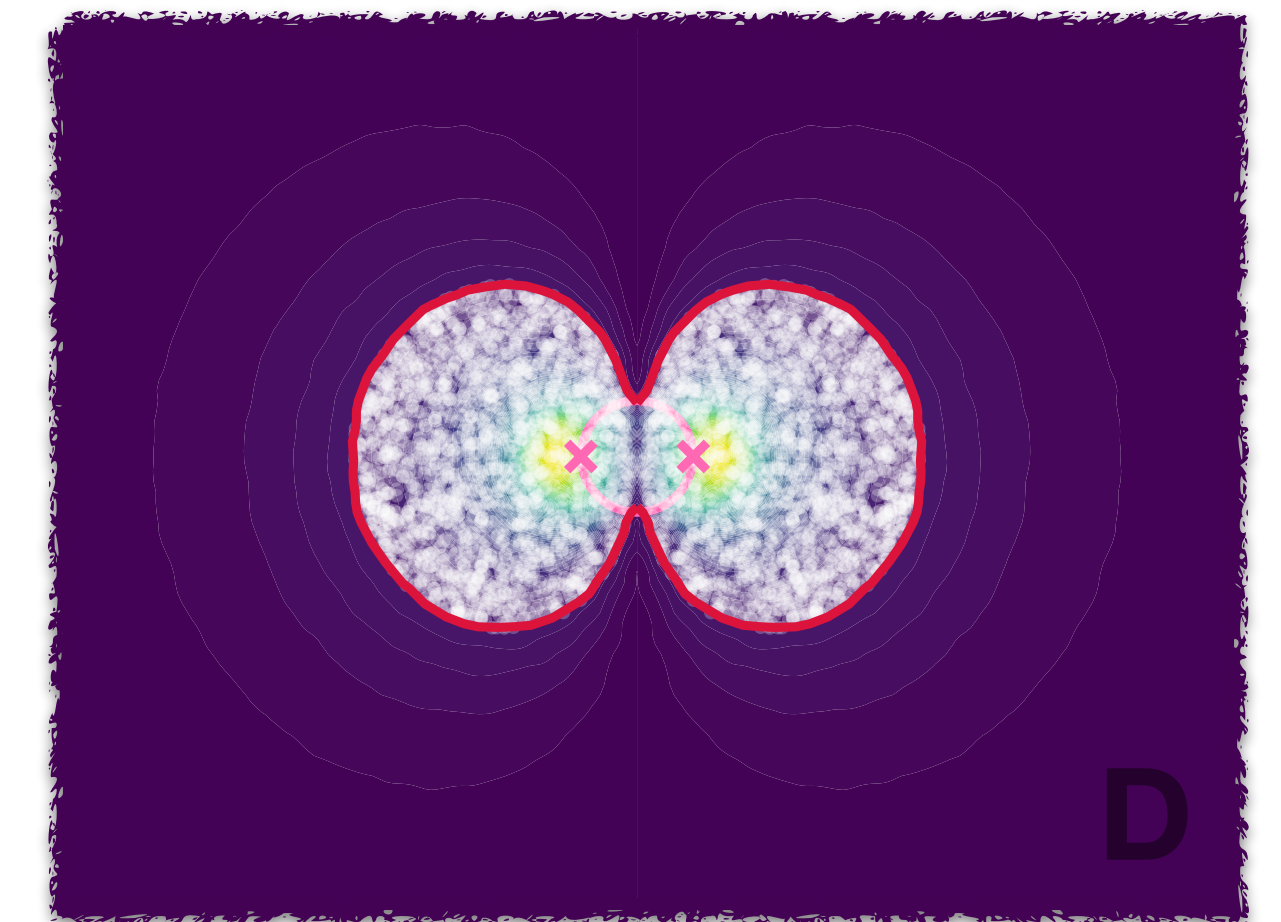
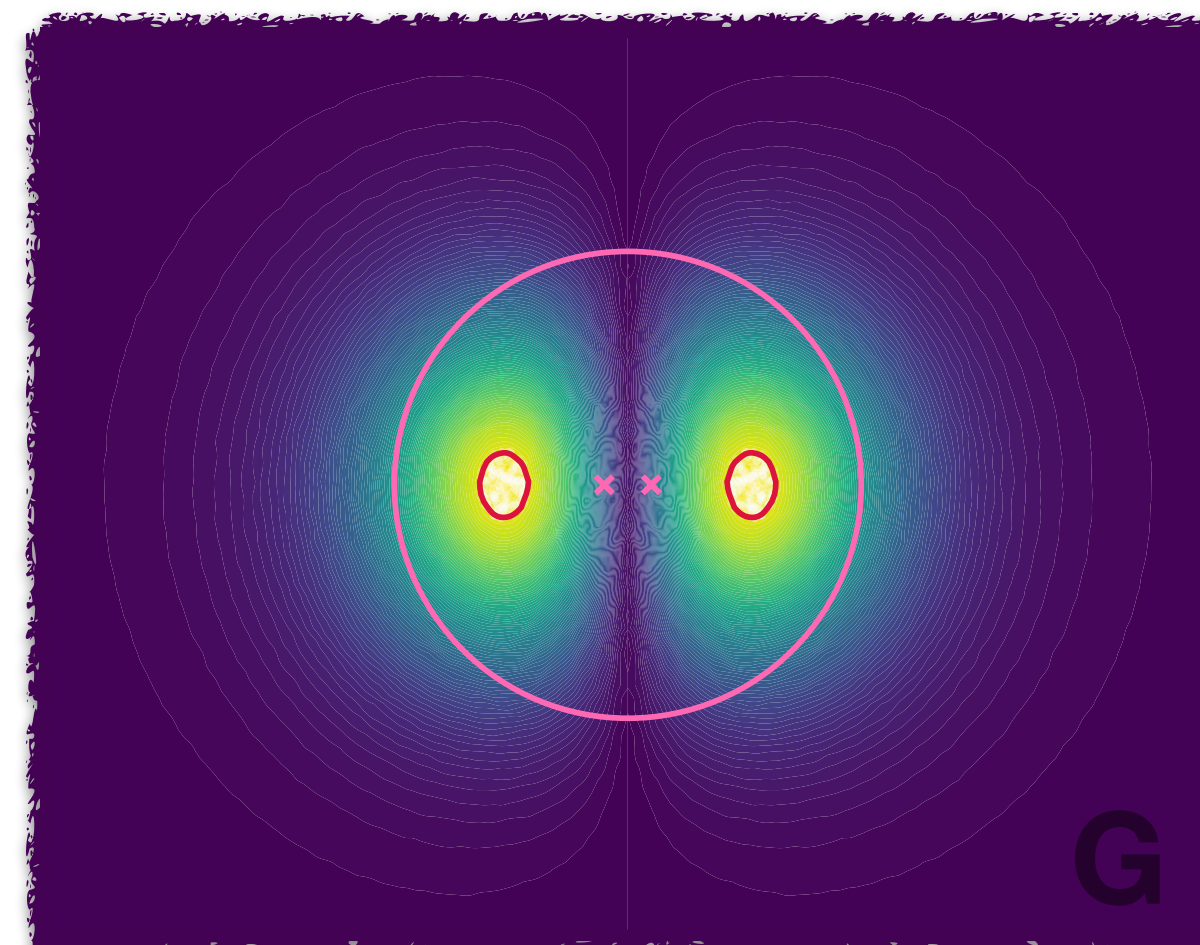
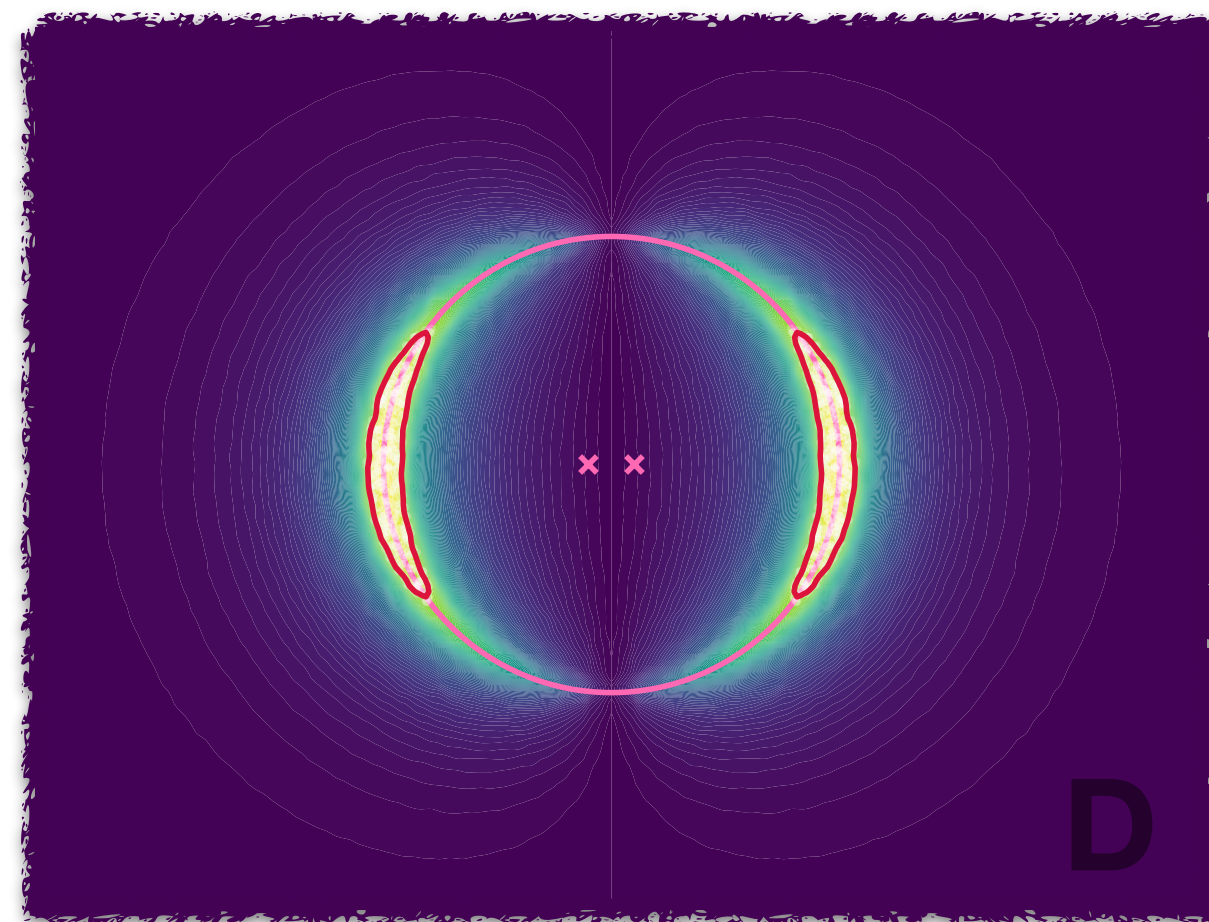
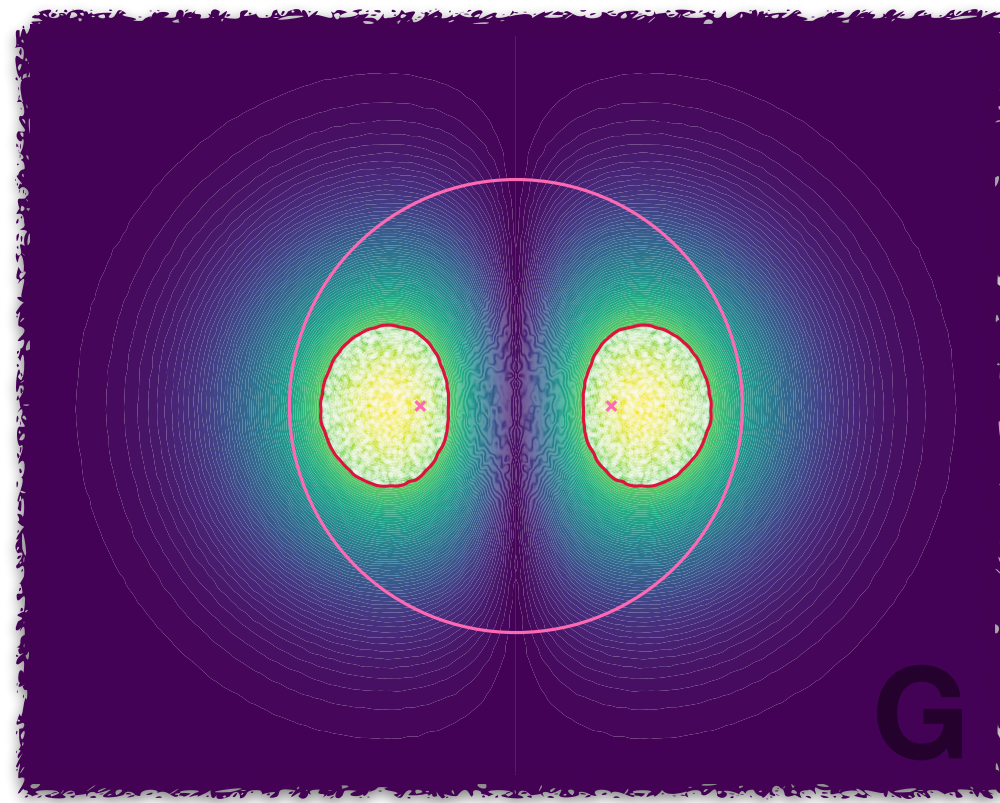
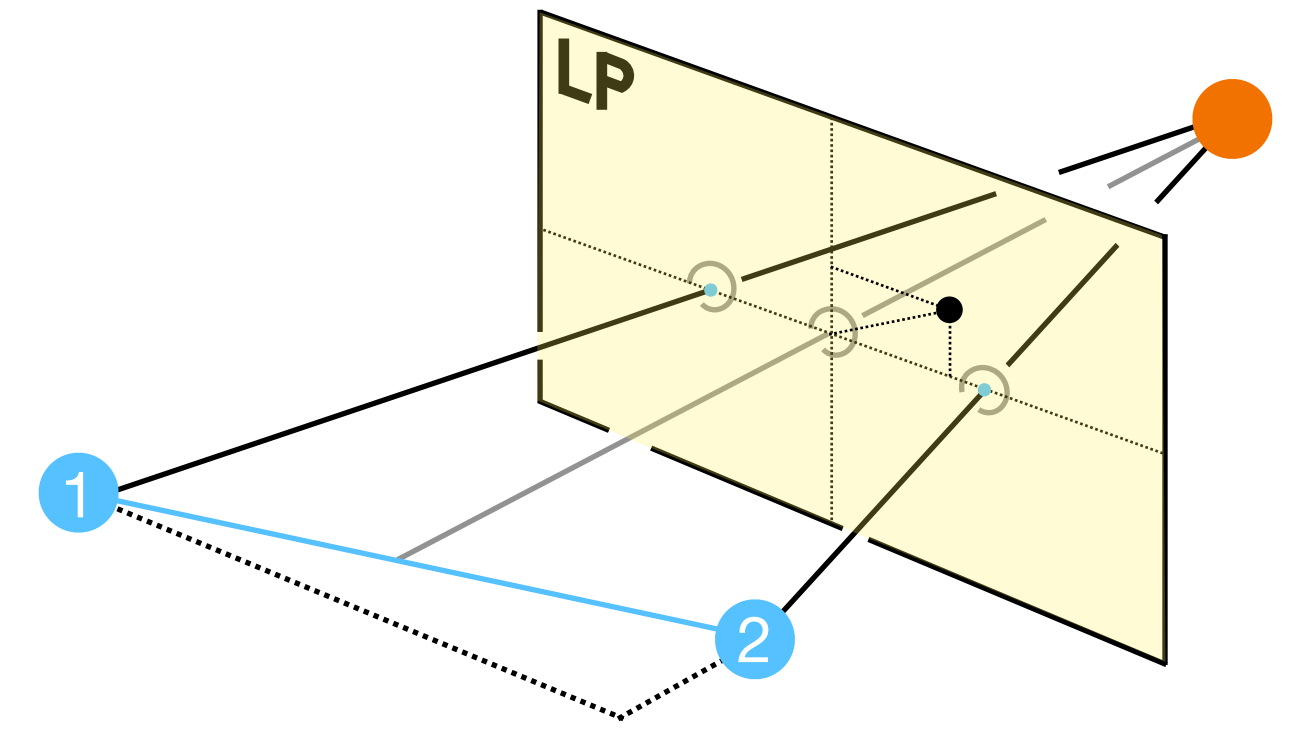
$$\propto | \bar{\mu}_1 - \bar{\mu}_2 |$$

**Need to convert this to a statement about
number of expected picolensing events**

Picolensing Cross-section σ

Region in lens plane where $\rho \geq \rho_*$, a threshold SNR.

Compute with Monte Carlo techniques.

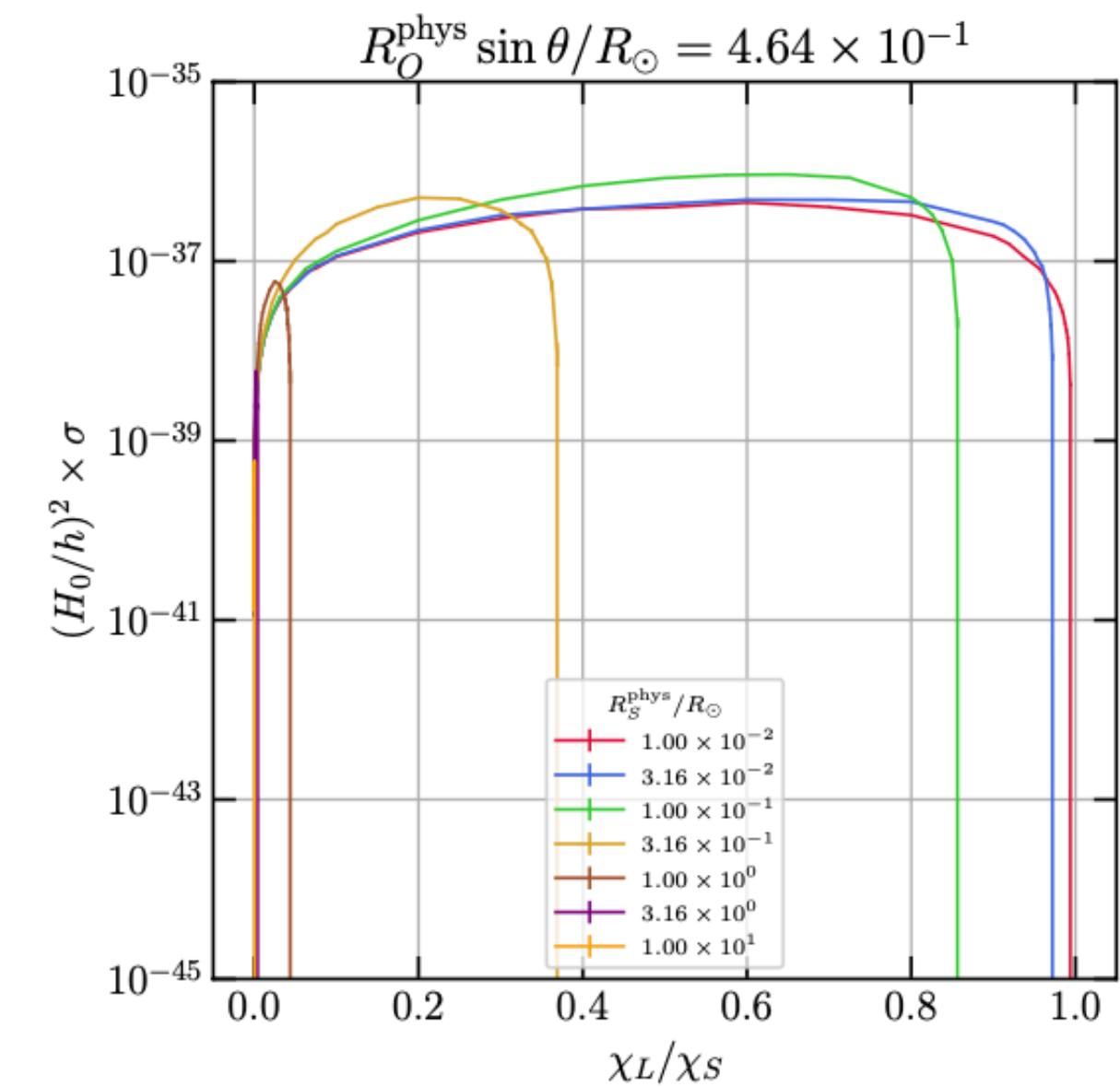


Picolensing volume & optical depth

$\sigma = \sigma(\chi_L, \dots)$ is a function of the distance to the lens plane χ_L

Co-moving picolensing volume:

$$\mathcal{V} = \int_0^{\chi_s} \sigma(\chi_L) d\chi_L$$



PBHs: uniform* co-moving lens number density n_0

Optical depth (= expected number of lenses) to source j is $\tau_j = n_0 \mathcal{V}_j$ (need $\tau_j \ll 1$ for validity)

N sources. Average lensing volume: $\overline{\mathcal{V}} \equiv \frac{1}{N} \sum_{j=1}^N \mathcal{V}_j \Rightarrow \bar{\tau} = n_0 \overline{\mathcal{V}}$ average optical depth

Straightforward Poisson statistics* to then get lensing probabilities and set limits / explore discovery space

*Jung and Kim [1908.00078] looked at clustering;
impact not significant for $\bar{\tau} \ll 1$. Poissonian stats OK.

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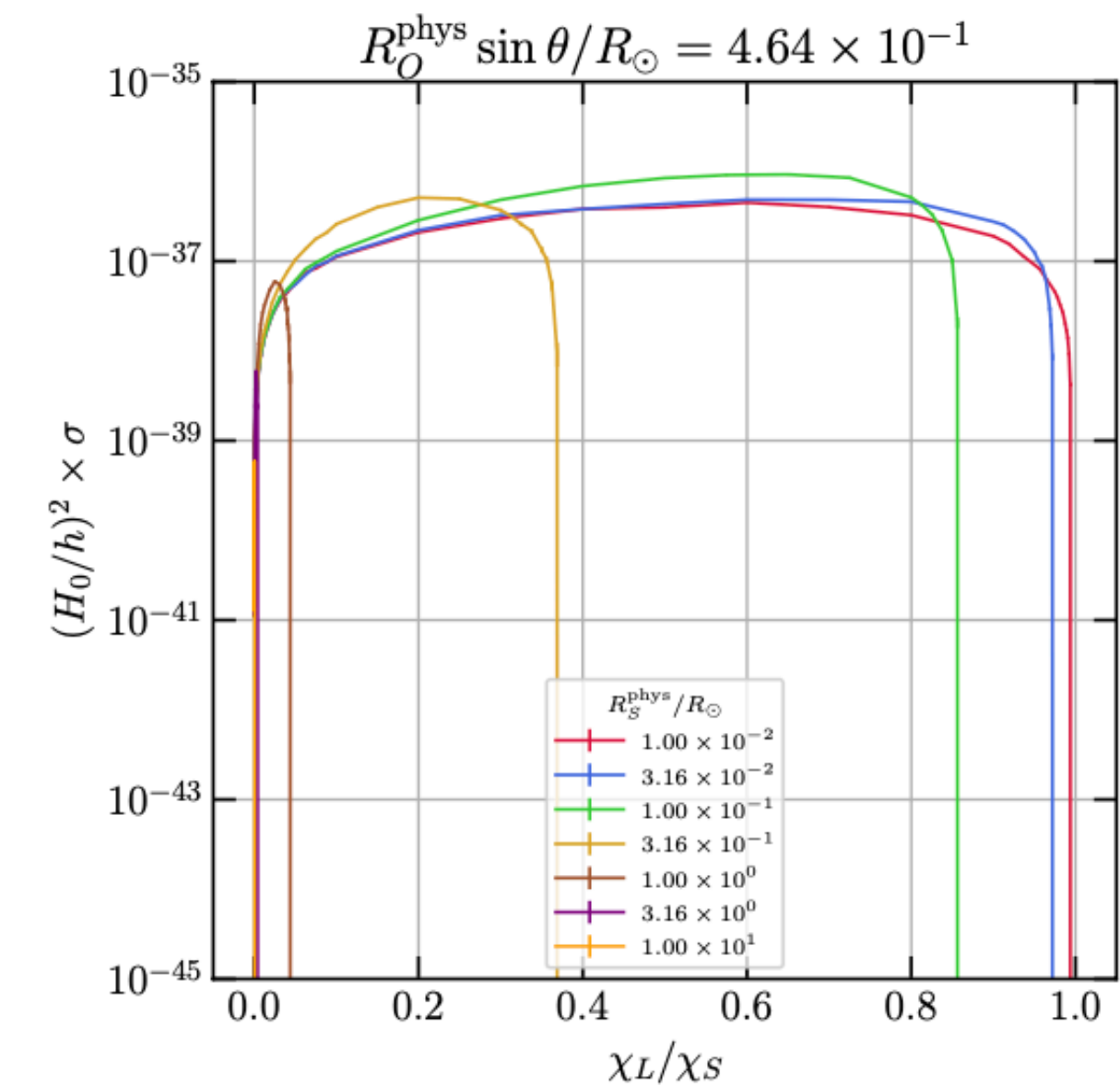
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$\sigma = \sigma(\chi_L, \dots)$ is a function of the distance to the lens plane χ_L

Co-moving picolensing volume:

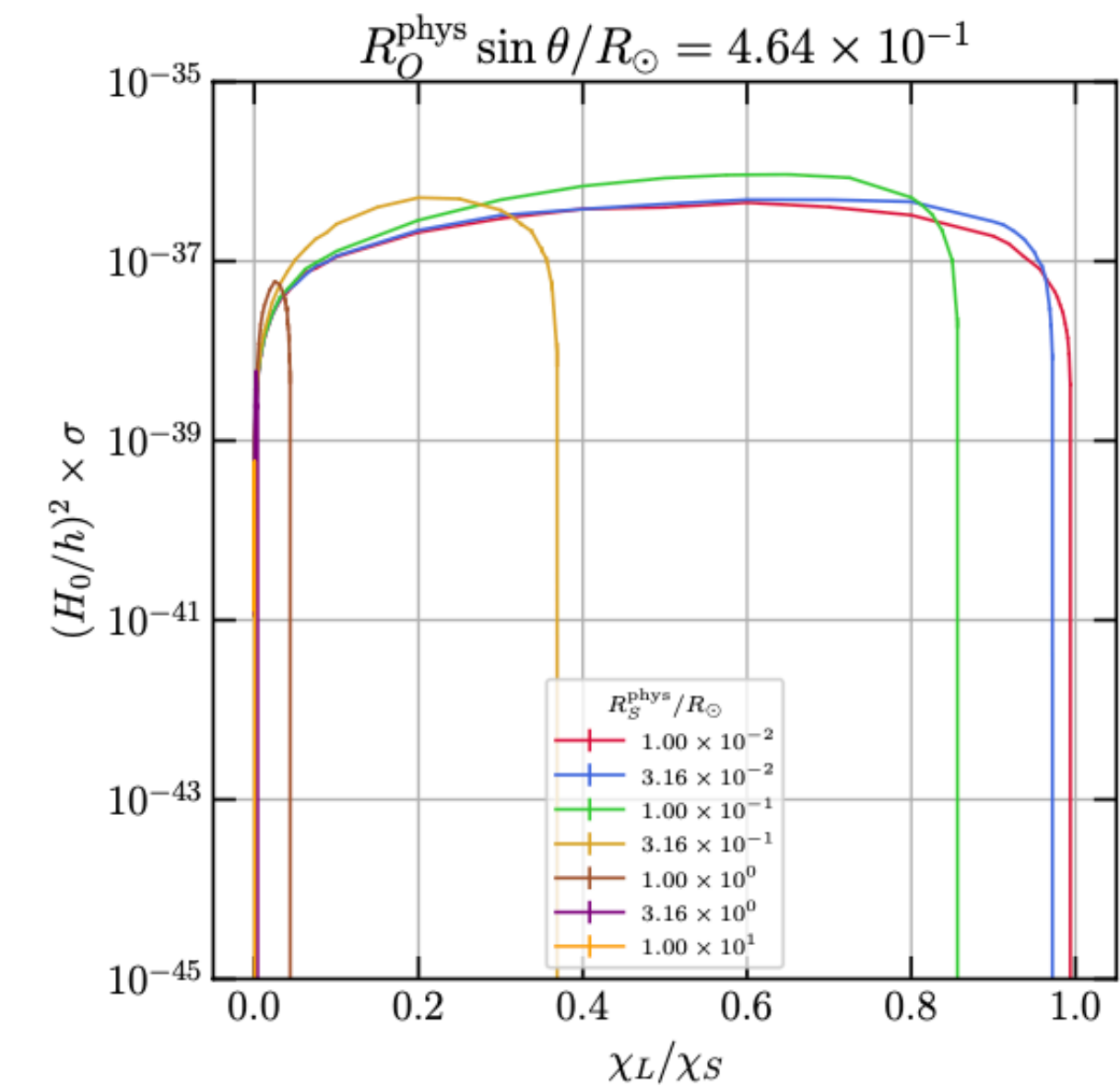
$$\mathcal{V} = \int_0^{\chi_S} \sigma(\chi_L) d\chi_L$$

PBHs: uniform* co-moving lens number density n_0

Optical depth (= expected number of lenses) to source j is $\tau_j = n_0 \mathcal{V}_j$ (need $\tau_j \ll 1$ for validity)

N sources. Average lensing volume: $\overline{\mathcal{V}} \equiv \frac{1}{N} \sum_{j=1}^N \mathcal{V}_j \Rightarrow \bar{\tau} = n_0 \overline{\mathcal{V}}$ average optical depth

Straightforward Poisson statistics* to then get lensing probabilities and set limits / explore discovery space



*Jung and Kim [1908.00078] looked at clustering;
impact not significant for $\bar{\tau} \ll 1$. Poissonian stats OK.

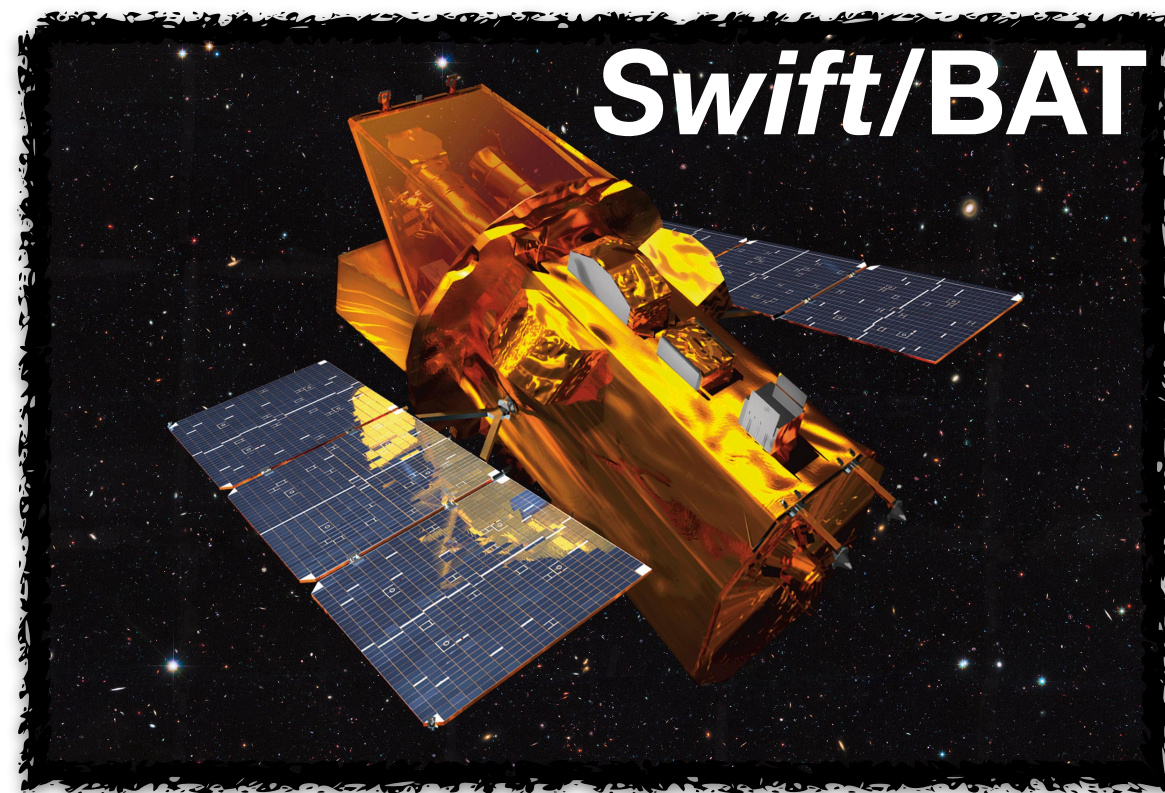
Sources: Gamma-Ray Bursts (GRBs)

LENSING:
GEOMETRICAL
OPTICS!

Transient. Two classes: long (> 2 s) and short (< 2 s)

Long: cosmologically distant ($\bar{z}_S \sim 2$) and **very** bright in x/ γ -rays.

Highly beamed emission: $\Gamma \sim 10^2$ $E_{\text{beam}} \sim 3 \times 10^{51} \text{ erg} \sim (2 - 3) \cdot E_{\text{SN Ia}}$



Gawade et al [2308.01775] looked at a possible future ISRO project Daksha ($2 \times \sim$ Swift/BAT-class detectors in space, but each with *Fermi*/GBM sky coverage)

For results today: **assume similar parameters to $2 \times$ Swift/BAT** ($\sim$ Daksha)

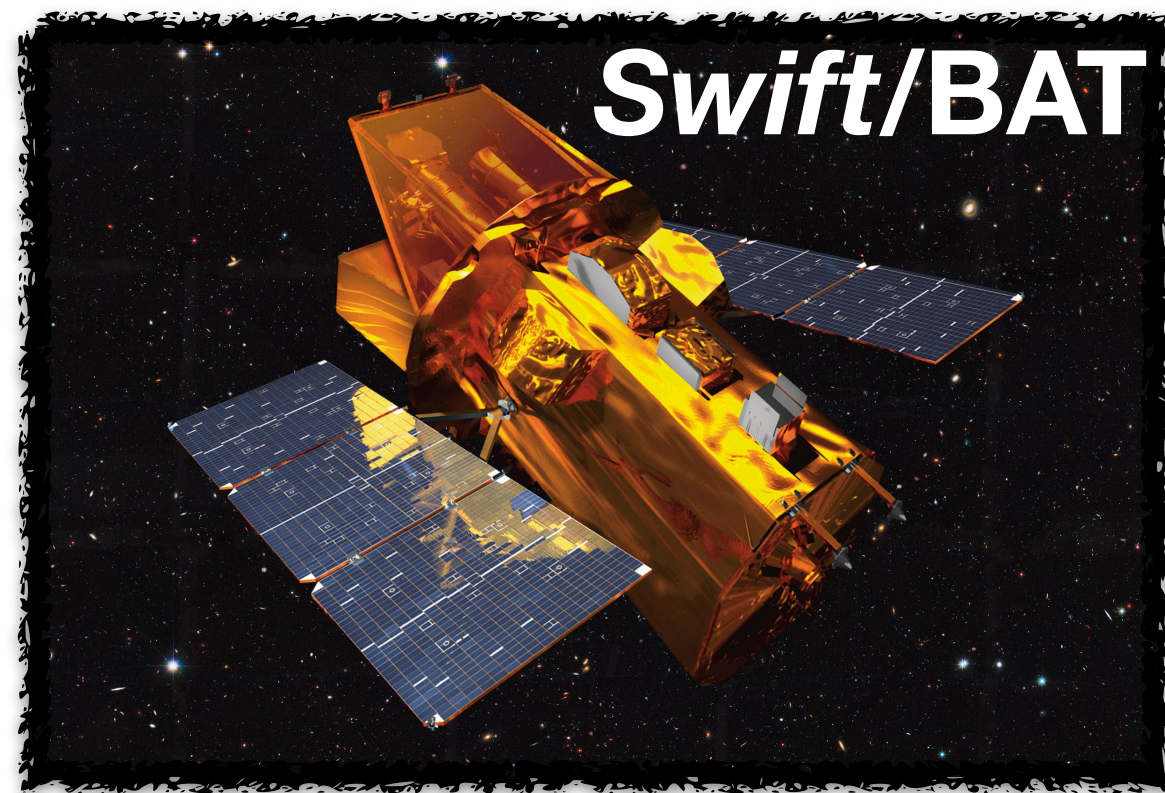
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Source characteristics: *Swift*/BAT catalogue

Use real GRBs to make population-informed projections.

For each GRB, need:

Duration: T_{90} — time for 90% of measured intensity after trigger

Distance: z_S — known for ~ 409 GRBs in the catalogue

Brightness: power law (PL) or cut-off power law (CPL) fit to energy spectrum

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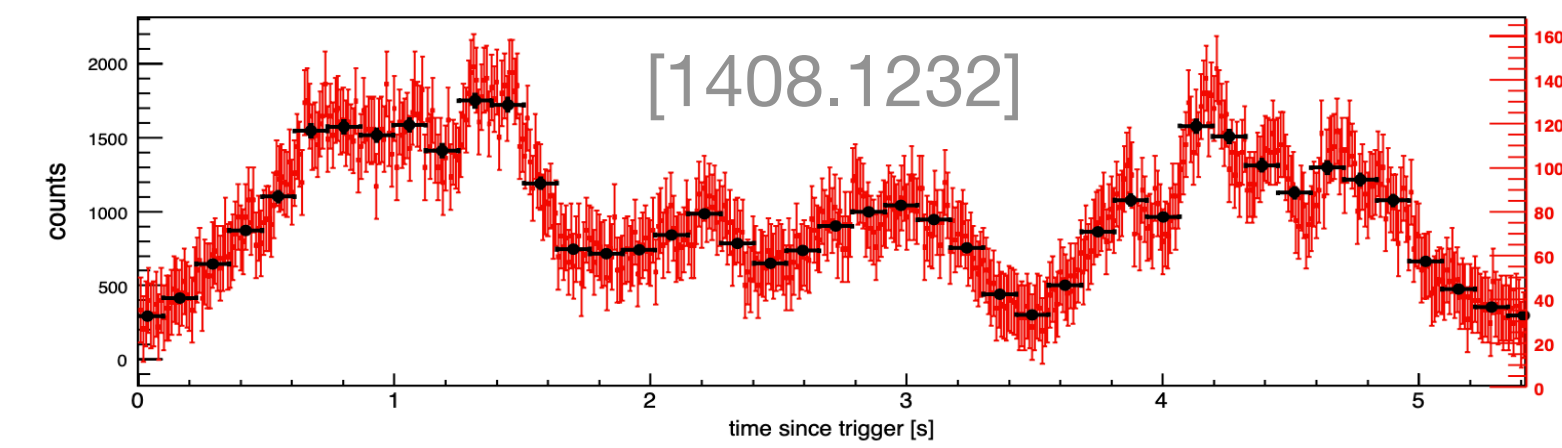
Duration: T_{90} — time for 90% of measured intensity after trigger

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Brightness: power law (PL) or cut-off power law (CPL) fit to energy spectrum

... AND: SIZE θ_S

GRB sizes



θ_S indirectly inferred from “minimum variability timescale” Δt_{var}

Gives estimate of the source light crossing time

$$D' \sim \frac{\Gamma^2 \times \Delta t_{\text{var}}}{1 + z_S}$$

Physical size of emission region

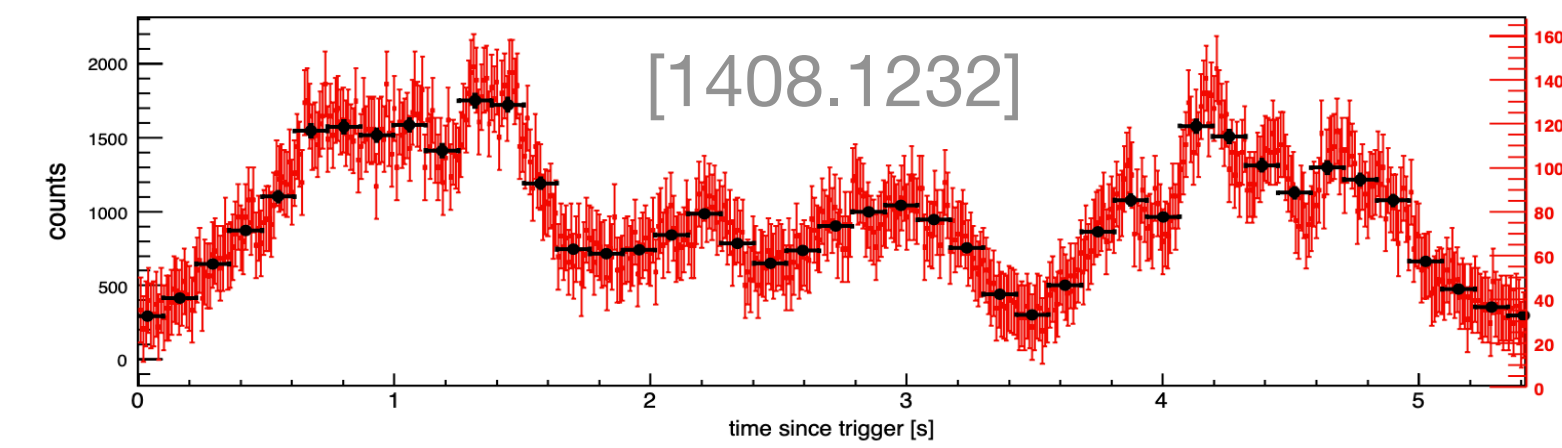
$$D_{\text{obs}} \sim \frac{D'}{\Gamma} \sim \frac{\Gamma \times \Delta t_{\text{var}}}{1 + z_S}$$

Observed size (beaming)

EMPIRICAL RELATIONSHIP: $T_{90} \sim \Gamma \Delta t_{\text{var}}$

Commonly used size estimate: $D_{\text{obs}} \sim \frac{T_{90}}{1 + z_S} \Rightarrow \theta_S = \frac{T_{90}}{\chi_S}$

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Commonly used size estimate: $D_{\text{obs}} \sim \frac{T_{90}}{1 + z_S} \Rightarrow$

Gawade, More, Bhalerao [2308.01775] used this

$$\theta_S = \frac{T_{90}}{\chi_S}$$

GRB size data are **very uncertain**

$$D_{\text{obs}} \sim \frac{D'}{\Gamma} \sim \frac{\Gamma \times \Delta t_{\text{var}}}{1 + z_S}$$

$$D' \sim \frac{\Gamma^2 \times \Delta t_{\text{var}}}{1 + z_S}$$

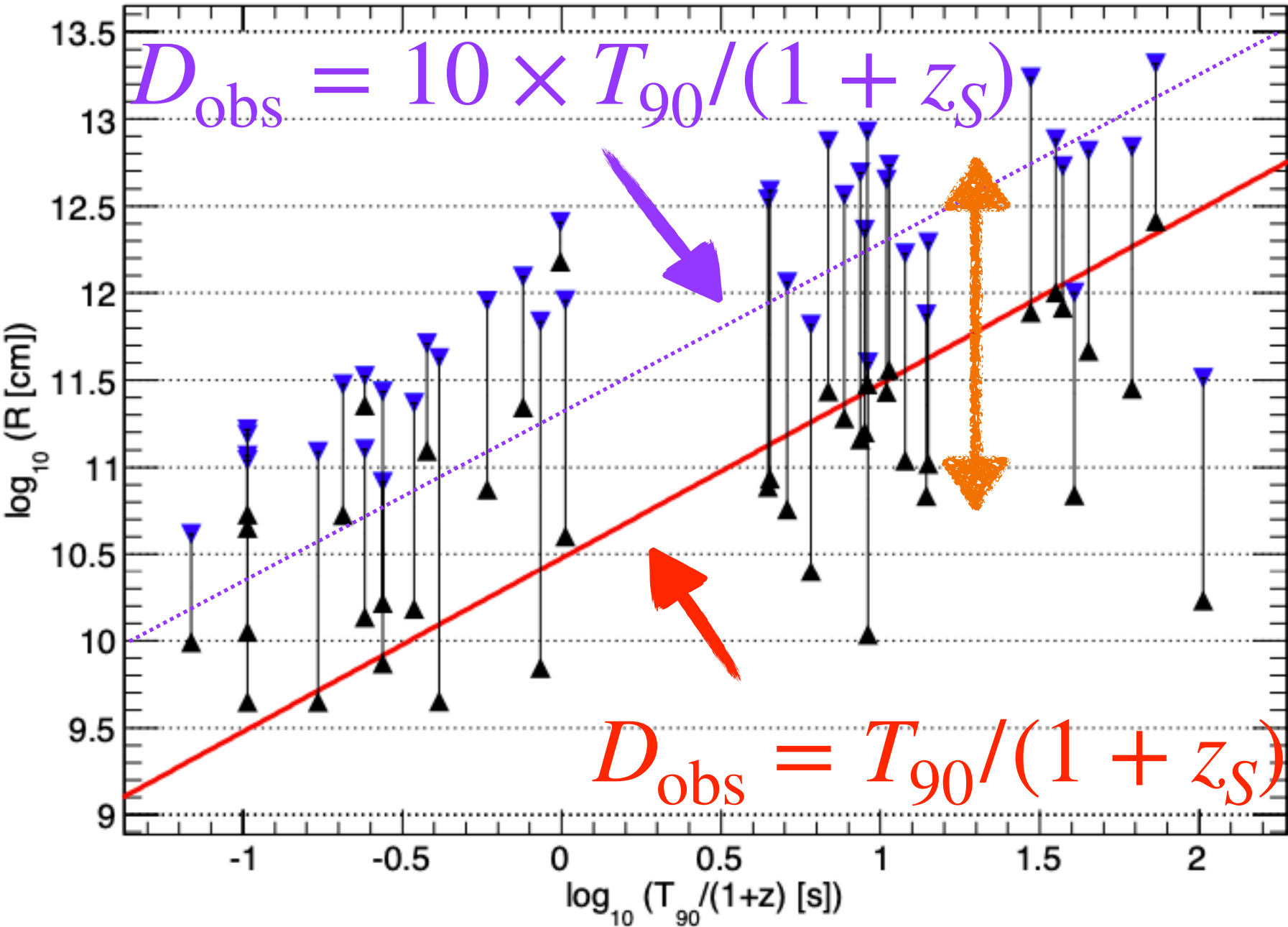
-Scatter?

-Offset?

-Trend?

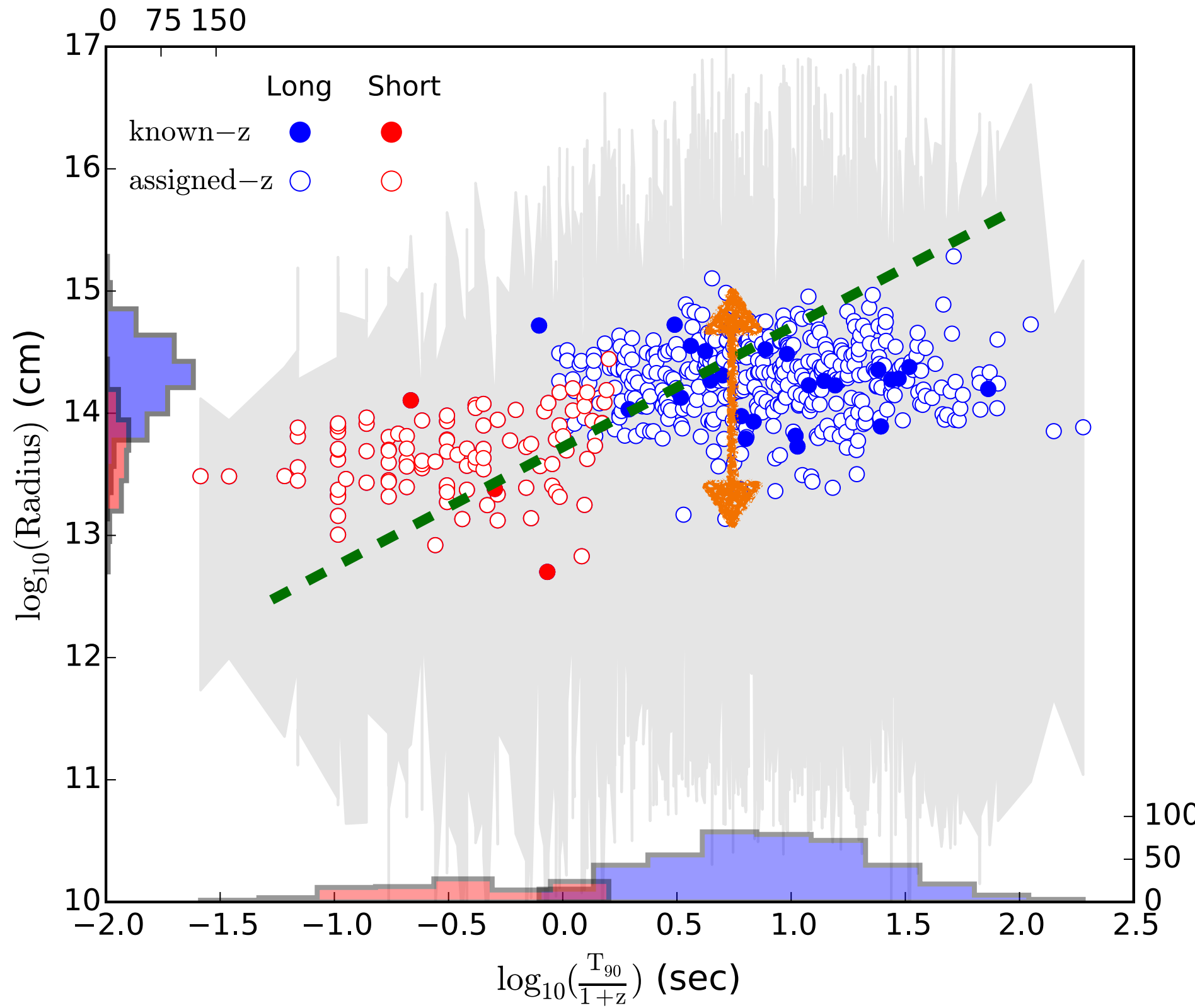


y-axes
differ by
 $\Gamma \sim 10^2$



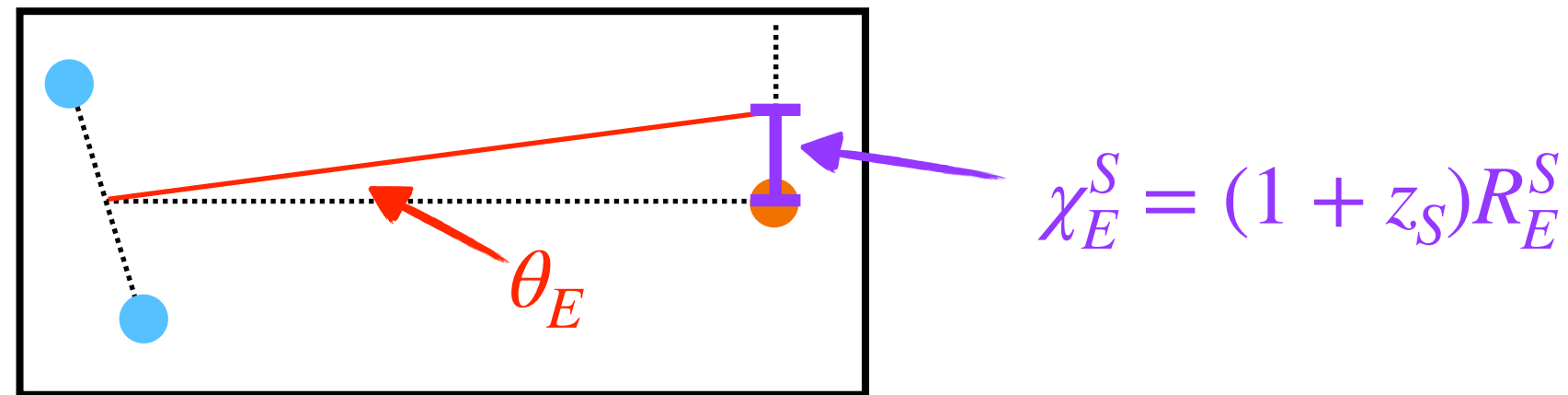
Note that this line is not attempting to fit the constraints from the data

Barnacka, Loeb [1409.1232]



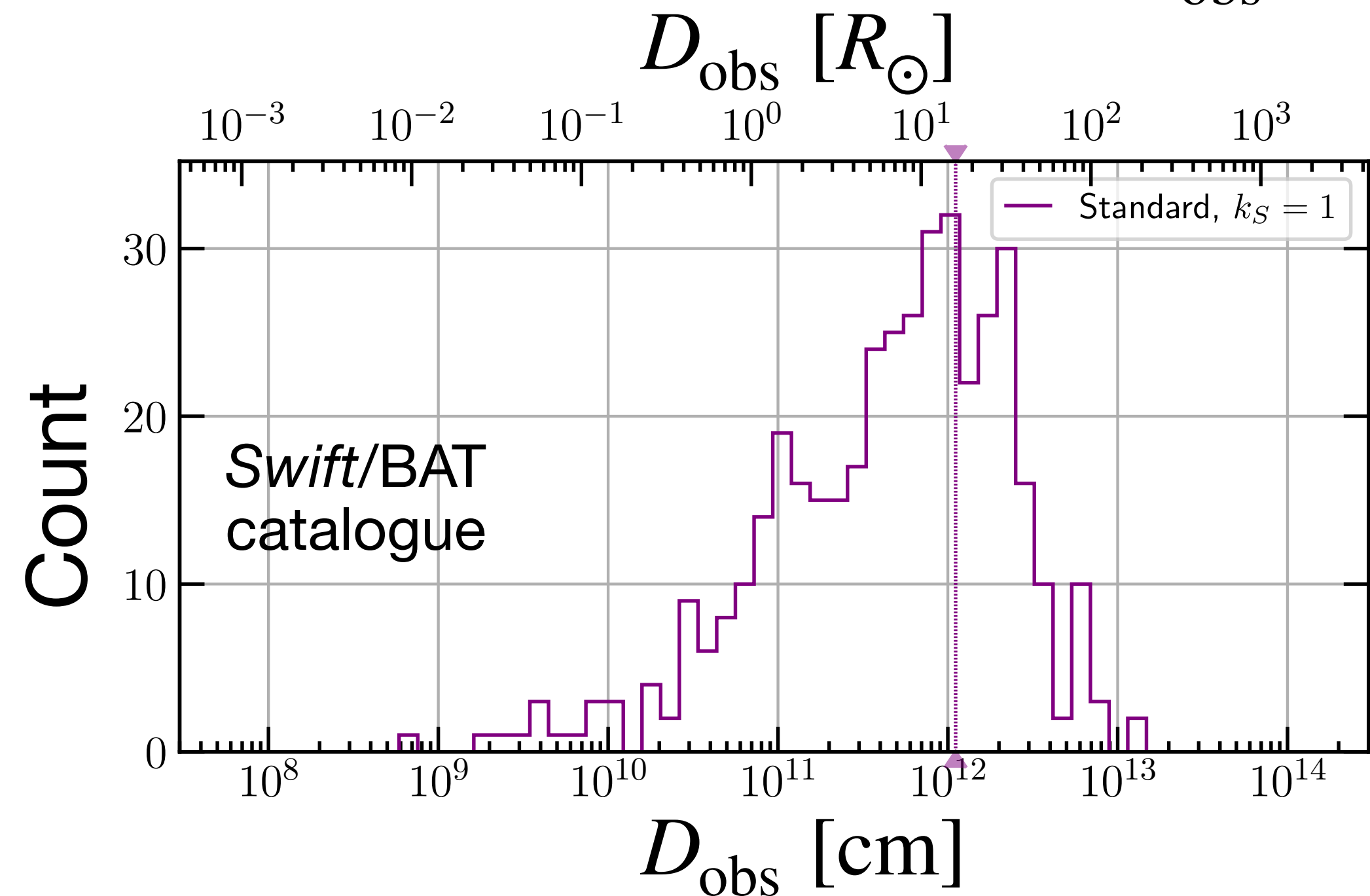
Golkhou et al [1501.05948]

This can matter!



$$R_E^S = \mathcal{D}_S \theta_E \sim R_\odot \times \sqrt{\frac{M}{10^{-12} M_\odot}}$$

$$D_{\text{obs}} \equiv \frac{k_S T_{90}}{1 + z_S}$$

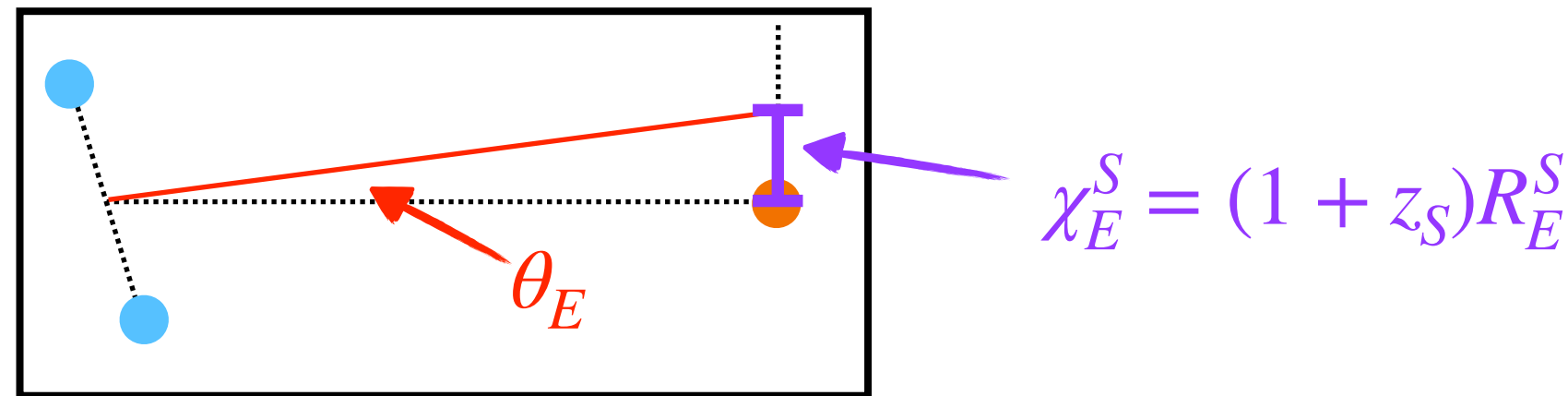


Source sizes can be large vs the Einstein radius in the source plane!

Magnifications are sensitive to source size uncertainties!

Enormous uncertainties, so we just bracket: $k_S = 0.1, 1, 10, 10^{\mathcal{U}[-1,1]}$

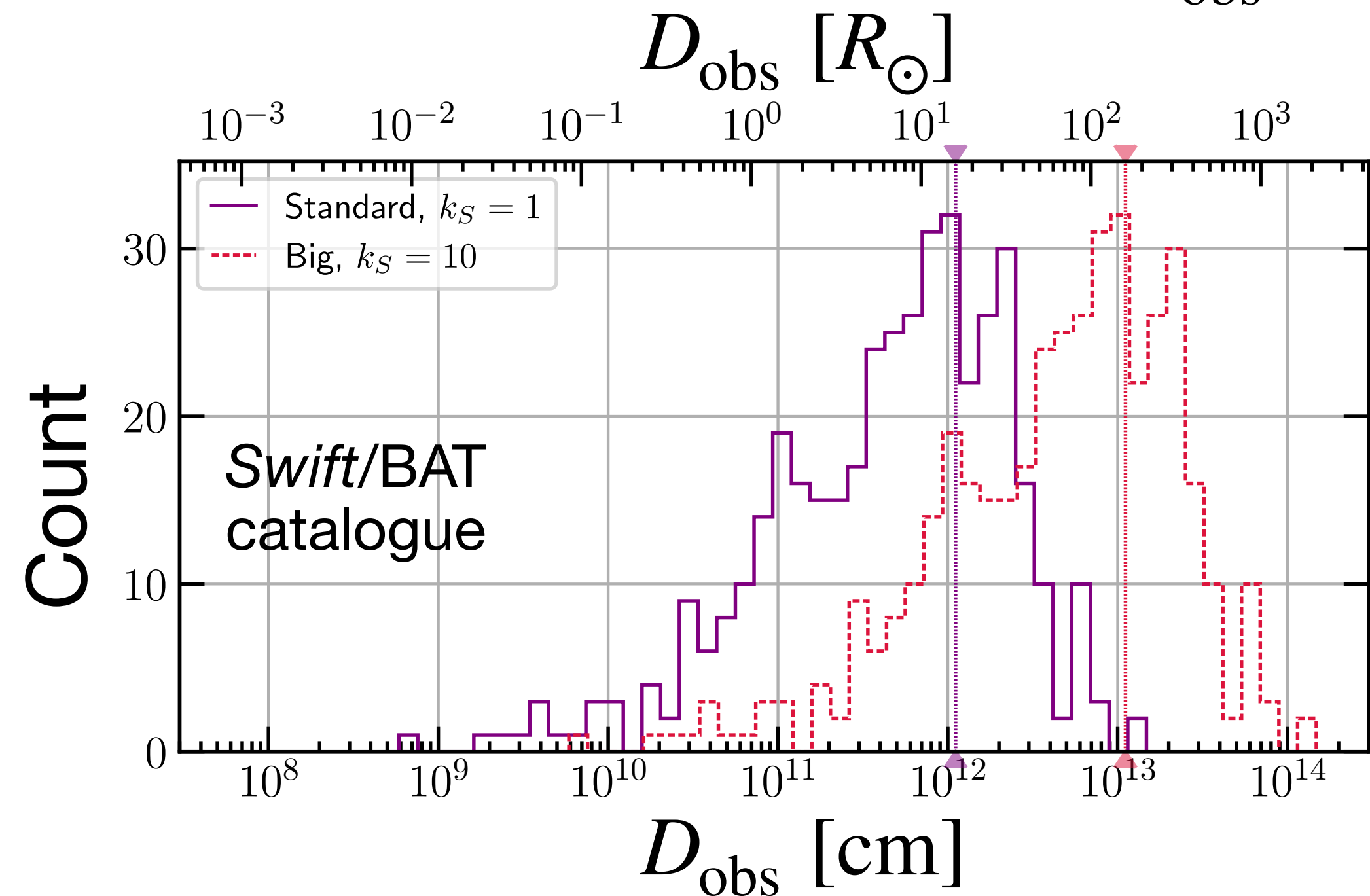
This can matter!



$$\chi_E^S = (1 + z_S) R_E^S$$

$$R_E^S = \mathcal{D}_S \theta_E \sim R_\odot \times \sqrt{\frac{M}{10^{-12} M_\odot}}$$

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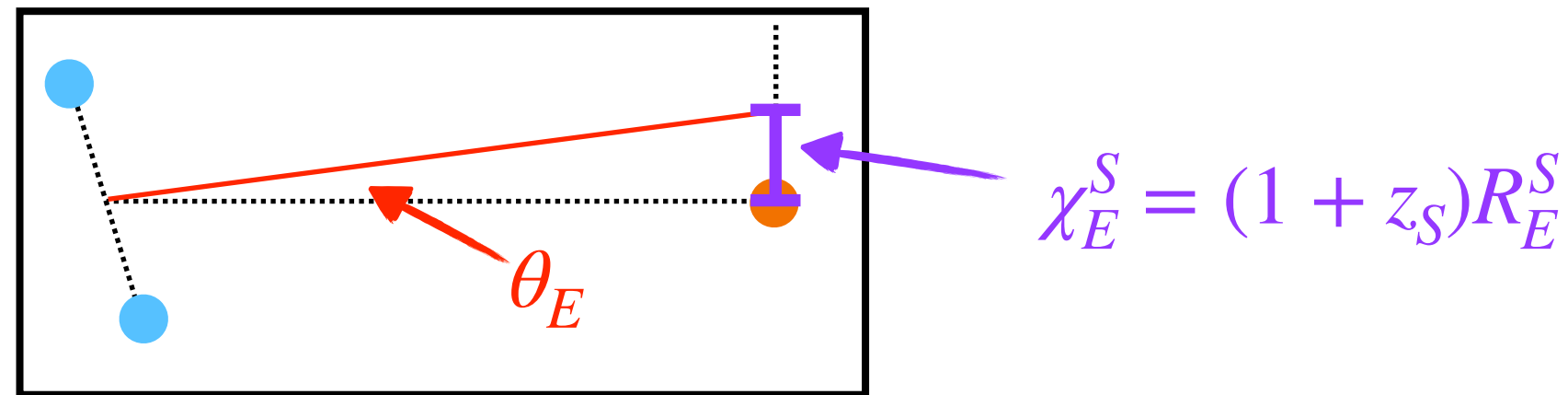


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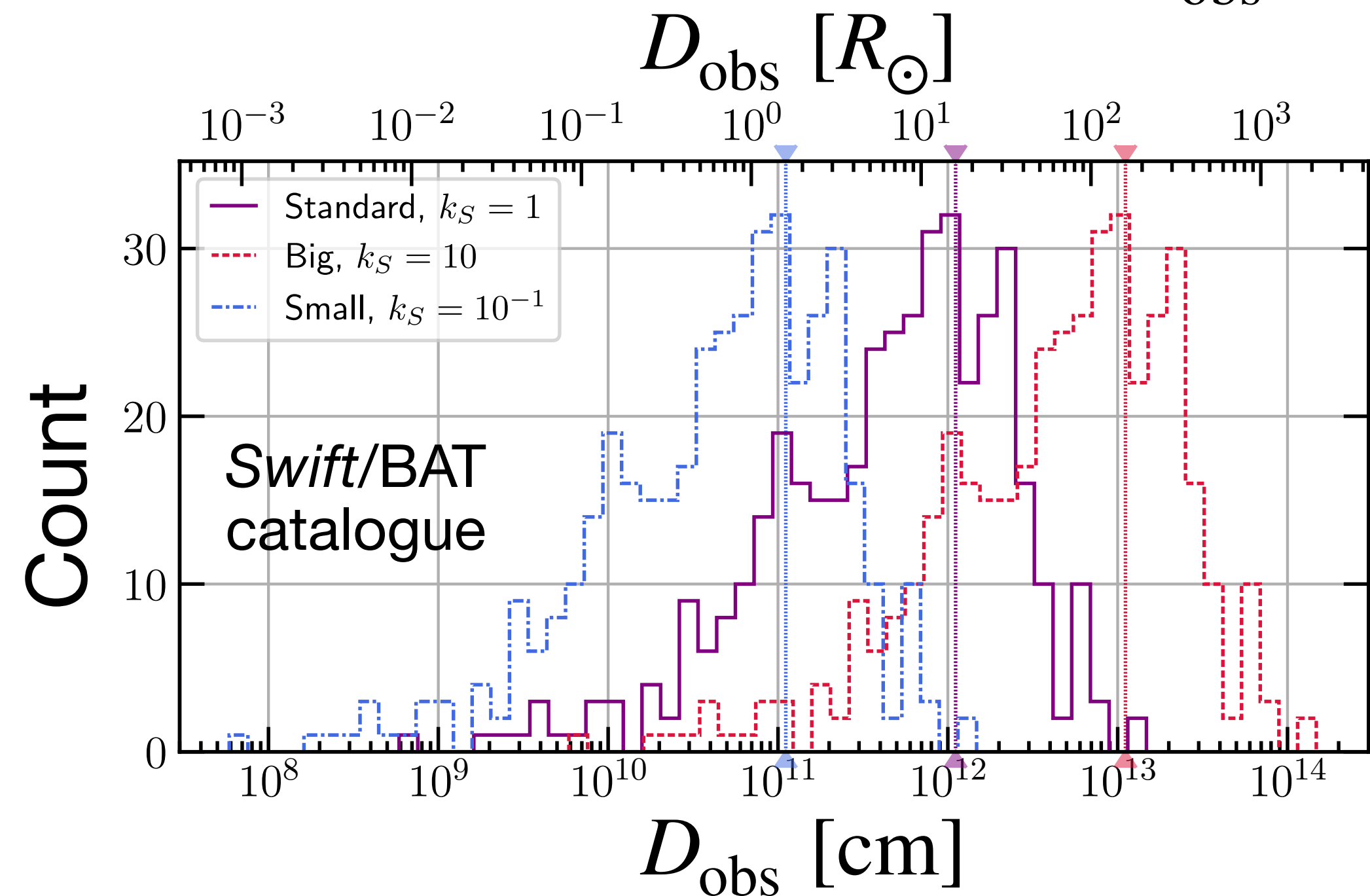
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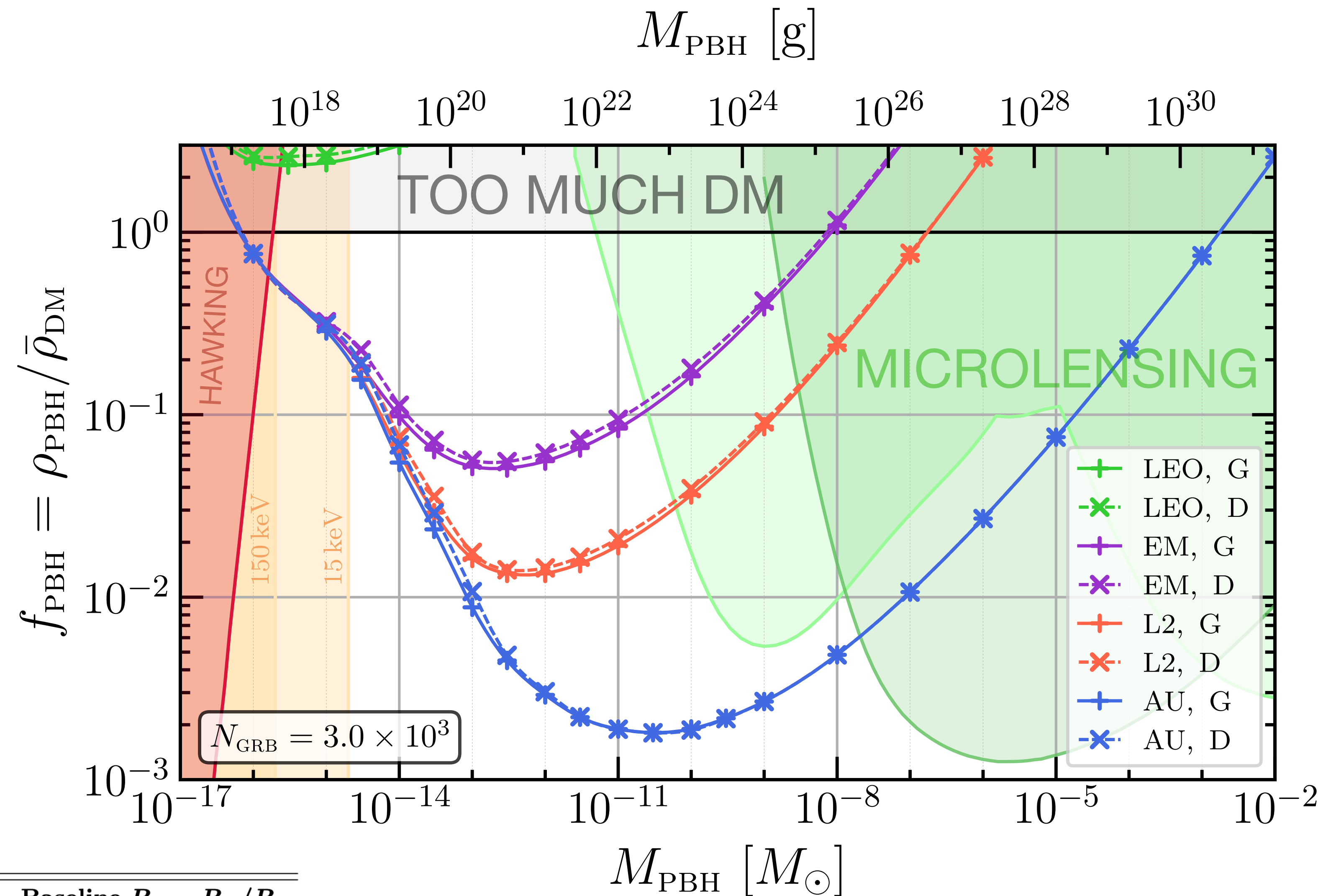
Enormous uncertainties, so we just bracket: $k_S = 0.1, 1, 10, 10^{\mathcal{U}[-1,1]}$

RESULTS

95% CL exclusion projections assuming no 5σ detections

Different baselines, different source profiles

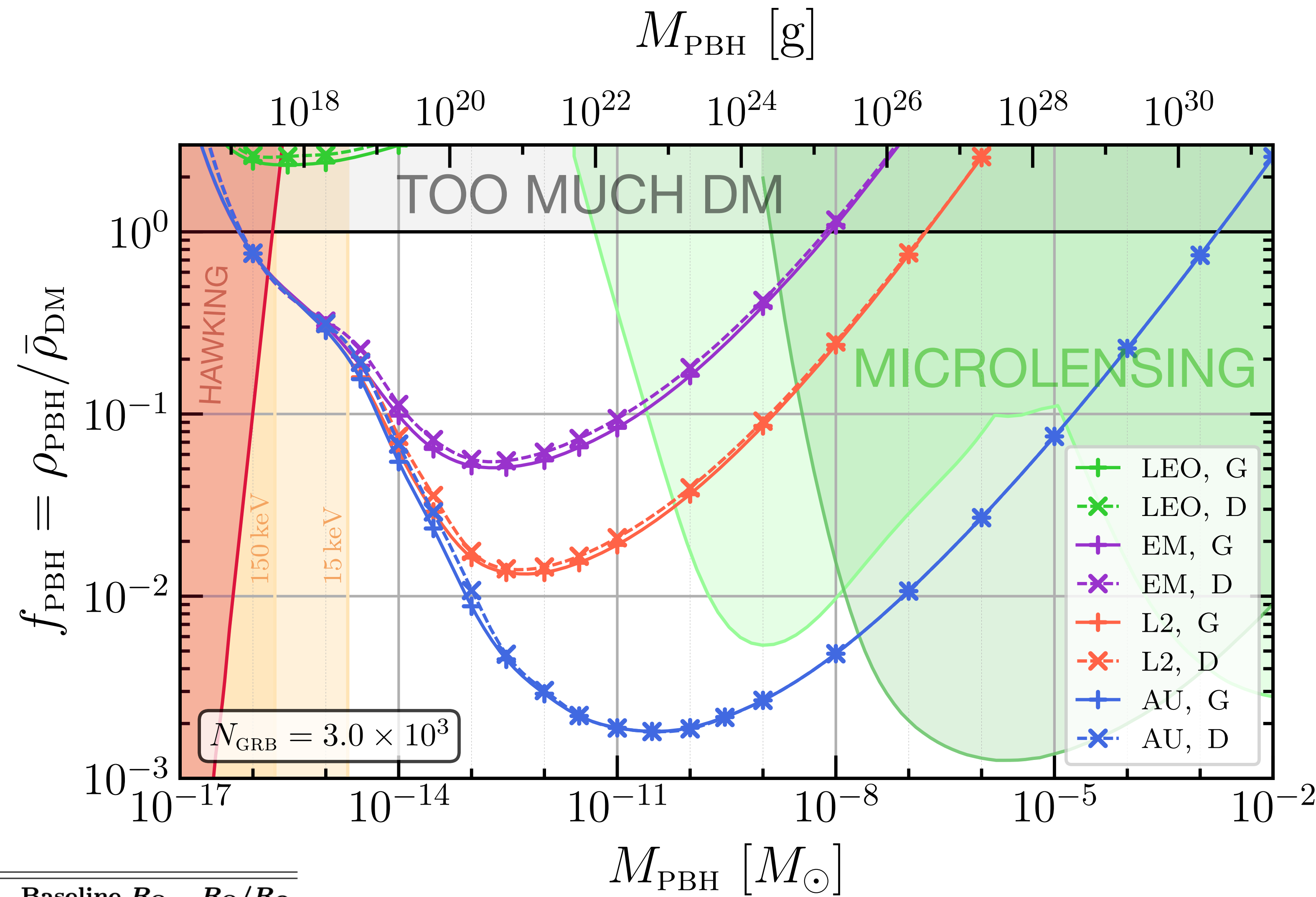
$$D_{\text{obs}} \equiv \frac{T_{90}}{1 + z_S}$$



Scenario	Abbrev.	Baseline R_O	$R_O/R_\odot$
Low Earth Orbit	LEO	1.40×10^4 km	0.020
Earth–Moon	EM	3.84×10^5 km	0.55
Lagrange Point 2	L2	1.50×10^6 km	2.15
Astronomical Unit	AU	1.50×10^8 km	215

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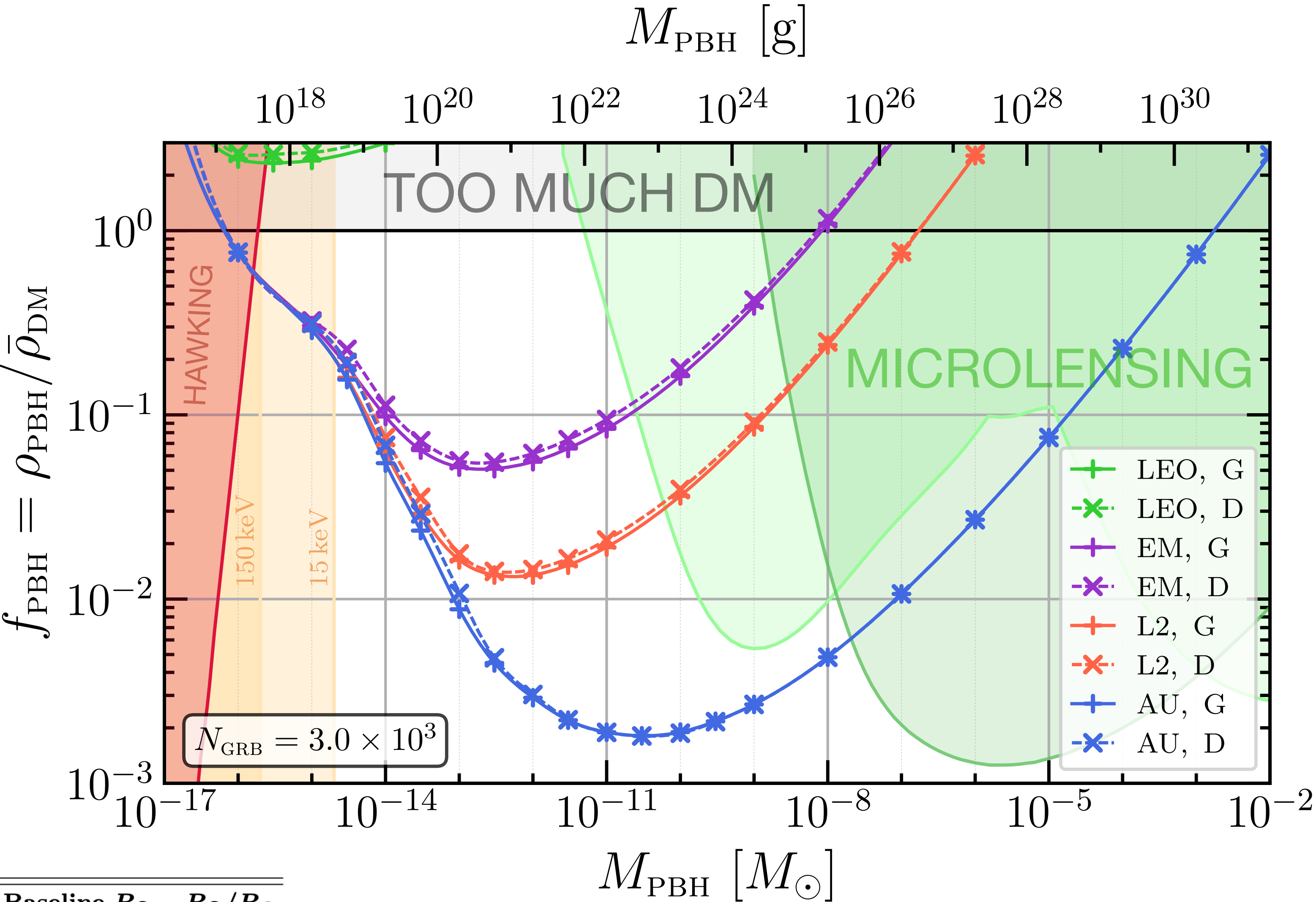


**Essentially no
difference in
Gaussian (G)
vs. Disk (D)
source profiles**

Scenario	Abbrev.	Baseline R_{O}	$R_{\text{O}}/R_{\odot}$
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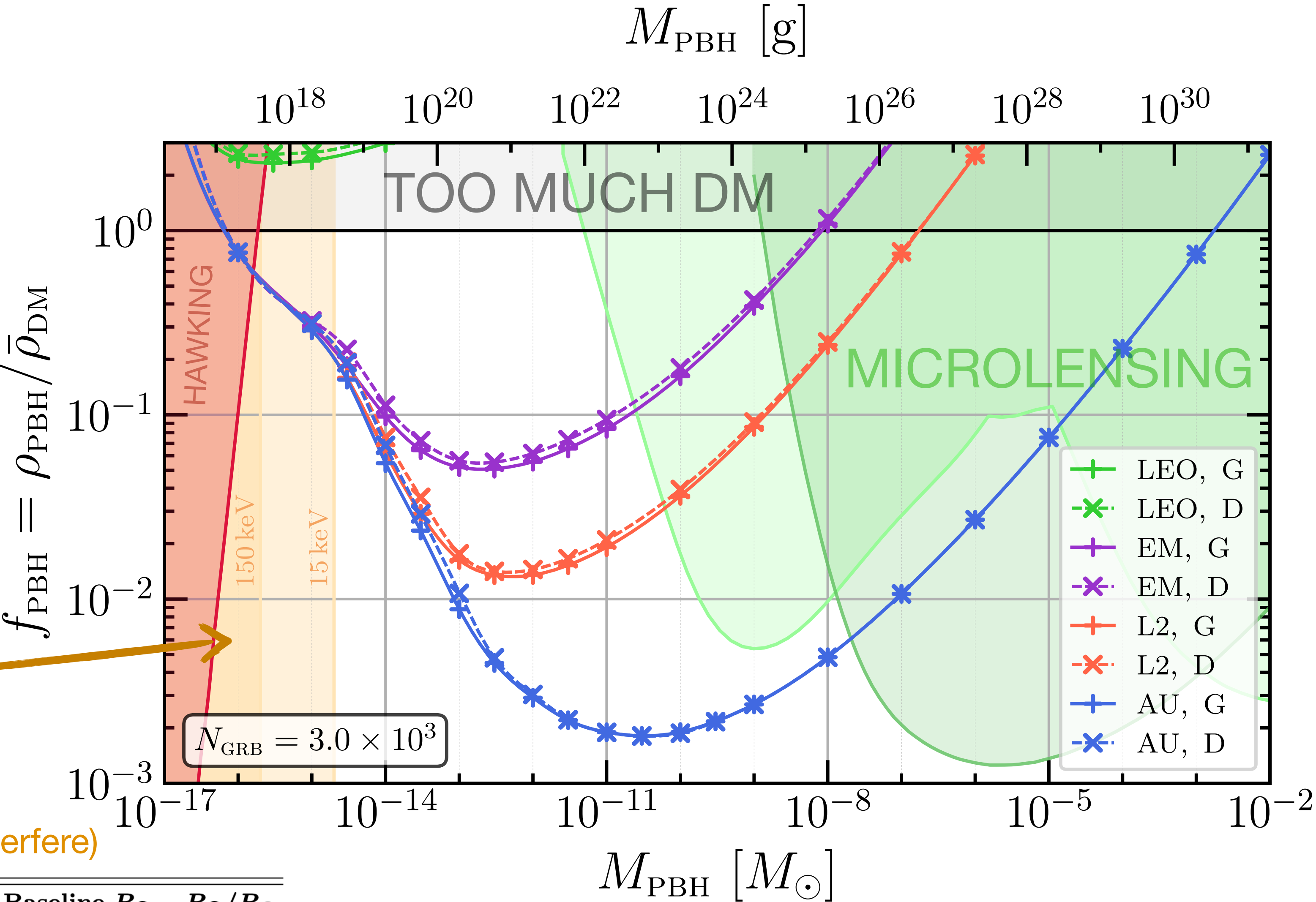
**AU baselines
can beat
microlensing!**

cf. Jung and Kim
[1908.00078]
(but for different
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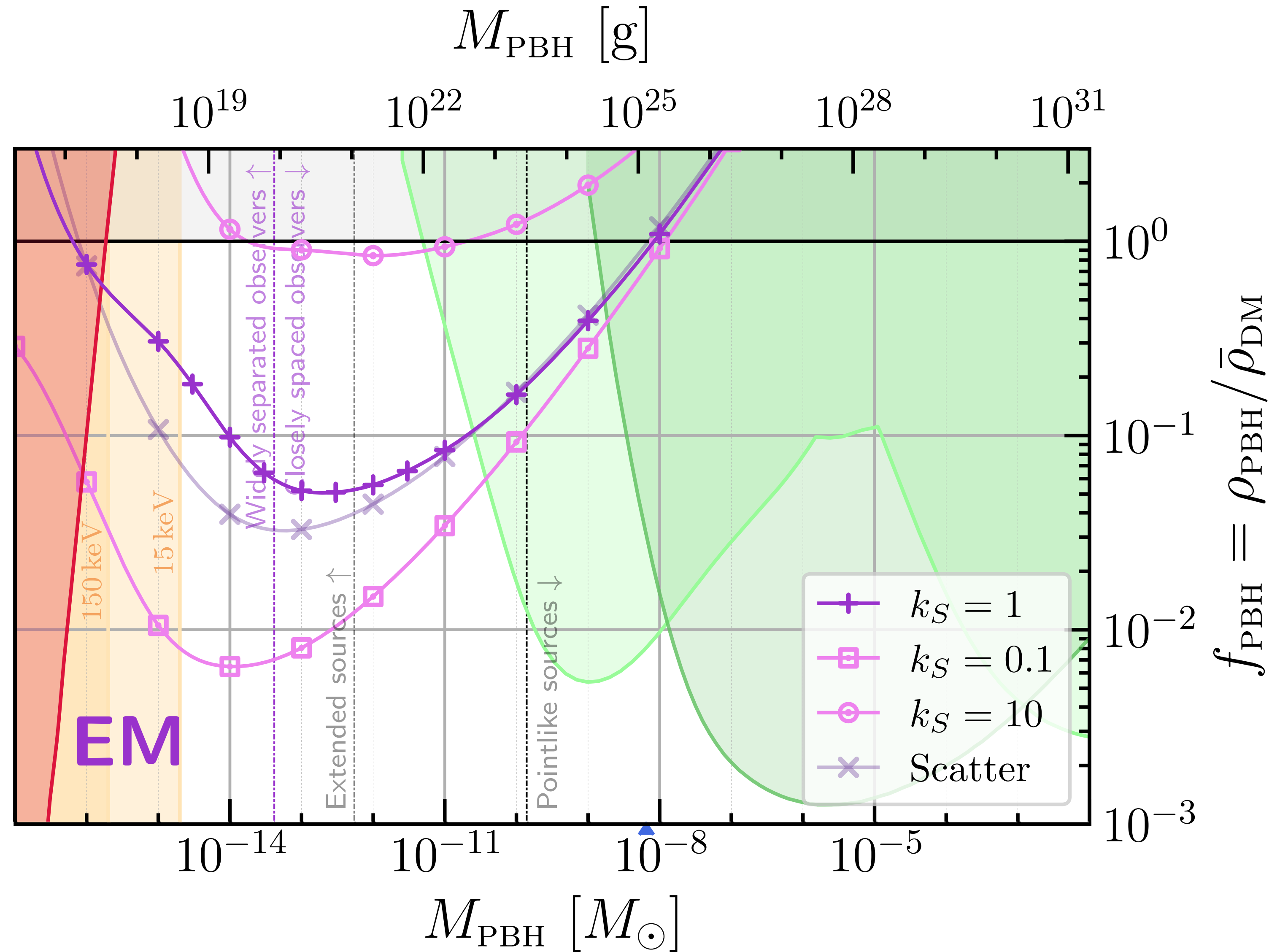
Geometrical optical fails
 $\omega R_s \lesssim 1$
 (two lensed images interfere)

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Vary the source sizes

$$N_{\text{GRB}} = 3 \times 10^3$$

$$D_{\text{obs}} \equiv \frac{k_S T_{90}}{1 + z_S}$$

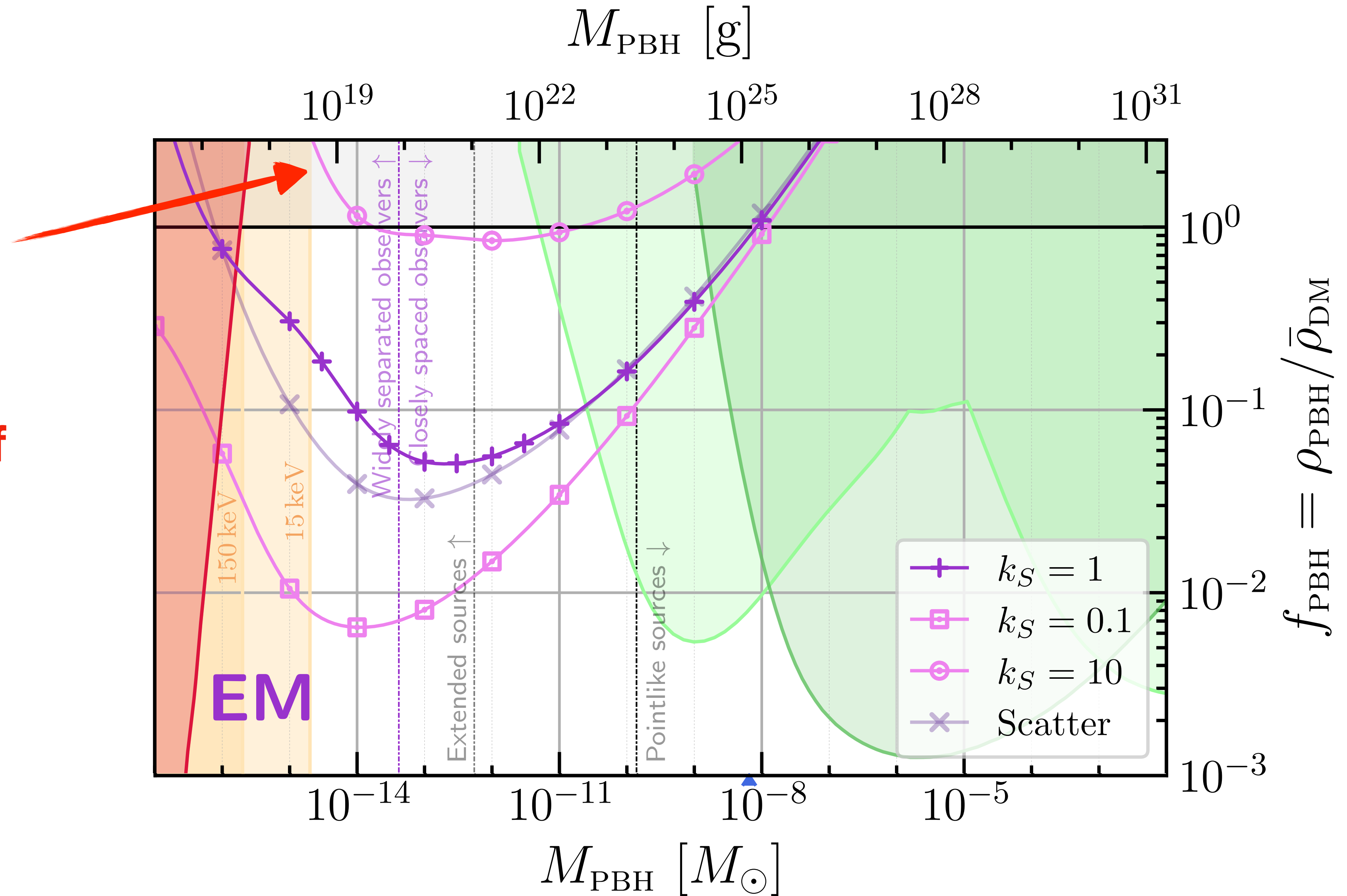


Vary the source sizes

$$N_{\text{GRB}} = 3 \times 10^3$$

Questionable
whether you can
robustly rule out
 $f_{\text{DM}} = 1$ with this
baseline if GRB
sizes are
systematically off

$$D_{\text{obs}} \equiv \frac{k_S T_{90}}{1 + z_S}$$



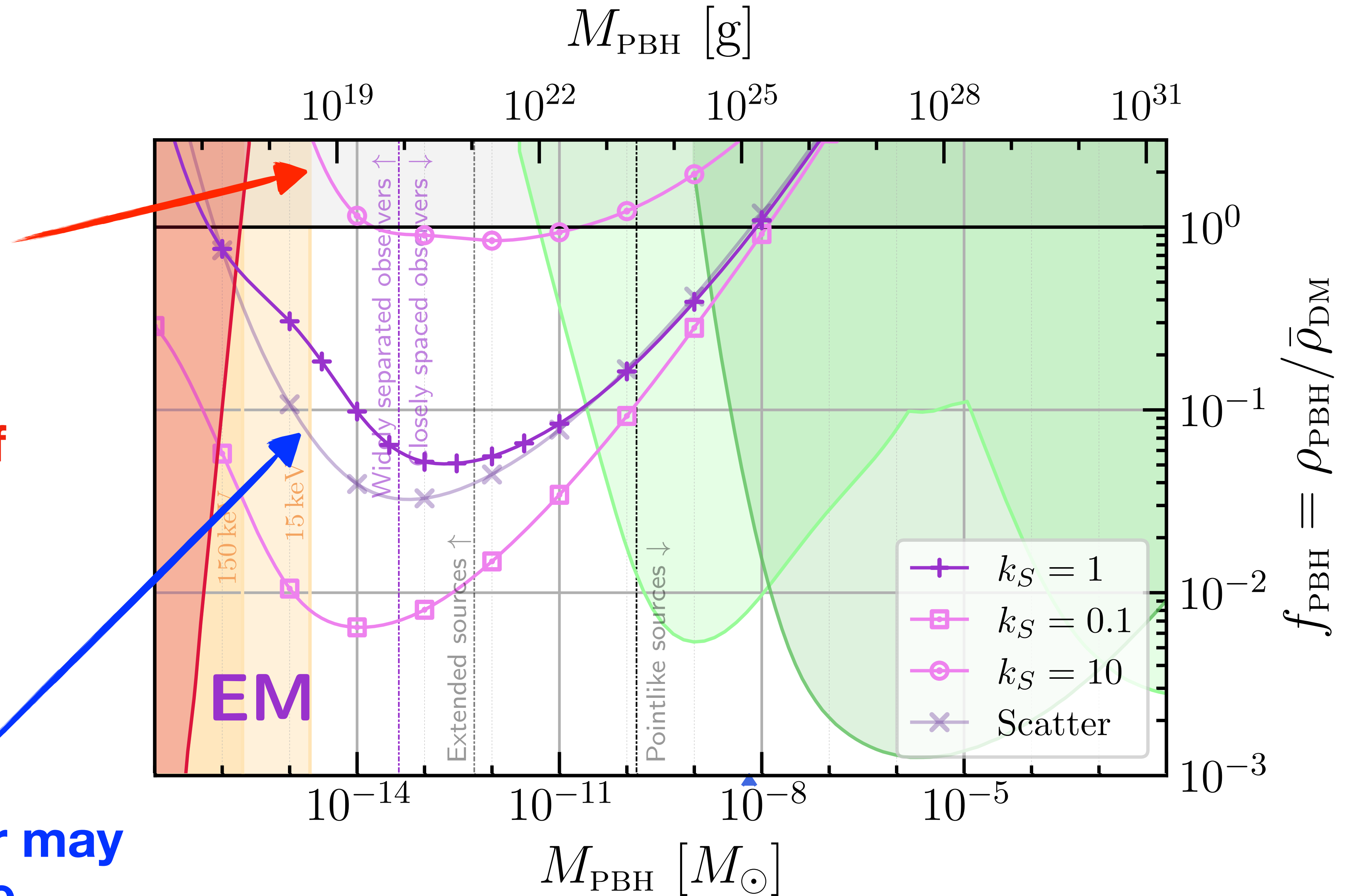
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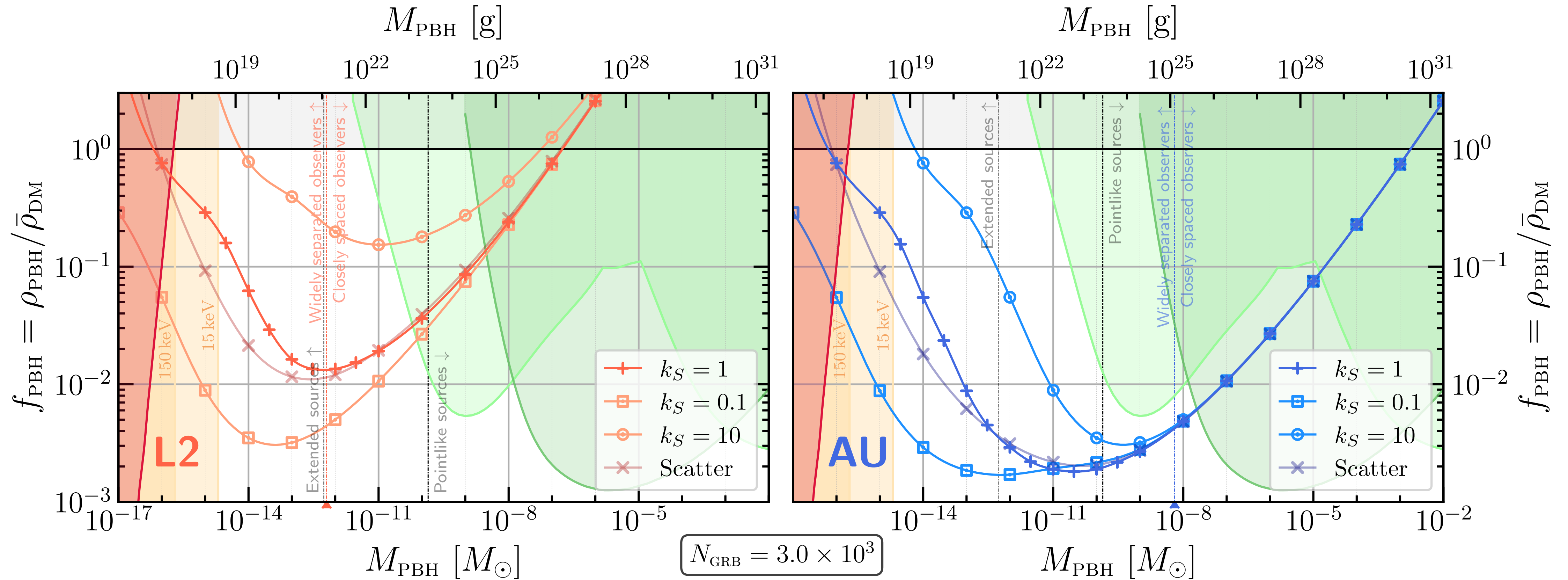
$$D_{\text{obs}} \equiv \frac{k_S T_{90}}{1 + z_S}$$

**But the scatter may
not be an issue**



Vary source sizes for each baseline

$$D_{\text{obs}} \equiv \frac{k_S T_{90}}{1 + z_S}$$

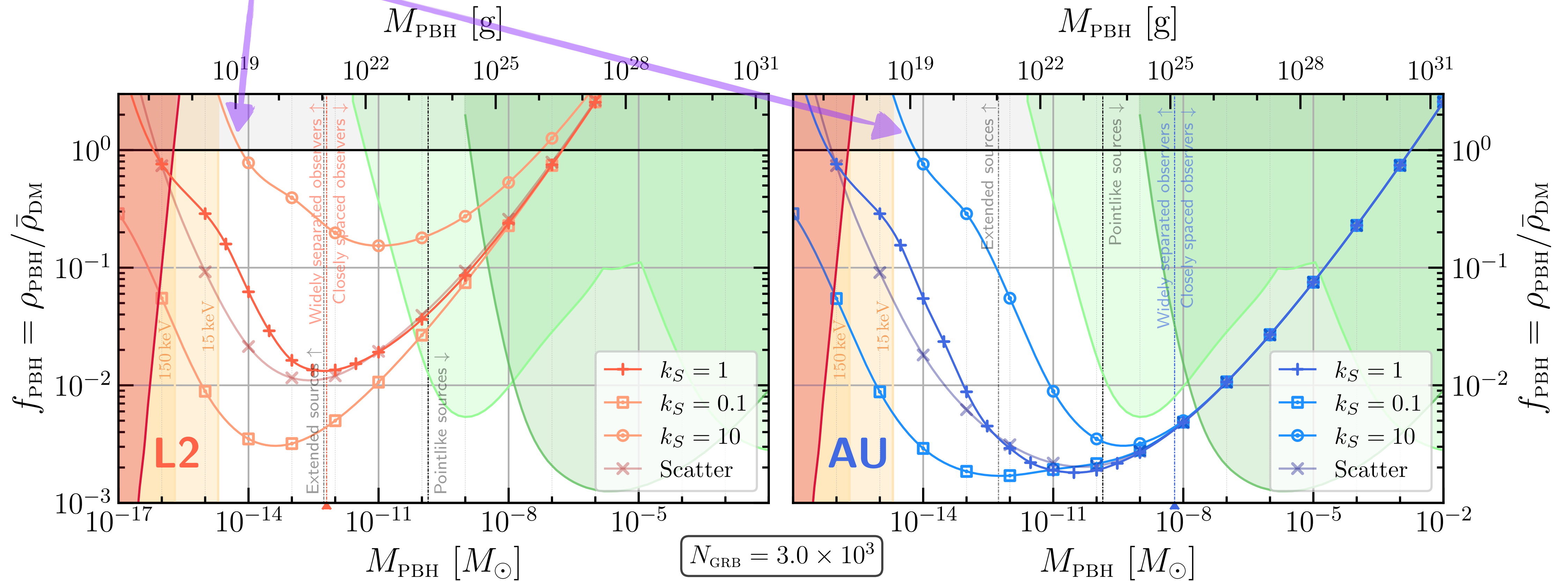


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ROBUST $f_{\text{DM}} = 1$ EXCLUSIONS FOR L2, AU

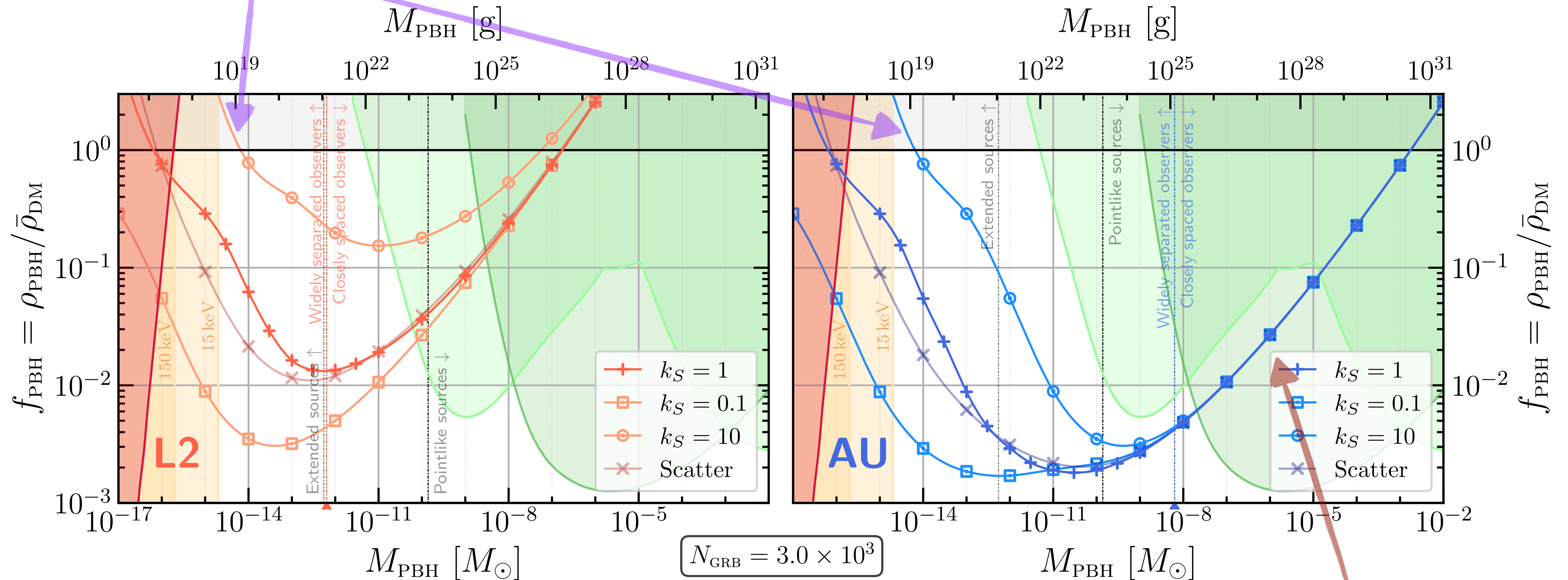


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ROBUST $f_{\text{DM}} = 1$ EXCLUSIONS FOR L2, AU



**LARGE-MASS REACH AT AU
INDEPENDENT OF SOURCE
UNCERTAINTIES!**

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Conclusions

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AU baselines would probe sub-component DM PBHs even above the window

Constraints on diffuse lenses look interesting (axion miniclusters, etc.)

Axion miniclusters

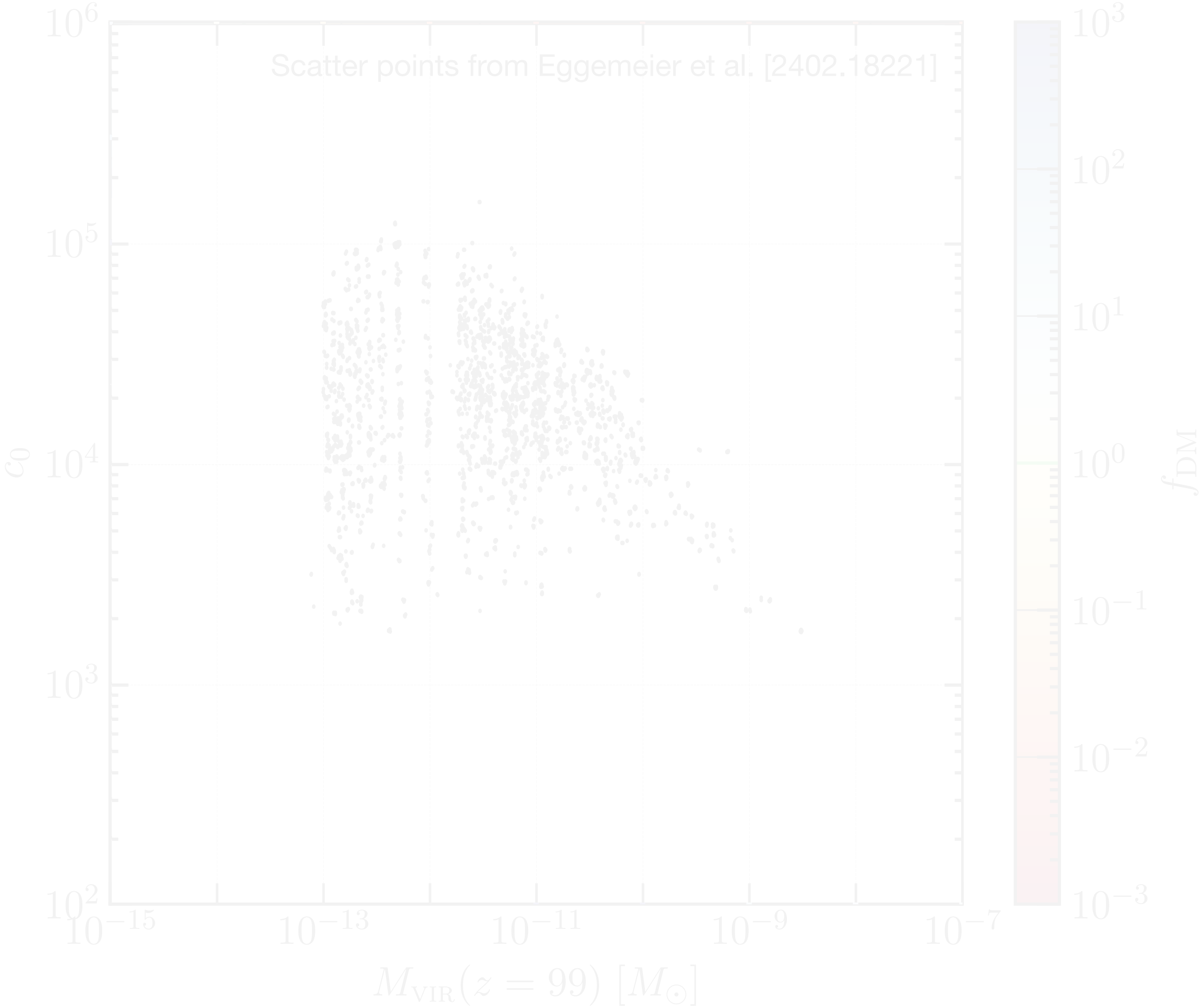
Dense clusters of QCD DM axions formed from large isocurvature fluctuations + self-interactions

Motivated by recent high-resolution QCD axion minicluster simulations

$$\rho_{\text{mNFW}}(r) = \frac{\rho_0}{(r/r_s)^2(1 + r/r_s)}$$
$$\rho_{\text{NFW}}(r) = \frac{\rho_0}{(r/r_s)(1 + r/r_s)^2}$$

Relax point-lens assumption

WORK NEARING COMPLETION



$$M_{\text{vir}} \equiv \frac{4\pi}{3} \Delta_{\text{vir}} \rho_c r_{\text{vir}}^3$$
$$c_0 \equiv \frac{r_{\text{vir}}}{r_s} \times (1 + z_{\text{vir}})$$

Axion miniclusters

Dense clusters of QCD DM axions formed from large isocurvature fluctuations + self-interactions

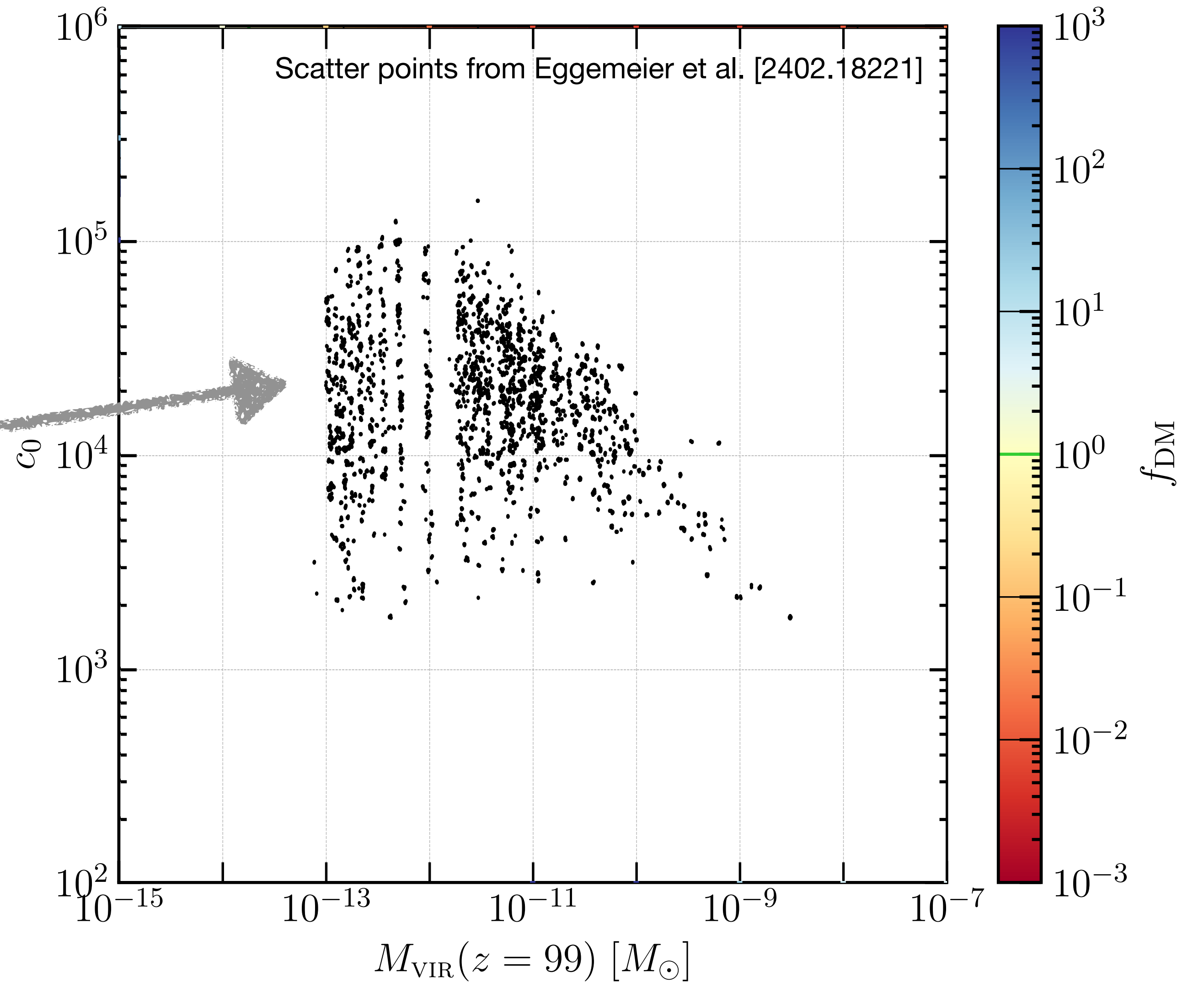
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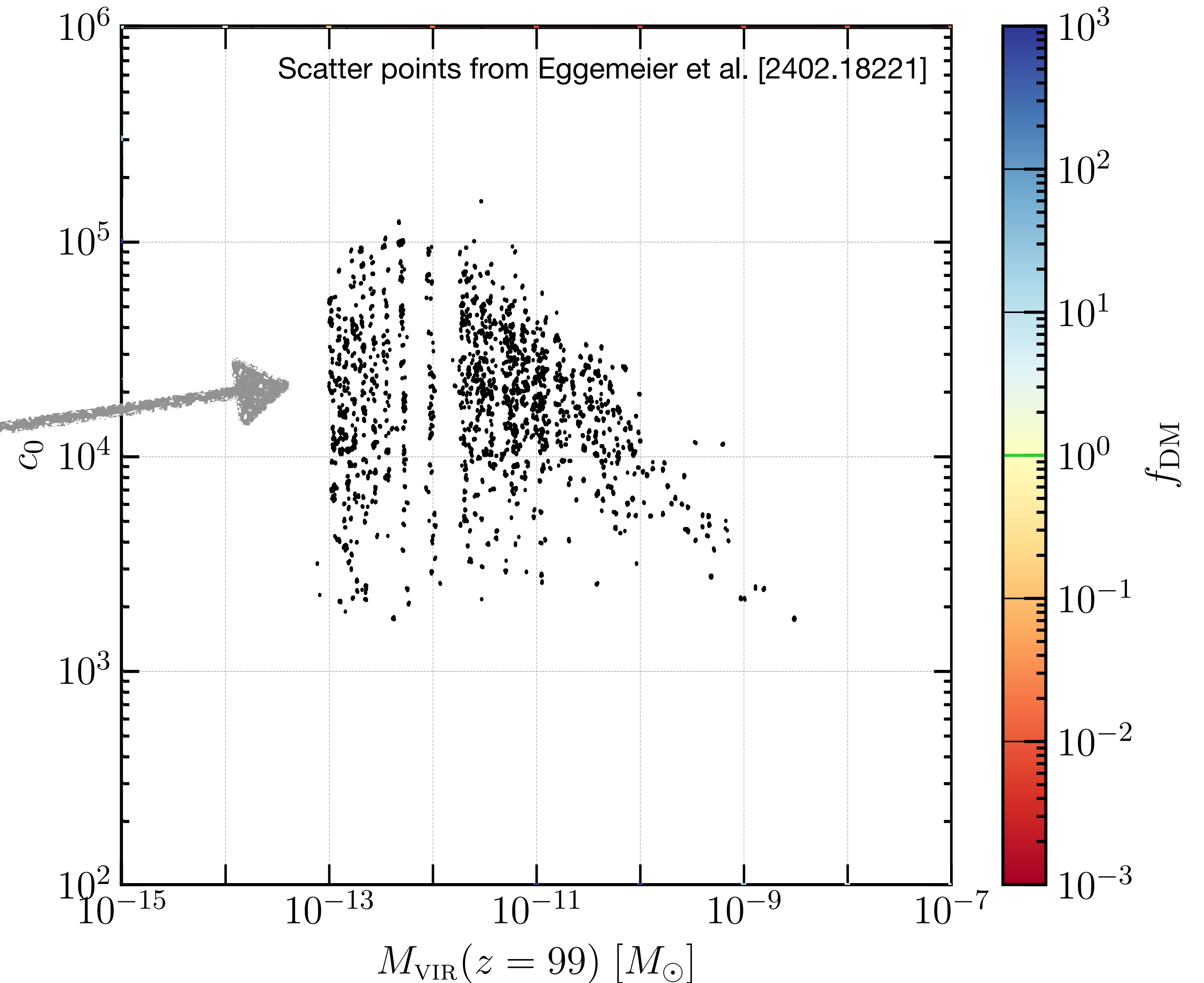
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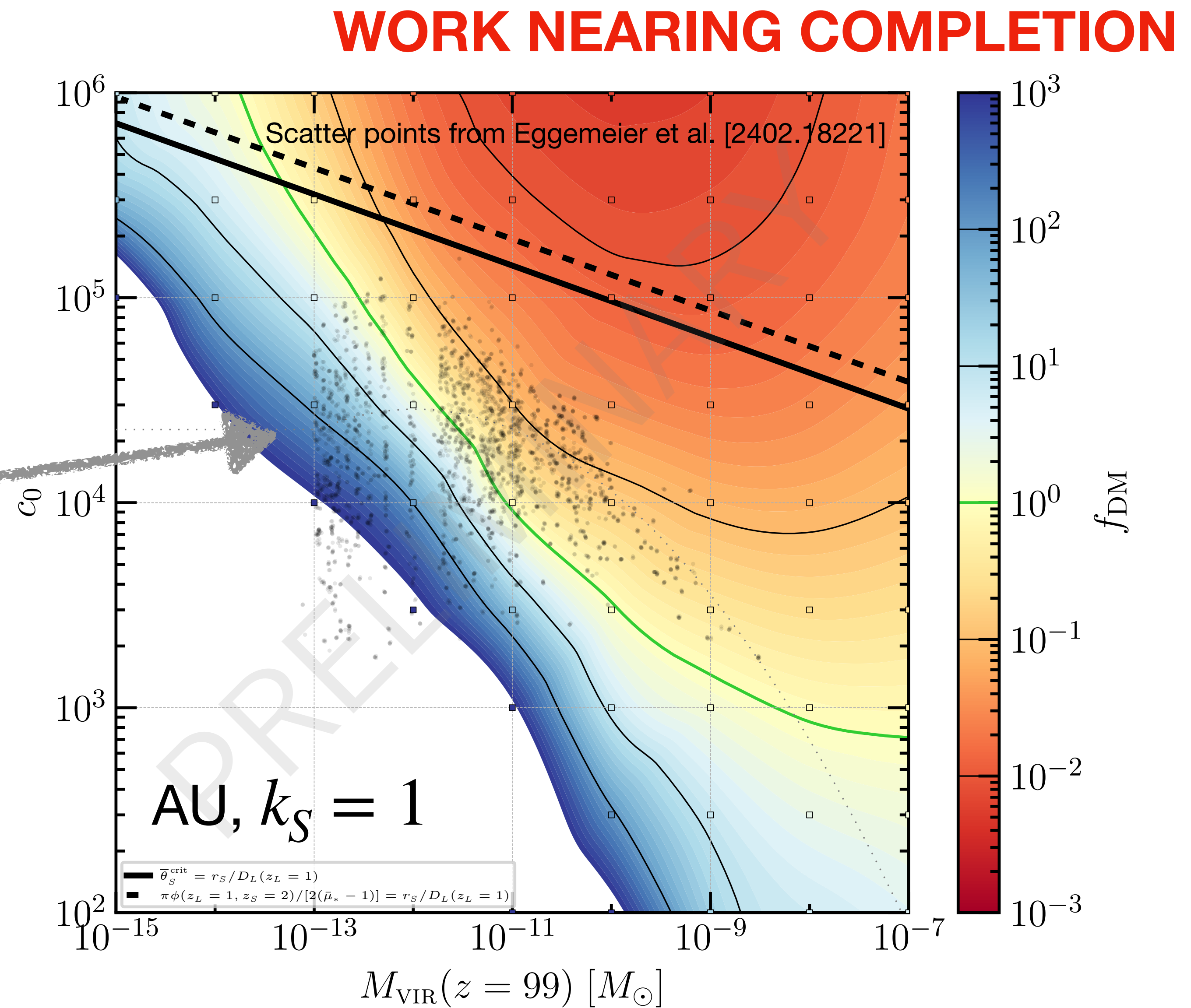
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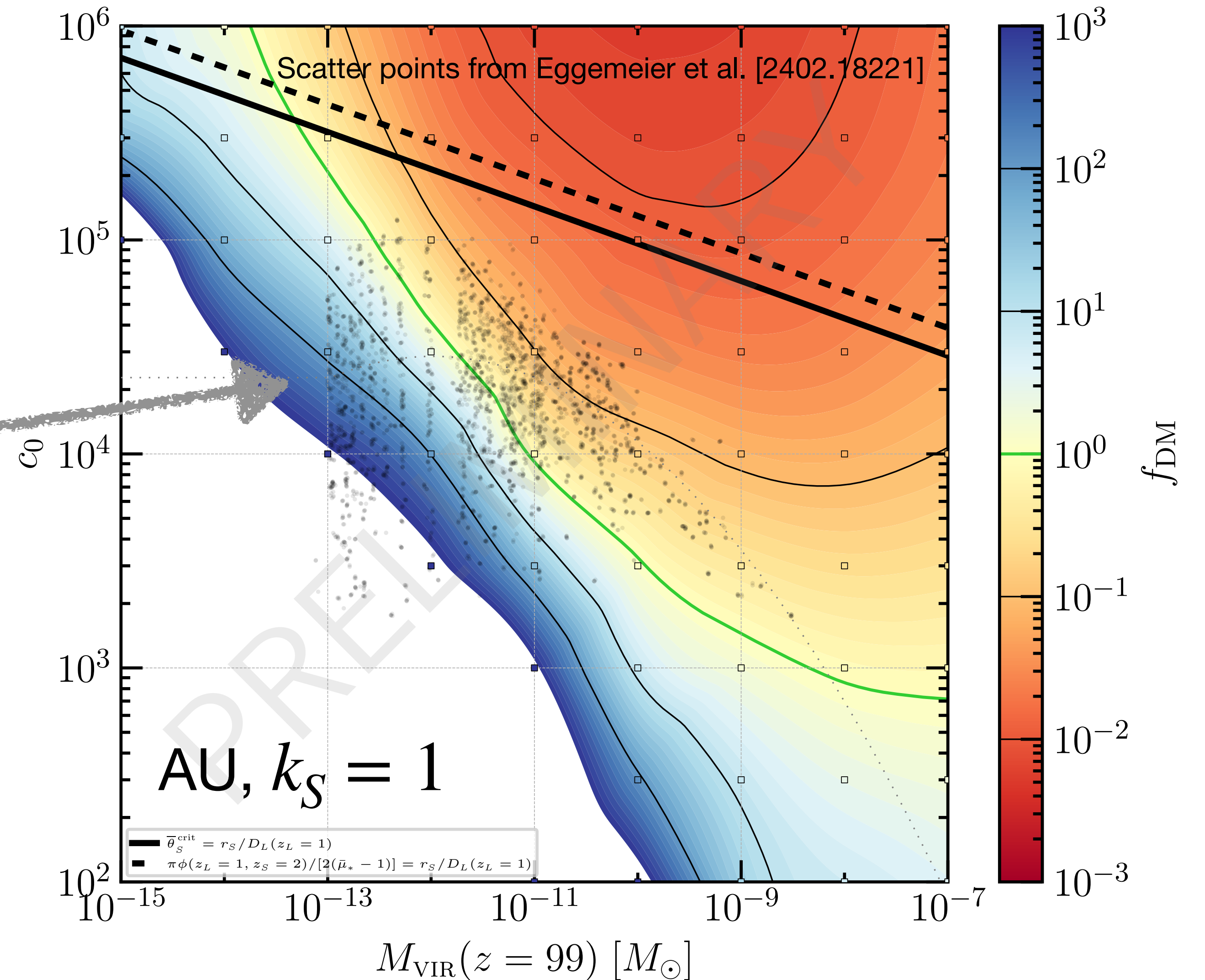
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Relax point-lens assumption

Picolensing may have sensitivity to see these...

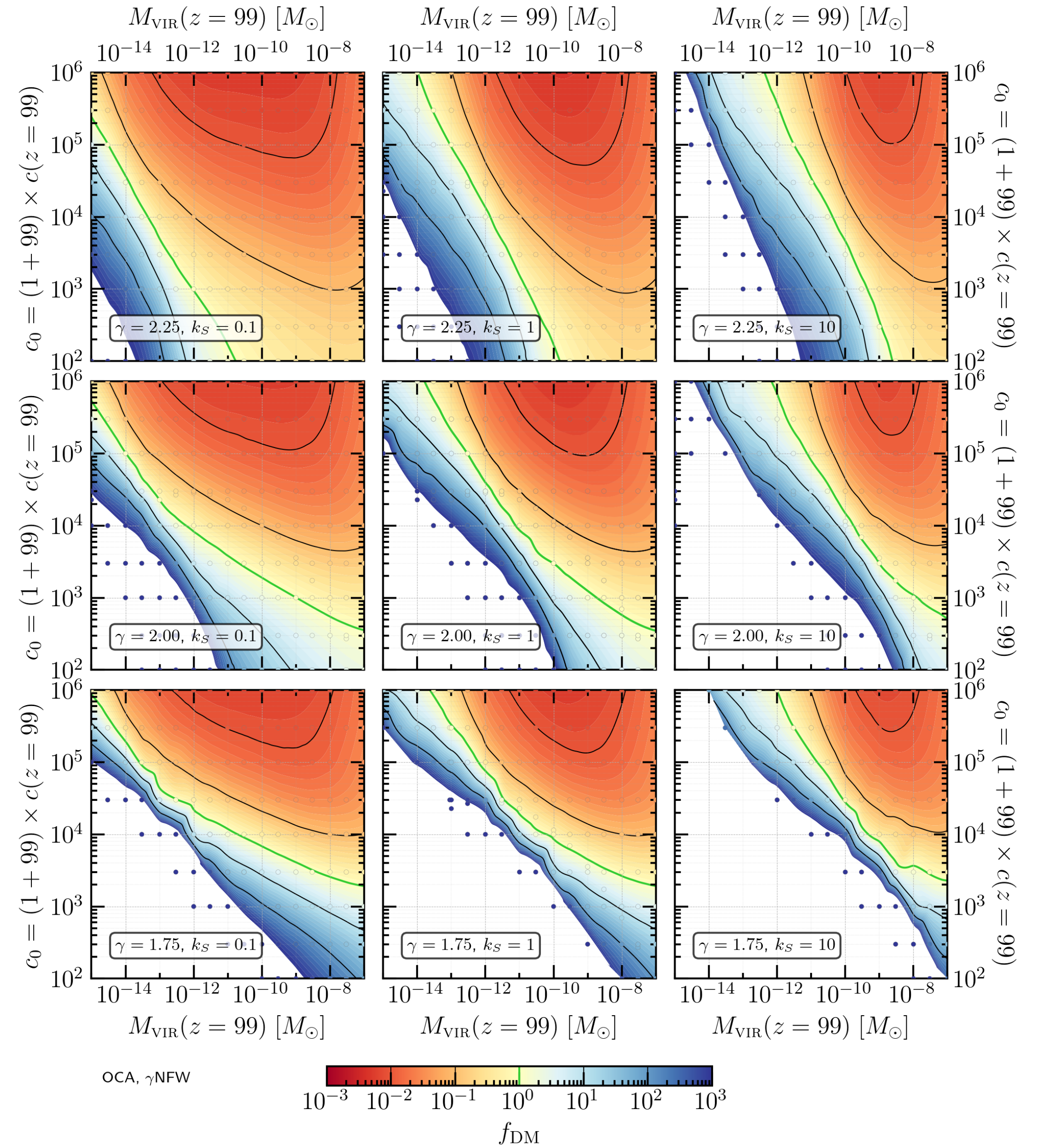
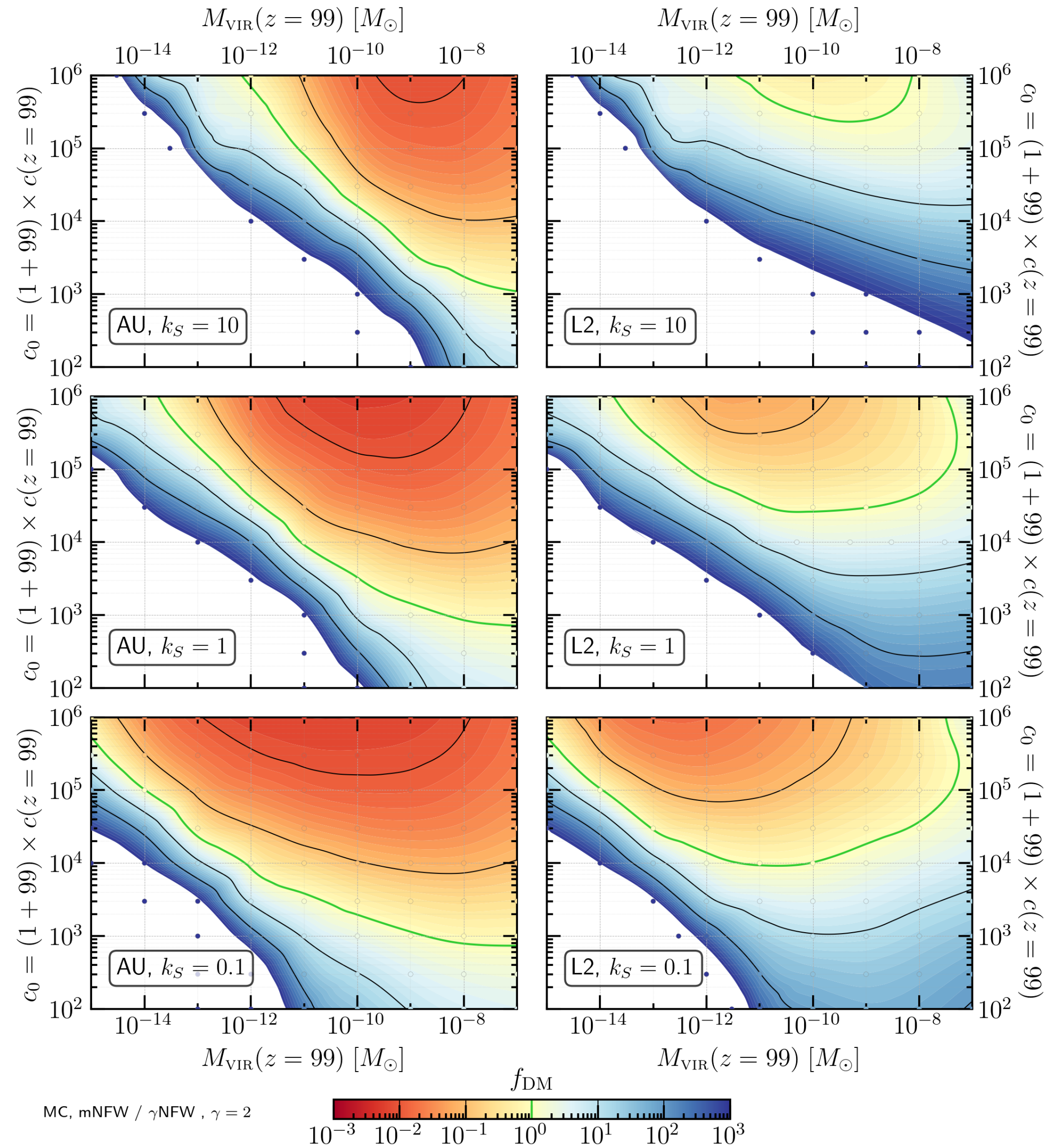
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Extended Lenses



Conclusions

Confirm that picolensing can probe asteroid mass window for PBH DM

GRB source size uncertainties are significant: most the **offset**

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Larger baselines (L2, AU) overcome systematics issues with source sizes

AU baselines would probe sub-component DM PBHs even above the window

Constraints on diffuse lenses look interesting (axion miniclusters, etc.)

BACKUP

Primordial black holes (PBH)

GRAVITATIONALLY COLLAPSED OBJECTS OF VERY
LOW MASS

Stephen Hawking

For the purposes of this talk: sub-solar mass black holes $M_{PBH} \ll M_{\odot}$

Production in the early universe via:

$$M \sim \frac{c^3 t}{G} \sim 10^{15} \left(\frac{t}{10^{-23} \text{ s}} \right) \text{ g}.$$

- ▶ Sharp features in the inflationary power spectrum. When these re-enter, direct collapse to BH ensues if density perturbation is large enough $\beta \approx \text{Erfc} \left[\frac{\delta_c}{\sqrt{2} \sigma} \right]$.
see also [2410.03451]
- ▶ Collisions of bubble walls from primordial first-order phase transitions
- ▶ ... many other variations on these themes

This talk will be **agnostic to the production mechanism.**

The key point for this study is...

Annu. Rev. Nucl. Part. Sci. 2020. 70:355–94

Bernard Carr¹ and Florian Kühnel²

Nucl. Phys. B 1003 (2024) 116494

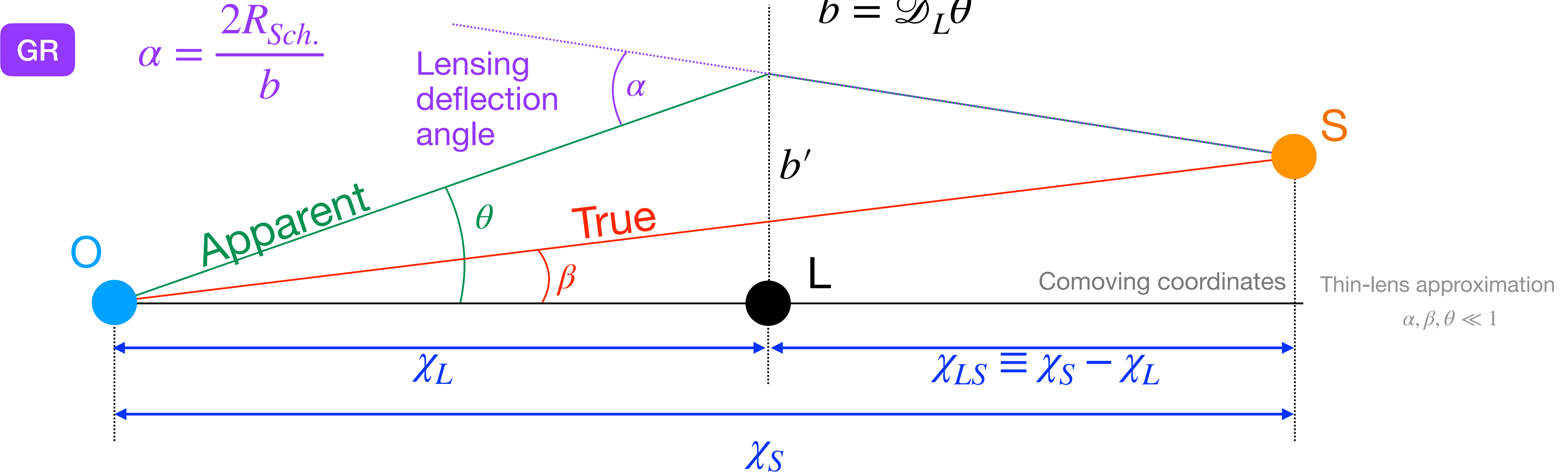
Anne M. Green

A. Escrivà, F. Kühnel and Y. Tada, *Primordial Black Holes*, [arXiv:2211.05767](https://arxiv.org/abs/2211.05767).

A. M. Green and B. J. Kavanagh, *Primordial Black Holes as a dark matter candidate*, *J. Phys. G* **48** (2021) 043001 [[arXiv:2007.10722](https://arxiv.org/abs/2007.10722)].

Gravitational Lensing 101

Angular diameter distance $\rightarrow \mathcal{D} = a\chi = \frac{\chi}{1+z}$ comoving distance



Null geodesics are straight lines in comoving coordinates

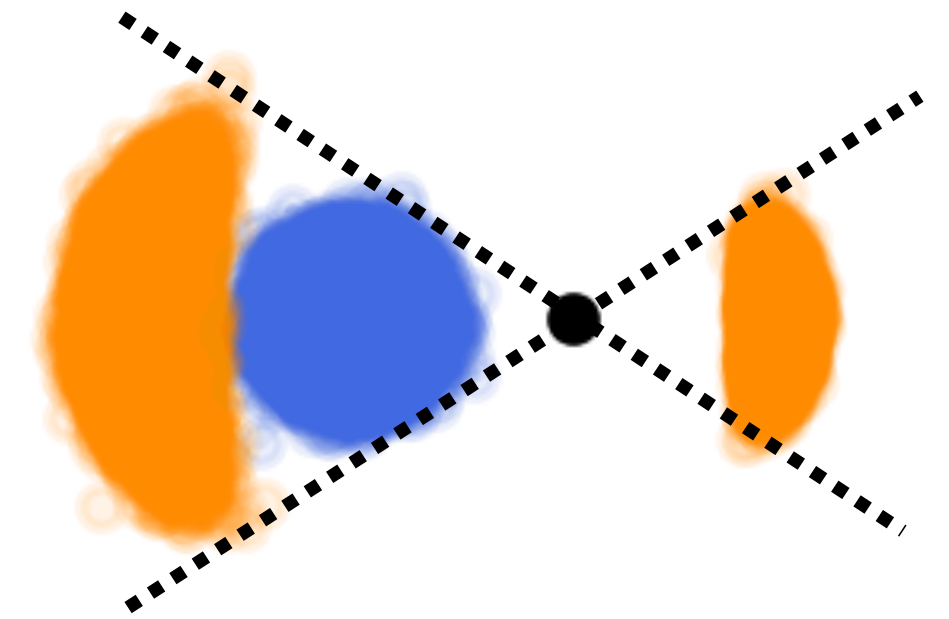
Basic geometry $\rightarrow$ Lens Equation: $\theta - \beta = \frac{\theta_E^2}{\theta}$, where $\theta_E \equiv \sqrt{\frac{4G_N M(1+z_L)\chi_{LS}}{\chi_S \chi_L}}$ Einstein angle

$R_{Sch.} = 2G_N M$

Gravitational Lensing 102

$$\theta_E^2 \equiv \frac{4G_N M(1+z_L)\chi_{LS}}{\chi_S \chi_L}$$

$$y \equiv \frac{\beta}{\theta_E} \quad ; \quad x \equiv \frac{\theta}{\theta_E} \quad \Rightarrow \quad x - y = \frac{1}{x} \quad \Rightarrow \quad x_{\pm} = \frac{1}{2} \left[y \pm \sqrt{y^2 + 4} \right]$$



Magnification? Gravitational lensing preserves surface brightness, so

$$\mu \equiv \frac{d\Omega_{\text{apparent}}}{d\Omega_{\text{true}}} = \frac{d \cos \theta}{d \cos \beta} \approx \frac{\theta d\theta}{\beta d\beta} = \frac{x dx}{y dy} \quad \Rightarrow \quad \mu_{\pm} = \frac{1}{2} \left| 1 \pm \frac{y^2 + 2}{y \sqrt{y^2 + 4}} \right|$$

Einstein angle is **EXTREMELY** small for a PBH: $\theta_E \sim 2 \text{ picoarcsec} \times \sqrt{\frac{M}{10^{-12} M_{\odot}}}$

Images are not resolved: $\mu \sim \mu_+ + \mu_- = \frac{y^2 + 2}{\sqrt{y^2(y^2 + 4)}}$

$[z_S = 1, \chi_L/\chi_S = 0.5 (z_L = 0.43)]$

... if geometrical optics holds

Finite Sources

So far, assumed both the lens and source are point-like.

But all sources have a finite extent!

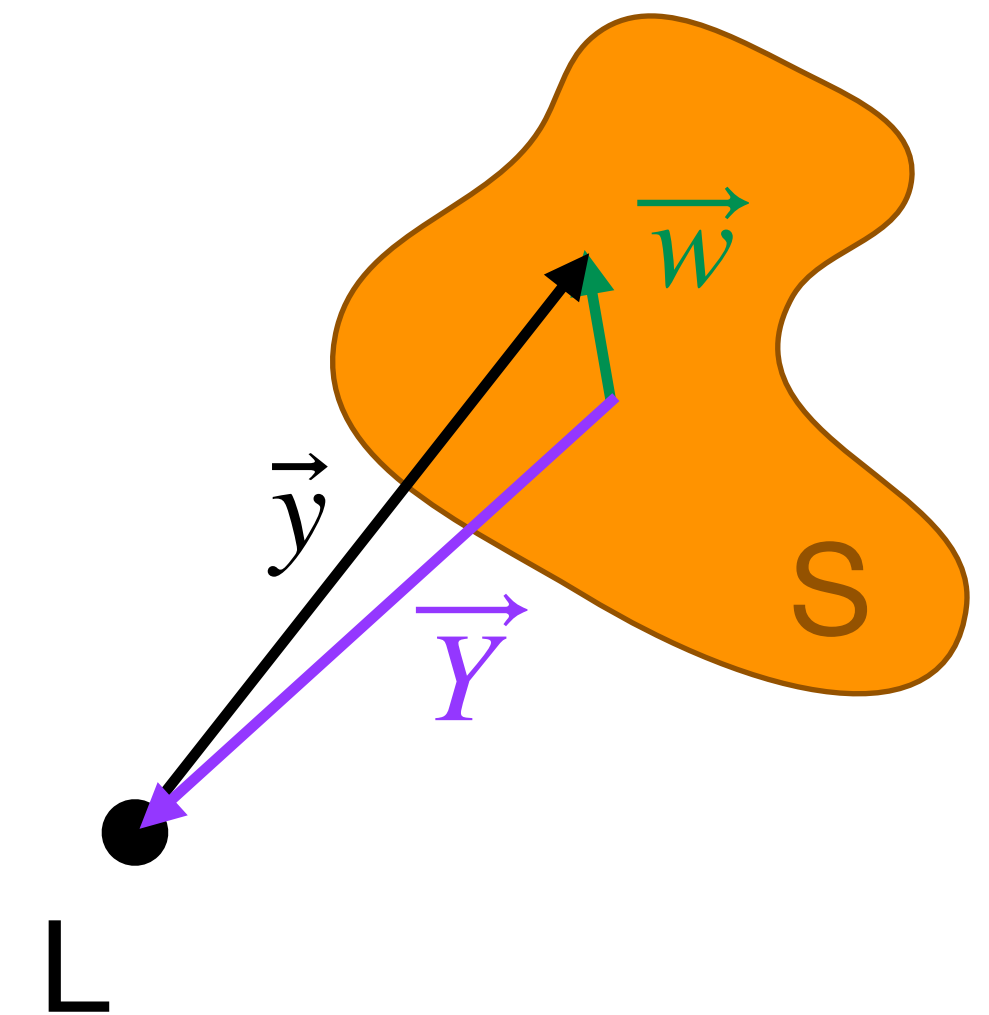
$$\mu(y) = \frac{y^2 + 2}{\sqrt{y^2(y^2 + 4)}}$$

$$\bar{\mu}(\vec{Y}) = \iint d^2w \mathcal{J}(\vec{w}) \cdot \mu(y = |\vec{Y} - \vec{w}|)$$

Source-averaged magnification for lens at location $\vec{Y}$ relative to source centroid

Source brightness profile at location $\vec{w}$ relative to the source centroid

small-angle/
flat-sky approx



We'll consider both Gaussian and flat disk source profiles

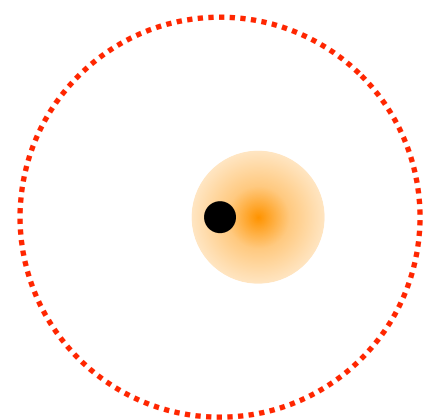
Gaussian source

$$\bar{\mu}(y', \delta) = \frac{e^{-(y')^2/2}}{\delta} \int_0^\infty dx \frac{e^{-x^2/2} [2 + (x\delta)^2] I_0(xy')}{\sqrt{4 + (x\delta)^2}}$$

not tractable in closed form

Point-like behaviour $\vartheta_S \ll \theta_E$

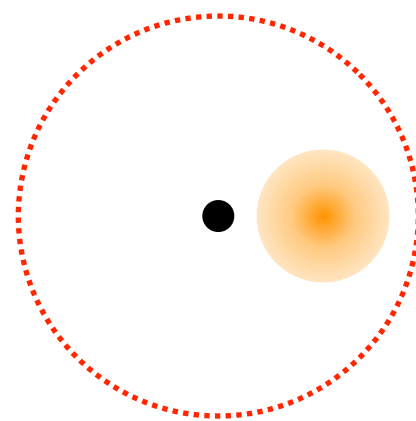
$$\bar{\mu}(\vartheta_L \lesssim \vartheta_S) \sim 1 + \sqrt{\frac{\pi k_G^2}{2}} \frac{\theta_E}{\vartheta_S} \gg 1$$



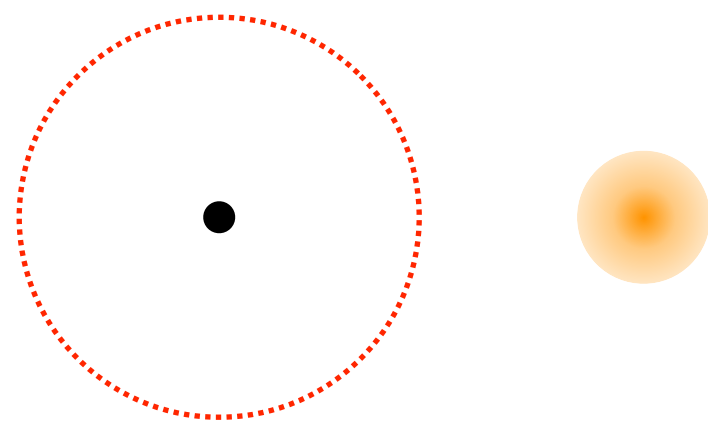
Regulated
from true
point-source
collinear
divergence

$$\mu \propto \frac{\theta_E}{\vartheta_L}$$

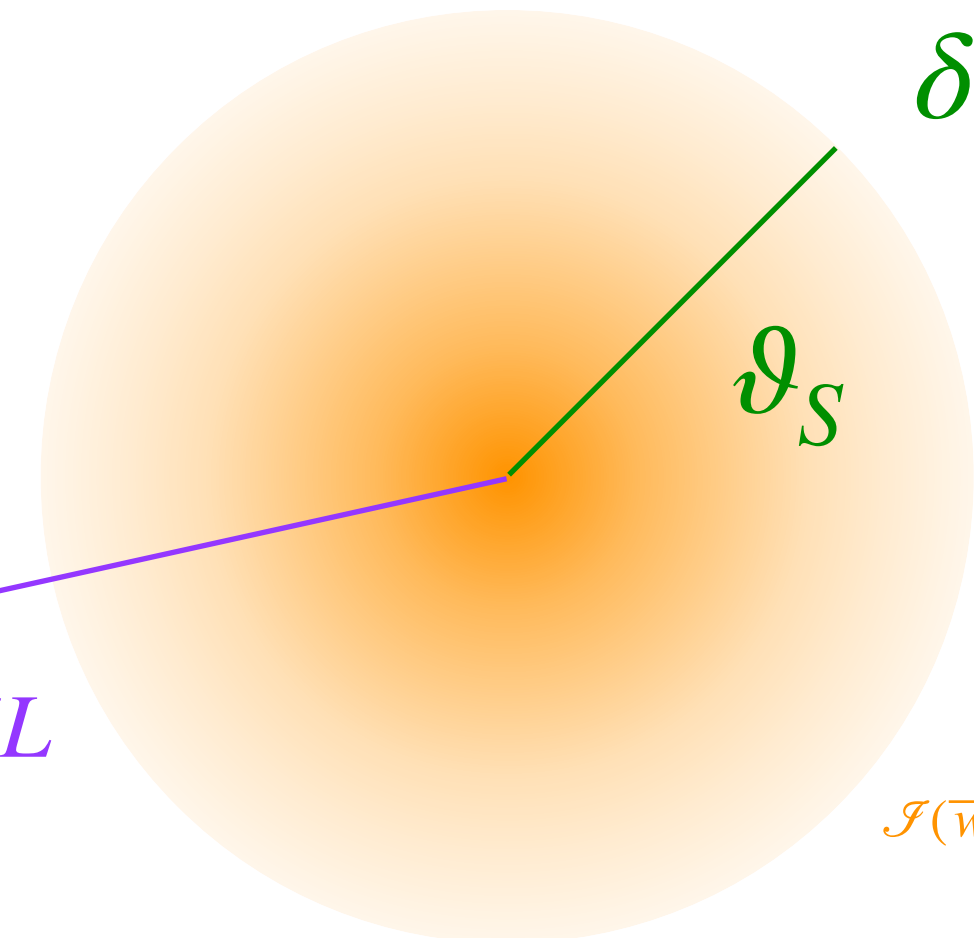
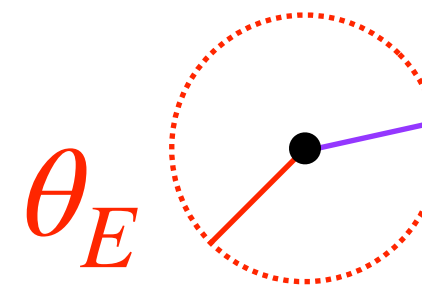
$$\bar{\mu}(\vartheta_S \ll \vartheta_L \ll \theta_E) \sim 1 + \frac{\theta_E}{\vartheta_L} \gg 1$$



$$\bar{\mu}(\vartheta_L \gg \theta_E) \sim 1 + 2 \left(\frac{\theta_E}{\vartheta_L} \right)^4 \sim 1$$



$$y' \equiv k_G \frac{\vartheta_L}{\vartheta_S}$$



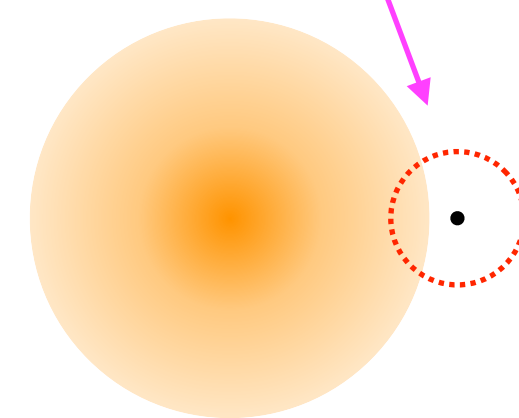
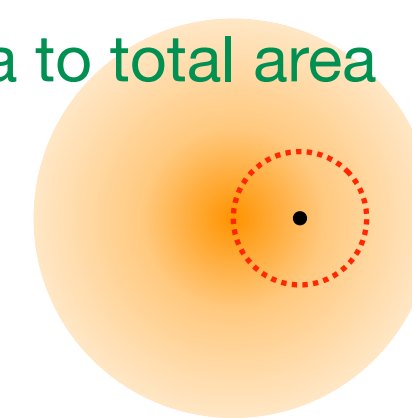
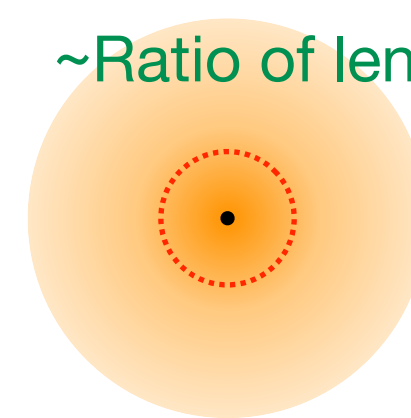
$$\delta \equiv \frac{1}{k_G} \frac{\vartheta_S}{\theta_E}$$

$$\mathcal{J}(\vec{w}) = \frac{1}{2\pi\delta^2} \exp \left[-\frac{|\vec{w}|^2}{2\delta^2} \right]$$

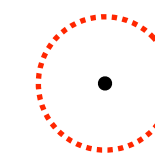
Extended source $\vartheta_S \gg \theta_E$ **

$$\bar{\mu}(\vartheta_L \lesssim \vartheta_S + \text{few} \times \theta_E) \sim 1 + \frac{k_G^2 \theta_E^2}{\vartheta_S^2} \exp \left[-\frac{k_G^2 \vartheta_L^2}{2\vartheta_S^2} \right]$$

~Ratio of lensed area to total area



$$\bar{\mu}(\vartheta_L \gg \theta_E, \vartheta_S) \sim 1 + 2 \left(\frac{\theta_E}{\vartheta_L} \right)^4 \sim 1$$



**** this is a slight oversimplification**

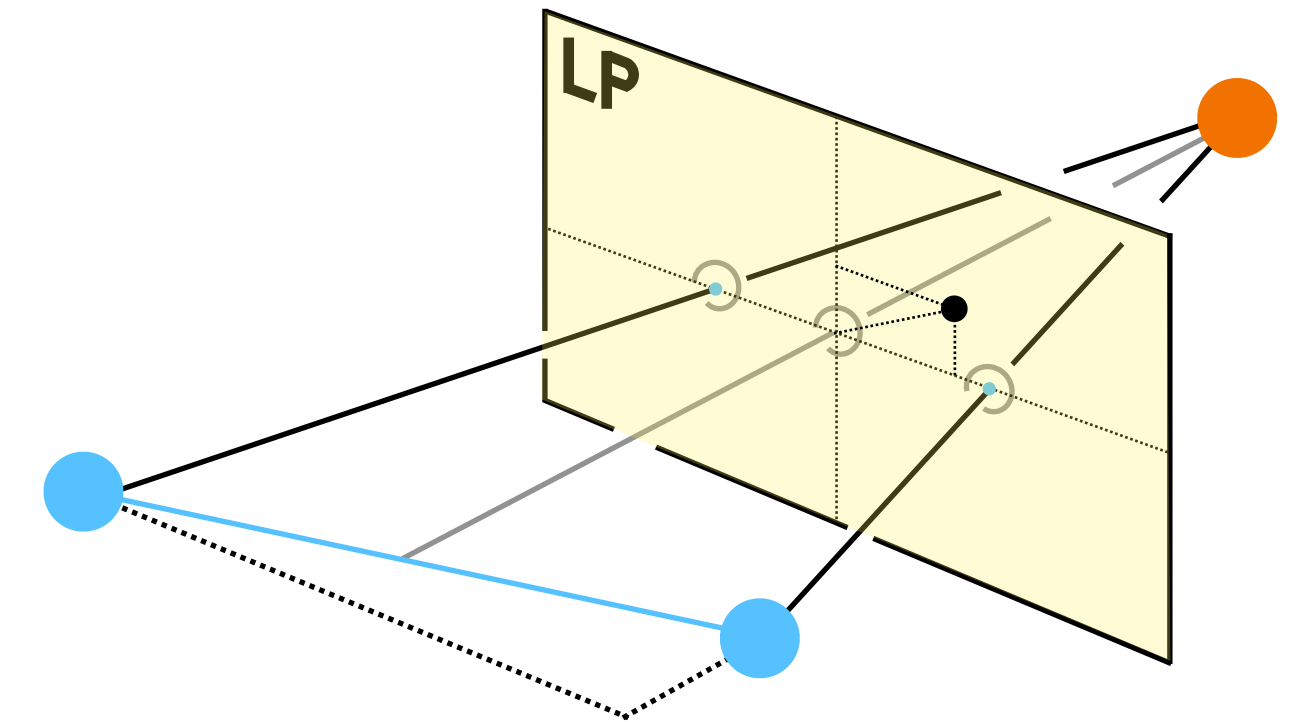
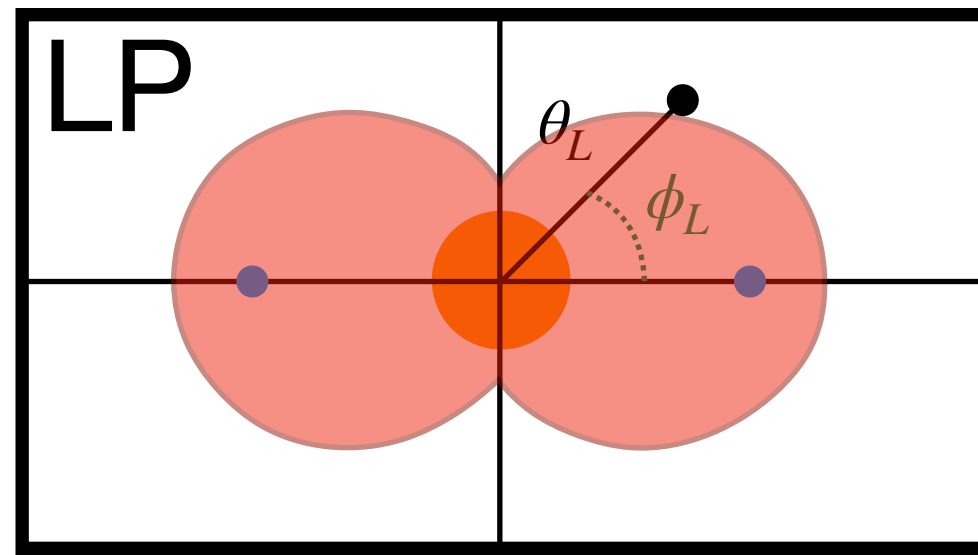
Michael A. Fedderke [Perimeter]

Picolensing cross-section

$$\rho = \rho(\{\theta_L, \phi_L, z_L, M\}, \{z_S, \theta_S, f_s\}, \{R_O, \theta_O, A_b, A_s, f_b, T\})$$

Fix: lens distance, lens mass, source parameters, observer parameters.

$$\text{SNR } \rho = \rho(\theta_L, \phi_L).$$



Is any lens detectable at some threshold SNR ρ_* ?

$\rho \geq \rho_*$ defines region in the lens plane where lensing is detectable. Can be multiple disjoint regions!

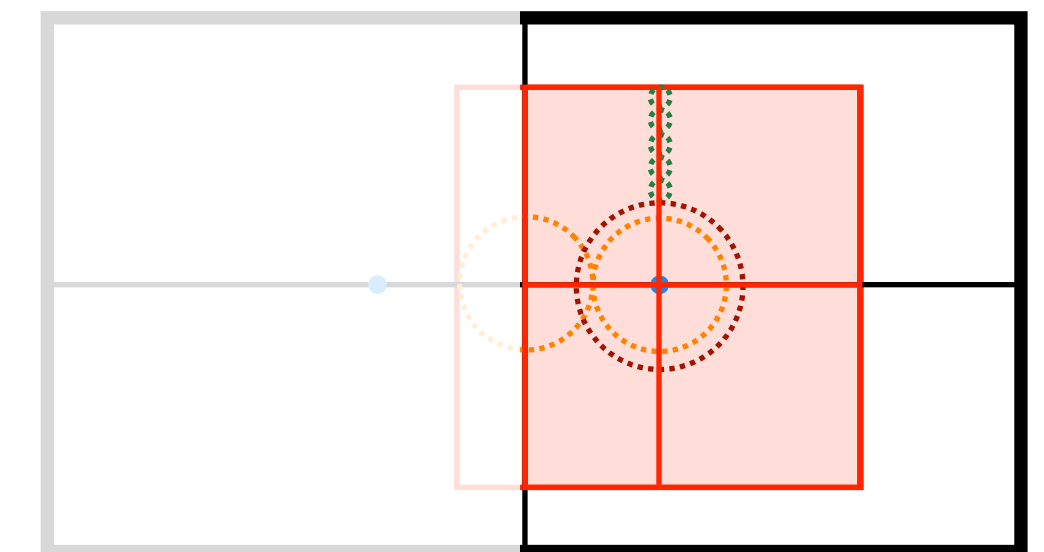
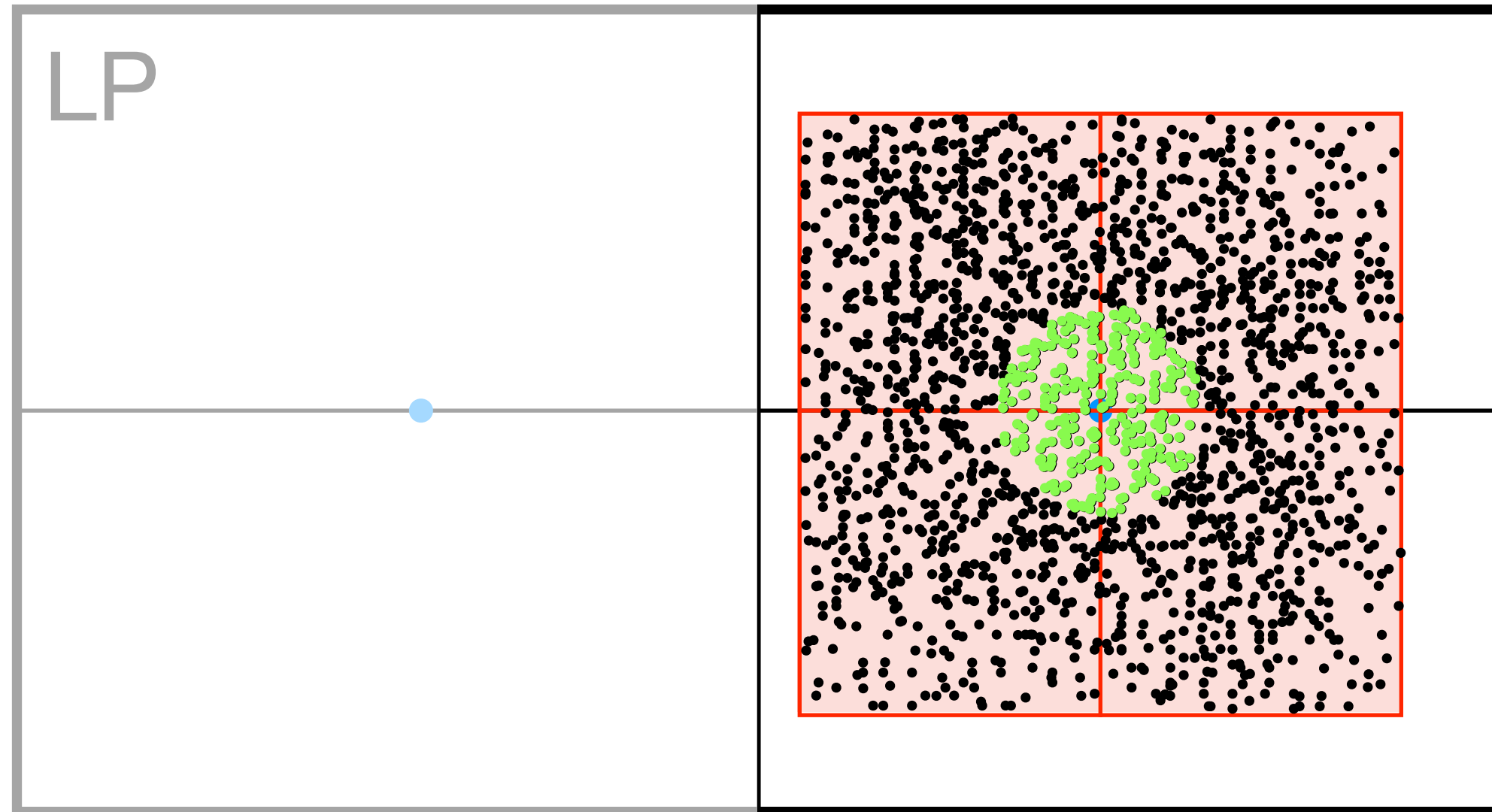
Lensing cross-section σ is the area of that region (comoving): $\sigma = \sigma(\rho_*)$

Compute this using Monte Carlo methods (sample LP area with lenses randomly)

Computing σ

Monte Carlo methods

Pick an appropriate sample area in the positive half-plane of the lens plane



A_{sample} reduced if the box crosses the symmetry axis

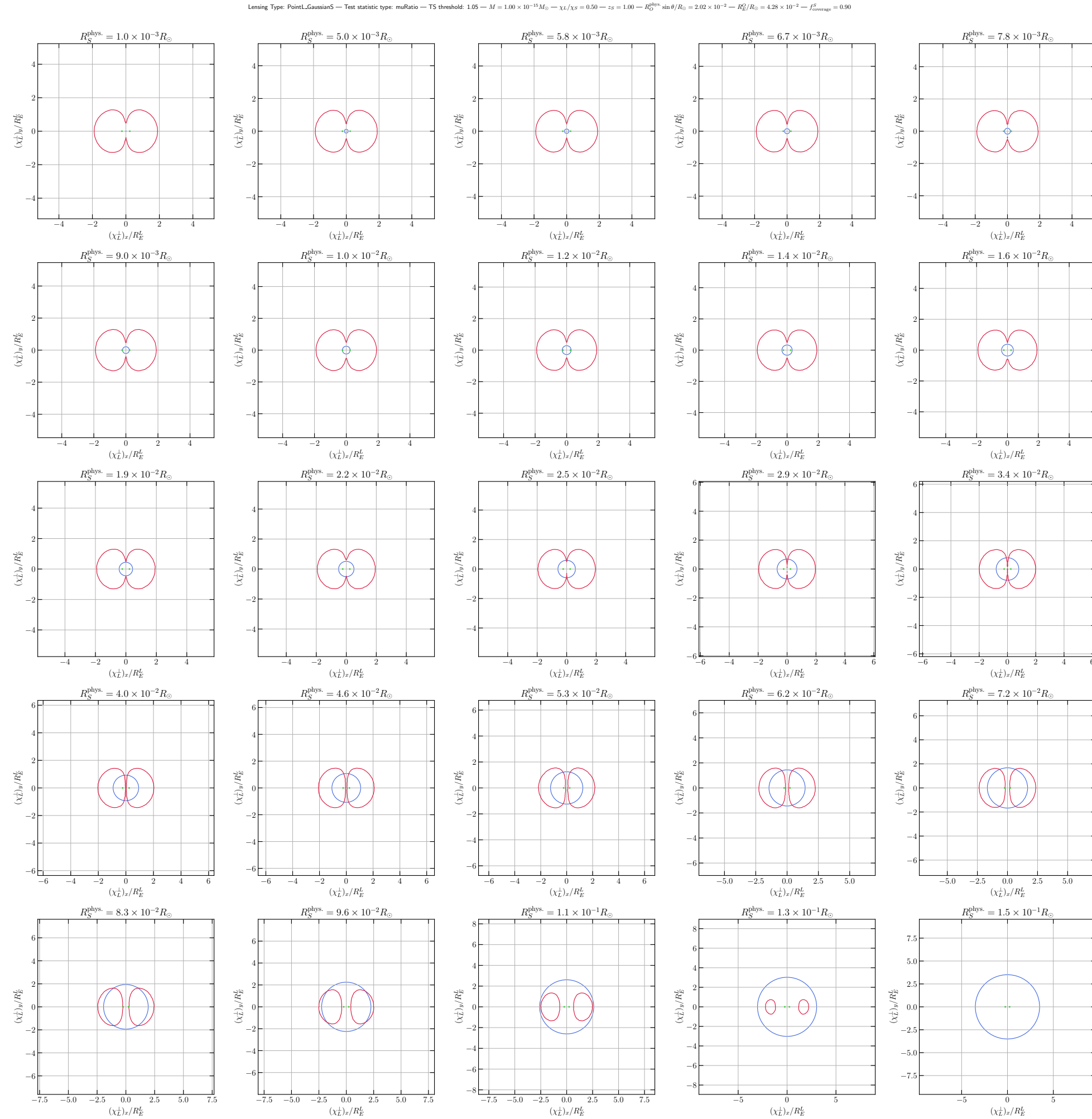
Populate with N_{lens} lenses randomly sampled in 2D; $N_{\text{lens}} = 20000$ (100000 if needed)

Compute $\bar{\mu}_i^k$ for $i = 1, 2$ and ρ^k for $k = 1, \dots, N_{\text{lens}}$. Count the fraction f_ρ with $\rho^k \geq \rho_*$

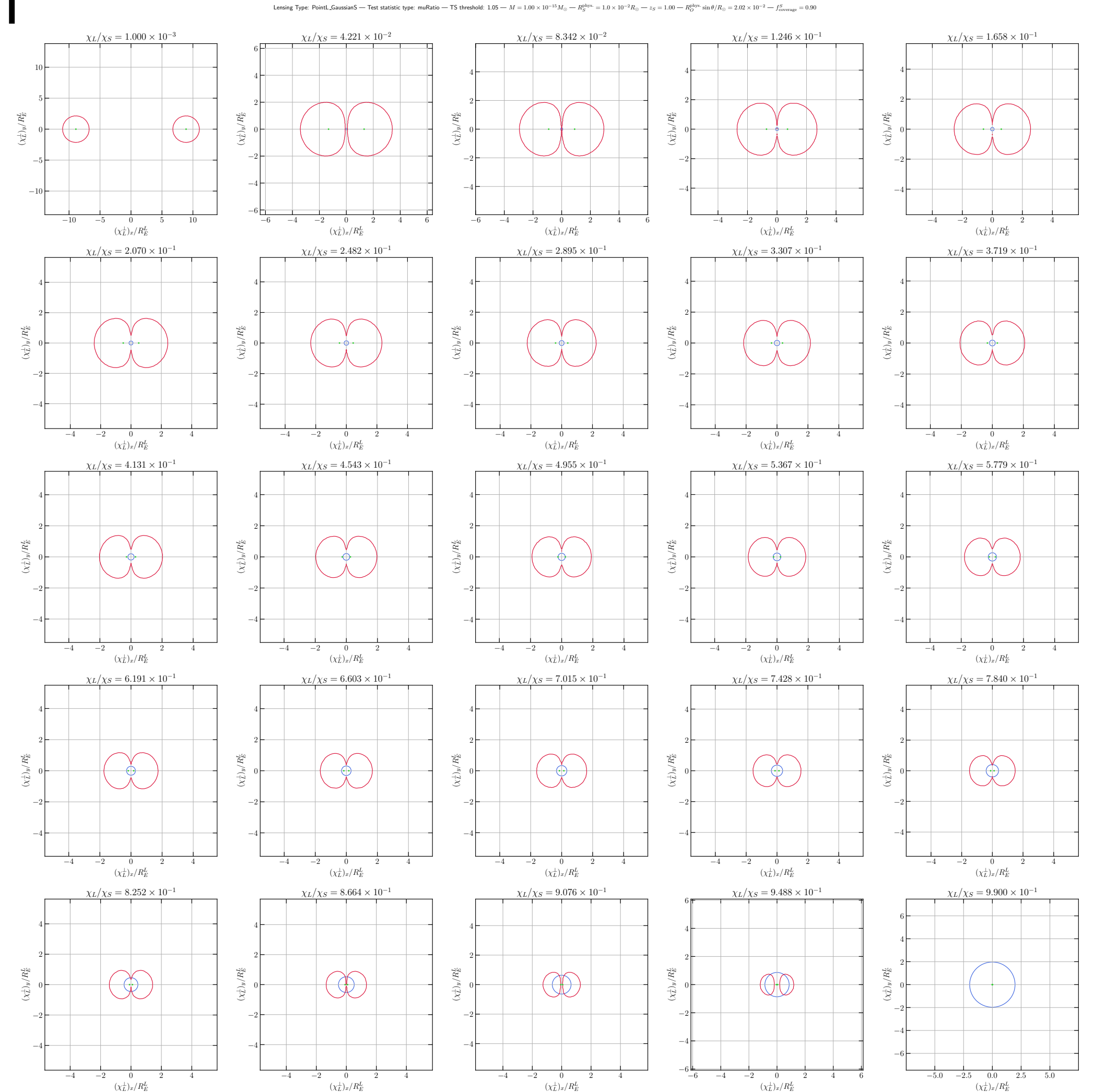
Cross-section is $\sigma = 2A_{\text{sample}} \times f_\rho$ (the 2 accounts for the reflection symmetry)

Picolensing cross-section

Gaussian



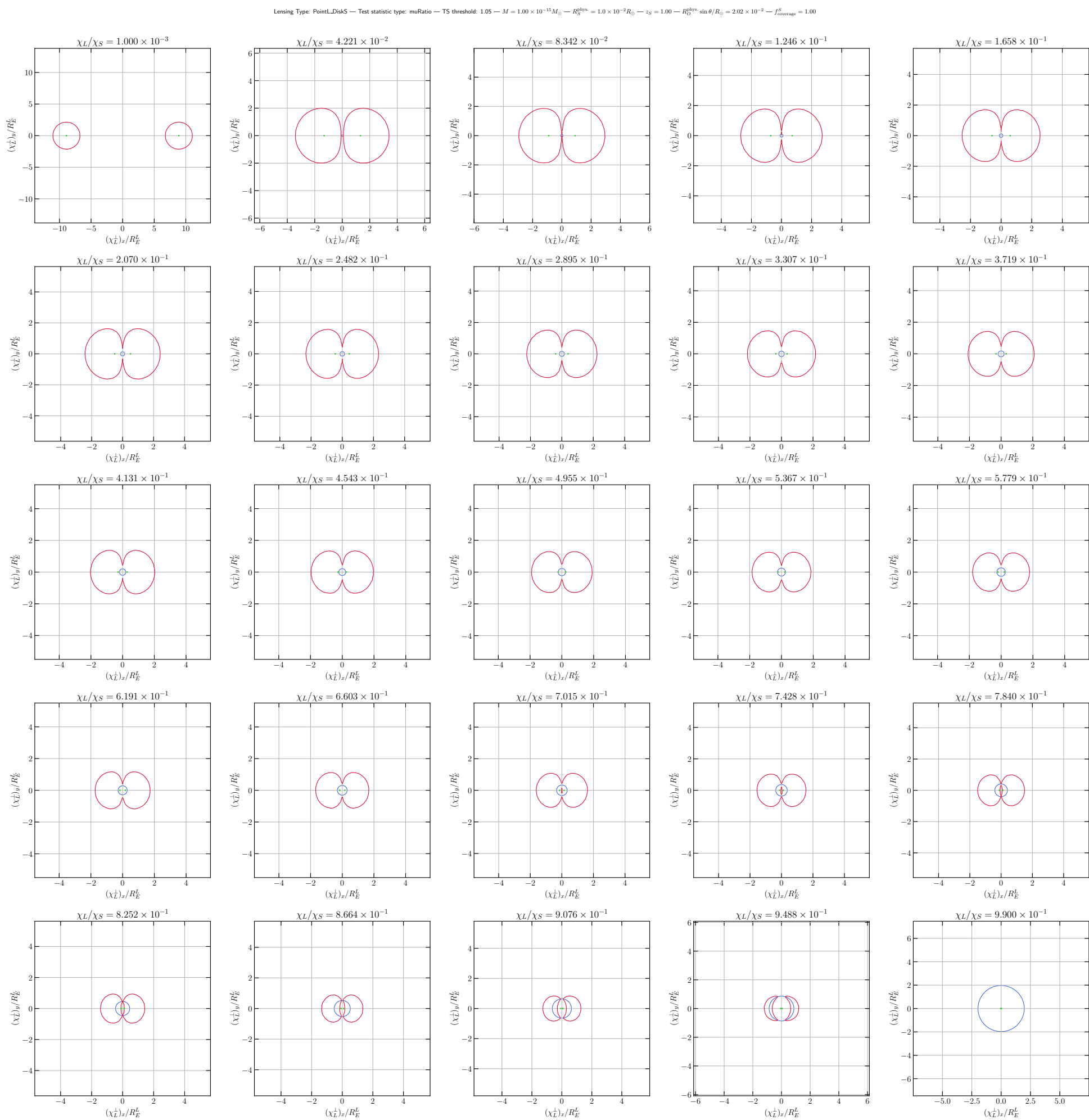
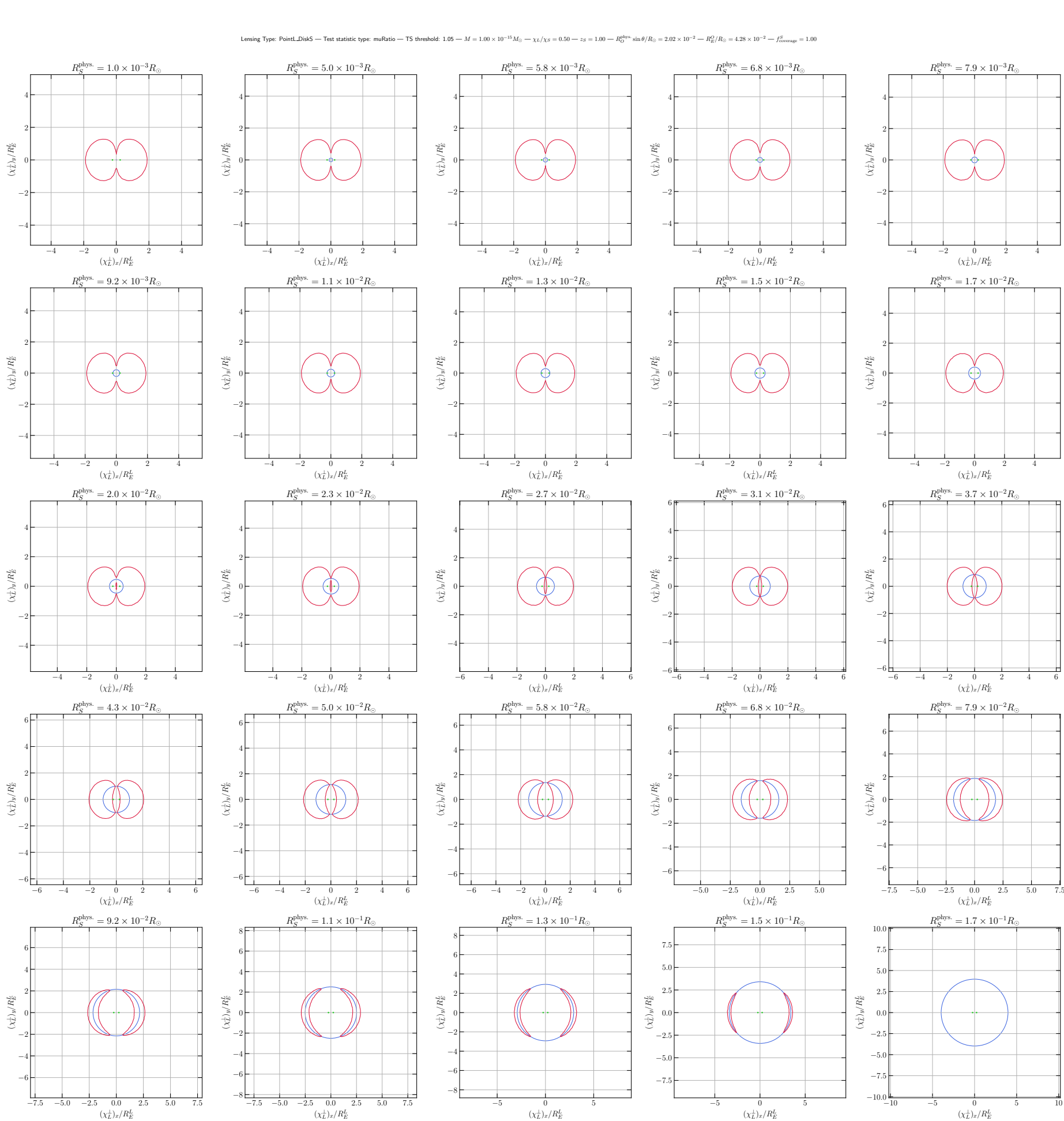
Changing source size



Changing lens distance

Picolensing cross-section

Disk



Changing source size

Changing lens distance

Source characteristics: *Swift*/BAT catalogue

Need to know: duration (T), distance (z_S), source fluence (f_S), source size (θ_S)

Duration: T_{90} , 90% of measured intensity

Distance: z_S , known for ~ 409 GRBs in the *Swift*/BAT catalogue

Source fluence: Band function in source frame. In detector frame, fit as a power law (PL) or cut-off power law (CPL):

$$f_{\text{PL}}(E) \equiv K_{50}^{\text{PL}} \left(\frac{E}{50 \text{ keV}} \right)^{\alpha_{\text{PL}}} \quad f_{\text{CPL}}(E) \equiv K_{50}^{\text{CPL}} \left(\frac{E}{50 \text{ keV}} \right)^{\alpha_{\text{CPL}}} \exp \left[-\frac{E(2 + \alpha_{\text{CPL}})}{E_{\text{CPL}}^{\text{peak}}} \right]$$

$$f_S = \int_{E_{\text{min}}}^{E_{\text{max}}} f_{(\text{C})\text{PL}}(E) dE$$

Optically thin regime?

$$\bar{\tau}(n_0) = n_0 \bar{\mathcal{V}} = -\frac{\ln(1-\alpha)}{\mathfrak{N}} \frac{n_0}{n_0^\alpha} = -\frac{\ln(1-\alpha)}{\mathfrak{N}} \frac{f_{\text{DM}}}{f_{\text{DM}}^\alpha}$$

For $\alpha = 0.95$, $-\ln(1-\alpha) \sim 3$

If we demand $\bar{\tau} \ll 1$, at...

... the limit: $\bar{\tau}(f_{\text{DM}} = f_{\text{DM}}^\alpha, \alpha = 0.95) = \frac{3}{\mathfrak{N}} \ll 1 \Rightarrow \mathfrak{N} \gg 3$



Automatic if many sources are needed to achieve the limit

... at all of the DM: $\bar{\tau}(f_{\text{DM}} = 1, \alpha = 0.95) = \frac{3}{\mathfrak{N} f_{\text{DM}}^{0.95}} \ll 1 \Rightarrow f_{\text{DM}}^{0.95} \gg \frac{3}{\mathfrak{N}}$



Need to make sure the limit is not too strong given the number of sources

Note that this is actually not dependent on $\mathfrak{N}$, but really a condition on the source population

$$\frac{3}{\mathfrak{N} f_{\text{DM}}^{0.95}} = \omega_{\text{DM}}^0 \times \frac{3 \bar{\mathcal{V}}}{4\pi(H_0/h)^{-3}} \frac{(H_0/h)^{-1}}{(2G_N M)}$$

$$\mathcal{V} \lesssim \chi_S^2 G_N M$$

PBH DM

$$\rho_i(z) = \rho_i^0(1+z)^{3(1+w_i)}$$

Assuming we are bounding the **unclustered PBH DM component**, physical mass density would be

$$\rho_{\text{phys}}(z) = f_{\text{DM}} \times \rho_{\text{DM}}^0 \times (1+z)^3 \equiv M n_{\text{phys}}(z) \Rightarrow n_{\text{phys}}(z) = f_{\text{DM}} \times \frac{\rho_{\text{DM}}^0}{M} \times (1+z)^3.$$

But comoving number density is $n_{\text{comov}} = n_{\text{phys}}(1+z)^{-3} = f_{\text{DM}} \times \frac{\rho_{\text{DM}}^0}{M} \equiv n_0.$

Optimism:

And $\rho_{\text{DM}}^0 = \Omega_{\text{DM}}^0 \rho_{\text{crit}} = 3\omega_{\text{DM}}^0 \frac{(H_0/h)^2}{8\pi G_N}$, so... $n_0 \equiv f_{\text{DM}} \times \frac{3\omega_{\text{DM}}^0 (H_0/h)^2}{4\pi(2G_N M)}$

DISCOVERY SPACE!

Limit: $f_{\text{DM}}^\alpha = -\frac{\ln(1-\alpha)}{\omega_{\text{DM}}^0 \mathfrak{N}} \boxed{\frac{4\pi(H_0/h)^{-3}}{3\mathcal{V}} \frac{(2G_N M)}{(H_0/h)^{-1}}}$

Ratio of Hubble volume to lensing volume (tiny!)... but you get back a single power to the Schwarzschild radius of the lens to the Hubble radius (large!)

for $\mathfrak{N}$ sources if I see NO picolensing at $\rho \geq \rho_*$

$$\mathcal{V} \sim \chi_S^2 G_N M$$

$$\boxed{\dots} \propto \left(\frac{\chi_S}{H_0^{-1}}\right)^{-2}$$

$$\omega_{\text{DM}}^0 \equiv \Omega_{\text{DM}}^0 h^2 \sim 0.12$$

Clustering

Jung and Kim
[1908.00078]

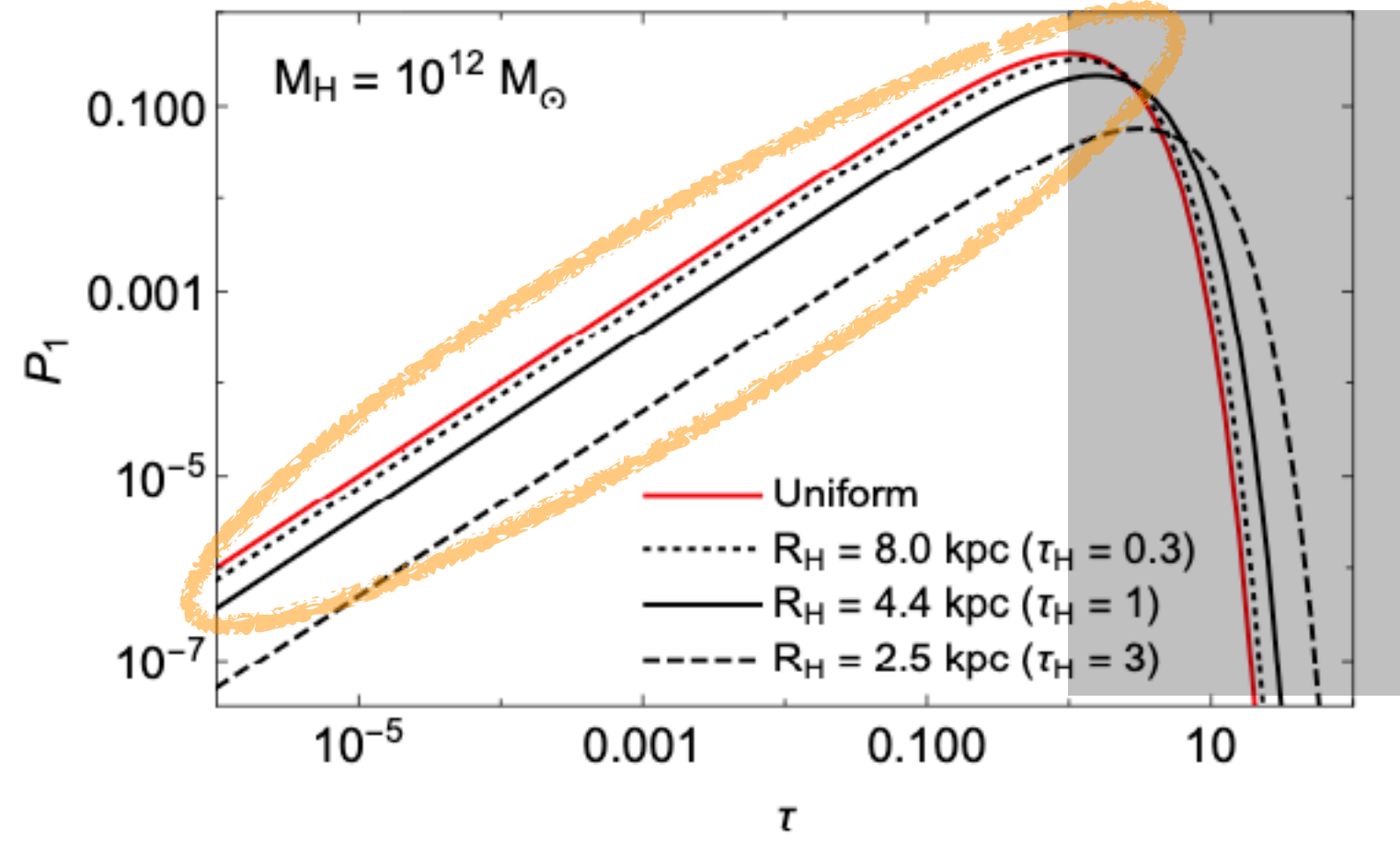


FIG. 3. Single lensing probability P_1 for selected values of R_H (equivalently, τ_H); the smaller the halo size R_H of the given mass $M_H = 10^{12} M_\odot$, the higher clustering of PBHs within the single halo yielding higher optical depth τ_H . The red line is for the uniform distribution of PBHs (no halos) giving the Poisson distribution of the number of lenses, while black lines represent clustered distributions which depart from the Poisson distribution. Note that the Milky Way has $R_H \sim 100$ kpc, yielding $\tau_H \ll 1$. $f = 10^{-2}$ and $k = 10$.

Appendix D: Effects of PBH clustering

We now demonstrate that assuming a uniform distribution of PBHs is decent in calculating lensing parallax, even though nearly half of DM is thought to be clustered around galaxies. As discussed in the paper, we (conservatively) consider only single-lensing events; multi-lensing can still lead to parallax, but is more complicated to calculate and analyze. Thus, we show that the single lensing probability P_1 does not change significantly for clustered PBHs in most regions of galaxies.

First of all, the clustering does not change the optical depth τ , as the average number of lenses within the V_L (the desired volume of PBH locations for lensing) remains the same. But P_1 may still change because once a lens is within V_L it is more likely that there are other clustered lenses within the same V_L so that multi-lensing occurs more often than single lensing. More quantitatively, the number of lenses within V_L no longer follows the Poisson distribution.

Suppose a LOS passes through N halos (the clustered PBH) and the optical depth within each halo is denoted by τ_H (the more clustered within a halo, the larger τ_H); then the expectation value of $\langle N \rangle = \tau/\tau_H$. The single-lensing probability for the case of N halos is

$$\begin{aligned} P_{1|N \text{ halos}} &= (\text{number of halos}) \\ &\times (\text{probability for one halo to give single lensing}) \\ &\times (\text{probability for other halos to give no lensing}) \\ &= N \times \tau_H e^{-\tau_H} \times (e^{-\tau_H})^{N-1}, \end{aligned} \quad (\text{D1})$$

where we assume that PBH DM is clustered but uniformly distributed within each halo. Summing N with its own Poisson distribution, we obtain the total probability for single lensing

$$P_1 = \sum_{N=1}^{\infty} P_{1|N \text{ halos}} \times \frac{(\tau/\tau_H)^N}{N!} e^{-\tau/\tau_H}. \quad (\text{D2})$$

Fig. 3 shows $P_1(\tau)$ for three selected values of τ_H . The deviation from the uniform distribution is sizable for $\tau_H \gtrsim 1$ irrespective of τ ; this is the manifestation of non-Poissonian properties in Eq. (D2). This is understandable because $\tau_H \gtrsim 1$ directly means that clustering is so high that there are likely multiple lenses within the V_L of a single halo.

The τ_H is related to the halo radius R_H with the mass M_H . The PBH number density within the halo is $n_H = (M_H f/M)/(4/3 \times \pi R_H^3)$. The V_L within the halo, denoted by $V_{L|H}$, is approximately the cylinder with a length R_H and a (Einstein) cross-section $\sigma = k \cdot \pi(D_L \theta_E)^2$, where k is determined by detection criteria and detector sensitivities (see below for realistic values of k). Thus we obtain τ_H as a function of R_H

$$\tau_H \approx 1.9 \times 10^{-22} \times k f \left(\frac{M_H}{M_\odot} \right) \left(\frac{D_L}{R_H} \right)^2 \left(\frac{1 \text{ Gpc}}{D_L D_S / D_{LS}} \right). \quad (\text{D3})$$

This allows us to interpret Fig. 3 for Milky-Way-like galaxies. For $M_H = 10^{12} M_\odot$, τ_H becomes $\gtrsim 1$ for $R_H \lesssim 4.4$ kpc. The Milky-Way is thought to have a much larger halo ~ 100 kpc. Thus, the PBH clustering is small enough not to affect our calculations based on the uniform distribution of PBH. For reference, the Milky-Way gives $\tau_H \lesssim 2 \times 10^{-3}$ with $R_H = 100$ kpc. In all these estimations, we use $k = 10$ and $f = 10^{-2}$; the values of k in our results are usually $\lesssim 10$ but grow with ϵ improvement so that the clustering and multi-lensing can become more relevant in the future.

Comparison to 2308.01775

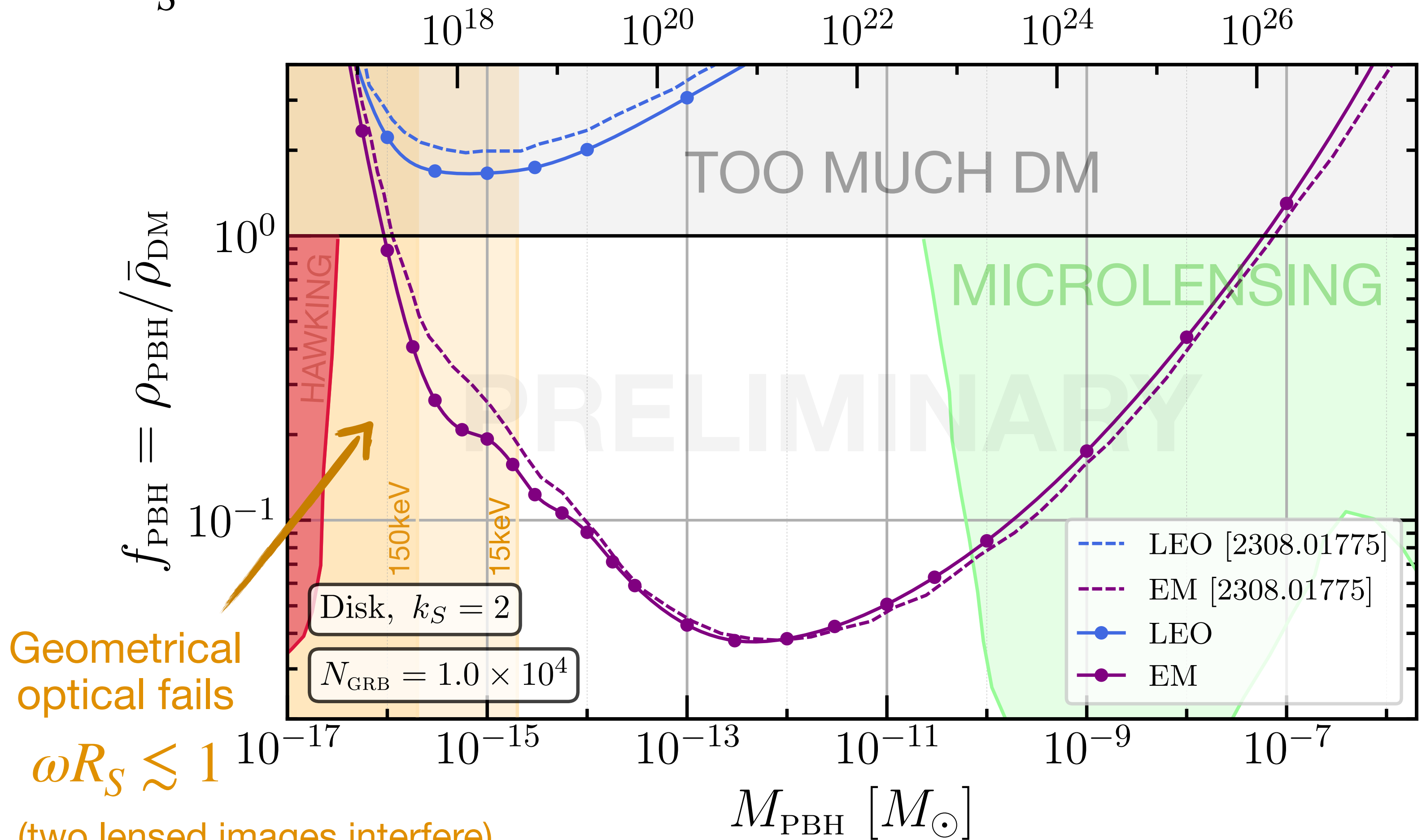
They took

$$k_S = 2$$

$$A_b = 2400 \text{ cm}^2 \quad A_s = 1300 \text{ cm}^2$$

$$f_b = 10 \text{ cm}^{-2} \text{ s}^{-1}$$

~Swift/BAT



Pretty good agreement (5-10%)

Validates our implementation

Confirms previous literature under their assumptions

Scenario	Abbrev.	Baseline R_O	$R_O/R_\odot$
Low Earth Orbit	LEO	$1.40 \times 10^4 \text{ km}$	0.020
Earth-Moon	EM	$3.84 \times 10^5 \text{ km}$	0.55

Minor difference in assumed energy range (15-150keV [us] vs. 20-200 keV [them]); makes little difference

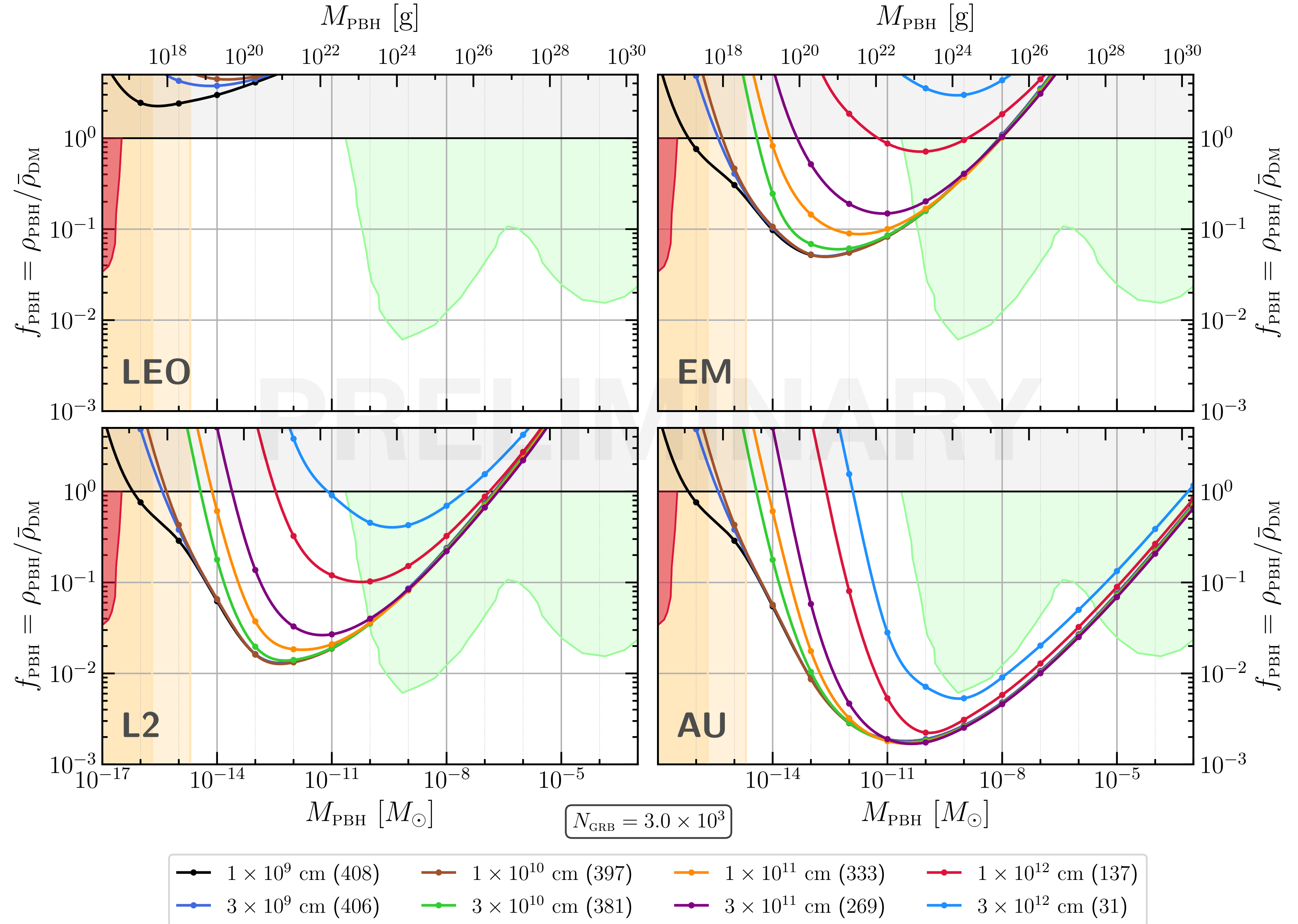
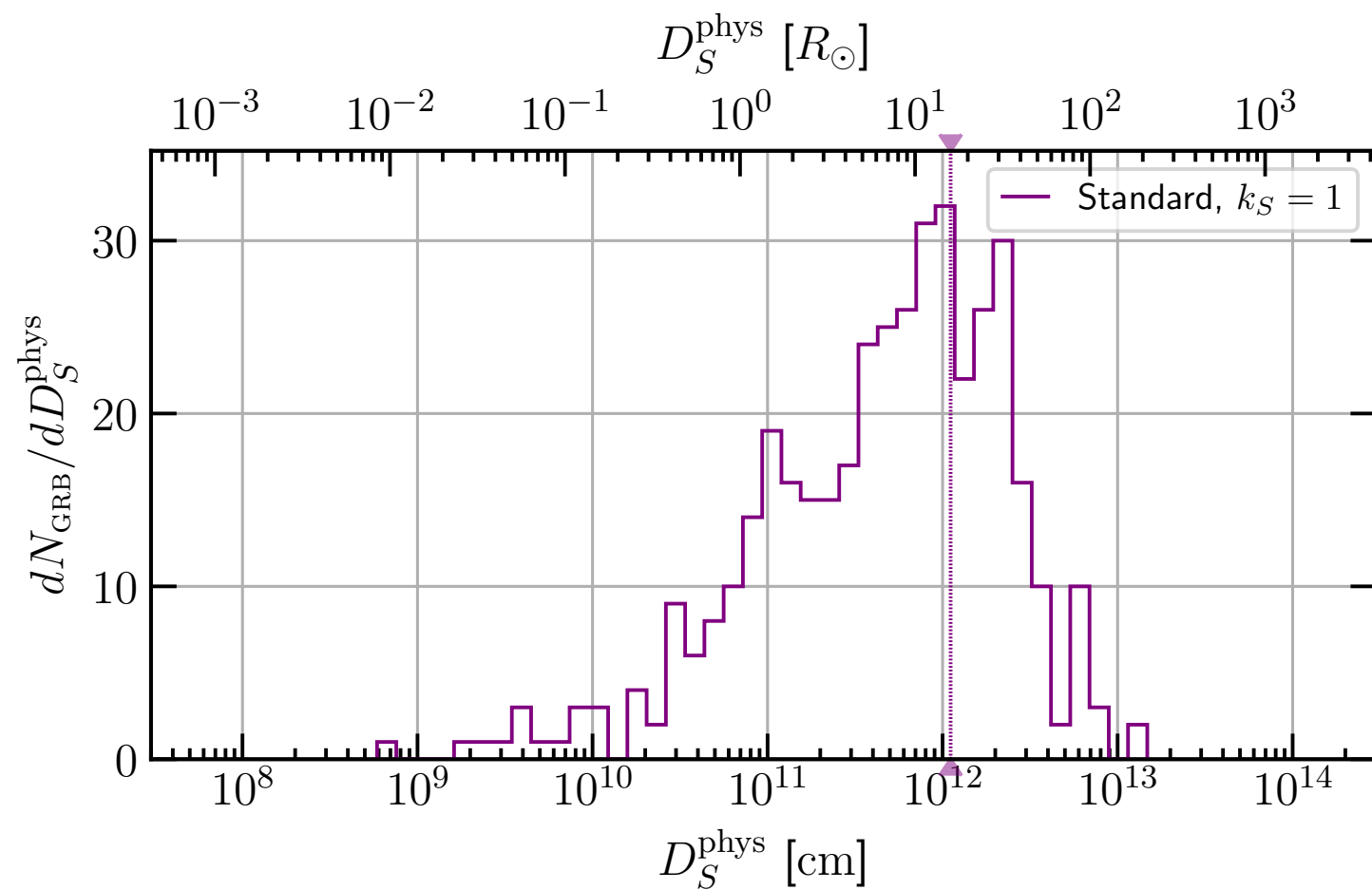
Vary minimum source size

$$k_S = 1 \quad D_{\text{obs}} \equiv \frac{k_S T_{90}}{1 + z_S}$$

Recall:

$$R_{\odot} \sim 7 \times 10^{10} \text{ cm}$$

Quite robust to
excluding
 $\lesssim R_{\odot}$ sources

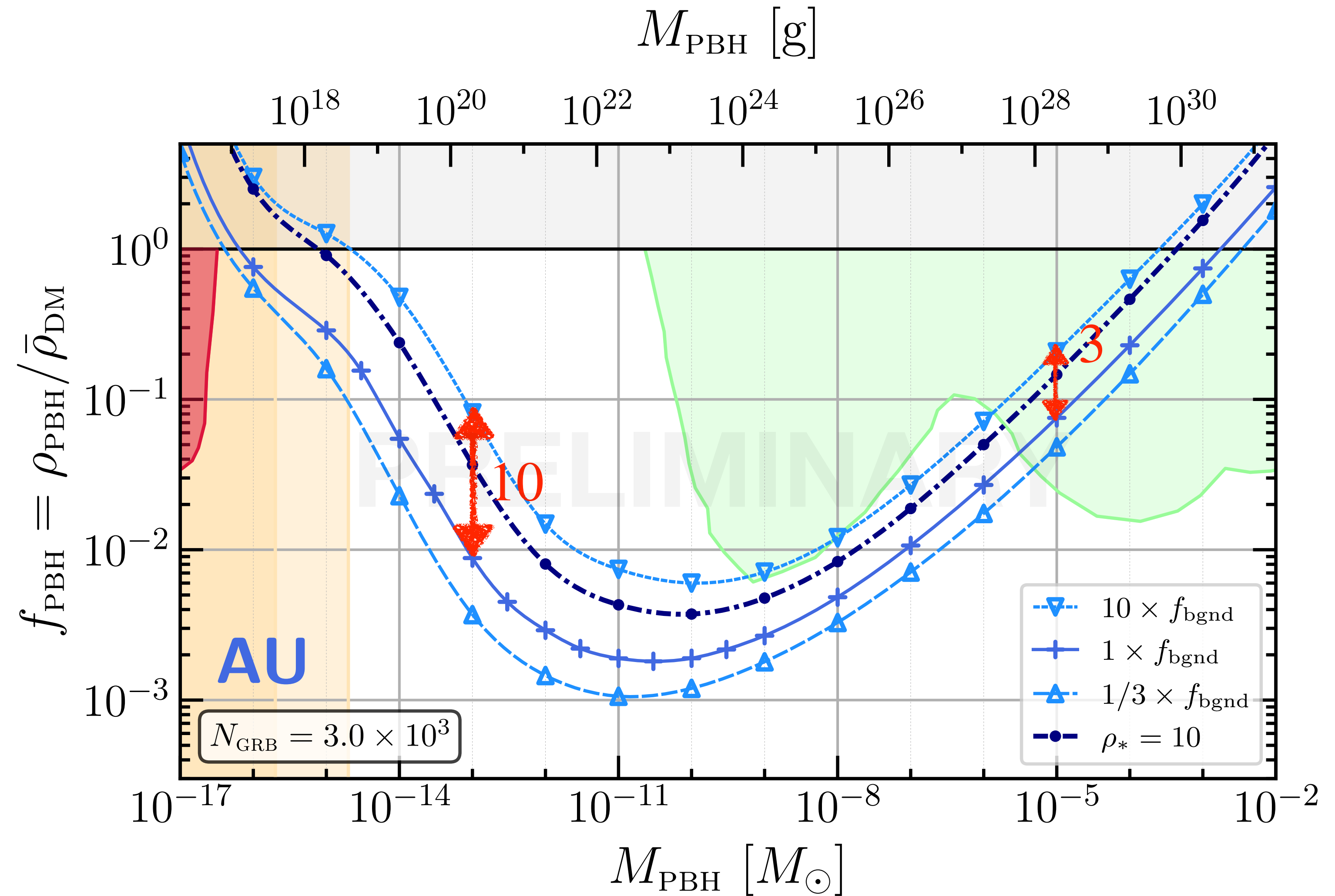


Vary the background level

Note: not always $\propto \sqrt{f_b}$

Higher
backgrounds
once out of
LEO?

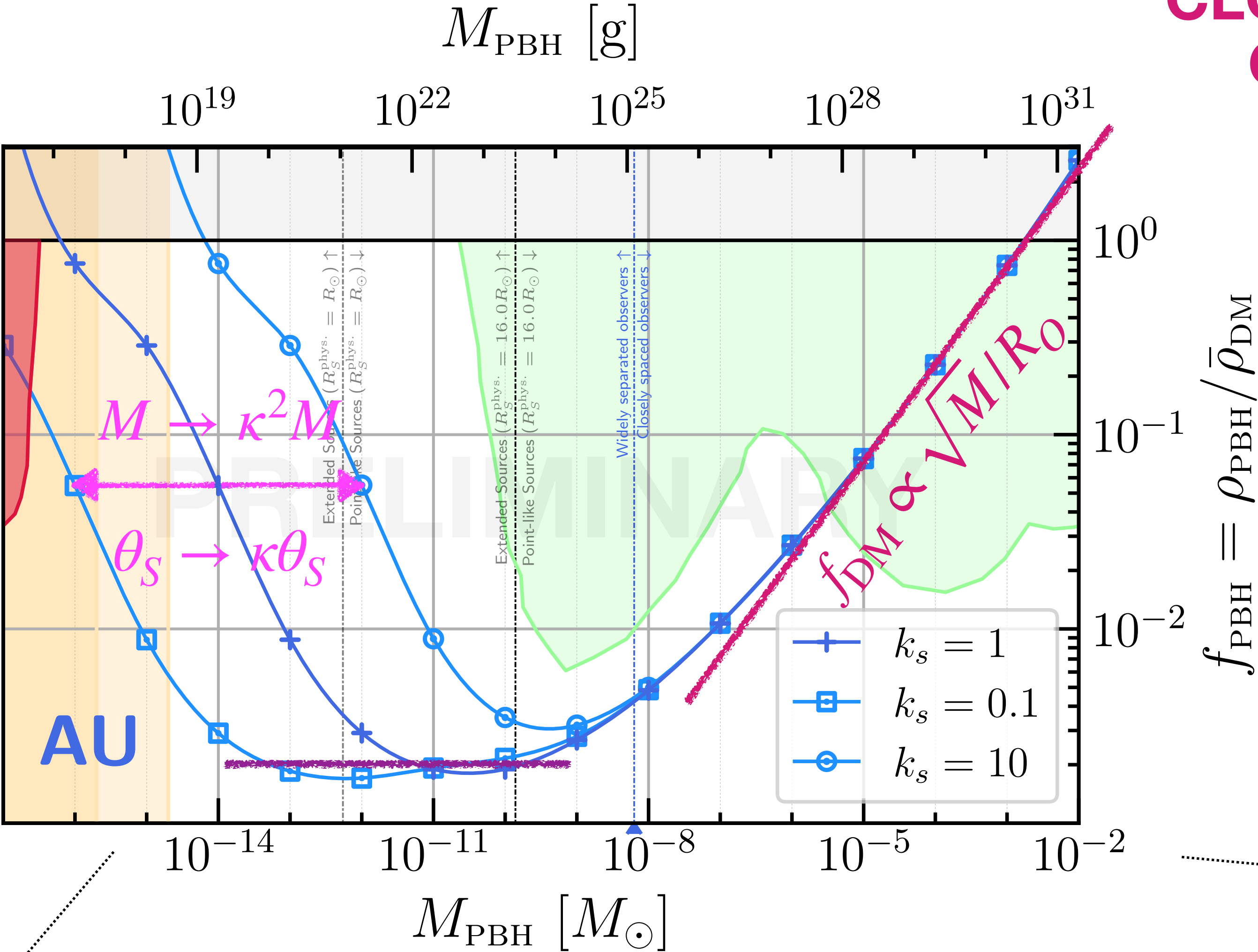
Some x-ray
detection
backgrounds
from HE
particles hitting
detector /
spacecraft



Scalings

EXTENDED
SOURCE, WIDELY
SEPARATED
OBSERVERS

POINT SOURCES,
CLOSELY SPACED
OBSERVERS

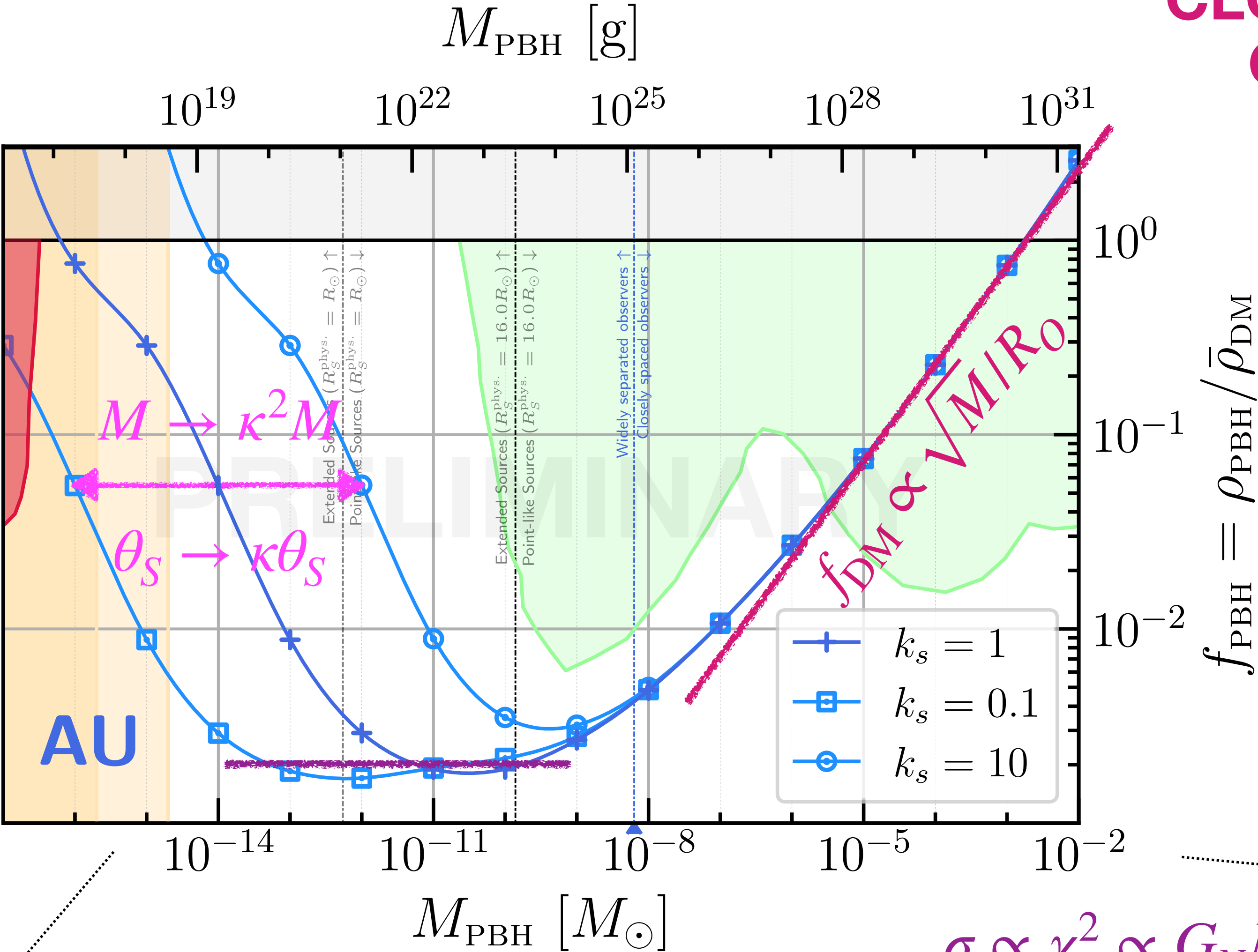


~POINT SOURCES, WIDELY
SEPARATED OBSERVERS

Scalings

EXTENDED
SOURCE, WIDELY
SEPARATED
OBSERVERS

POINT SOURCES,
CLOSELY SPACED
OBSERVERS



~POINT SOURCES, WIDELY
SEPARATED OBSERVERS

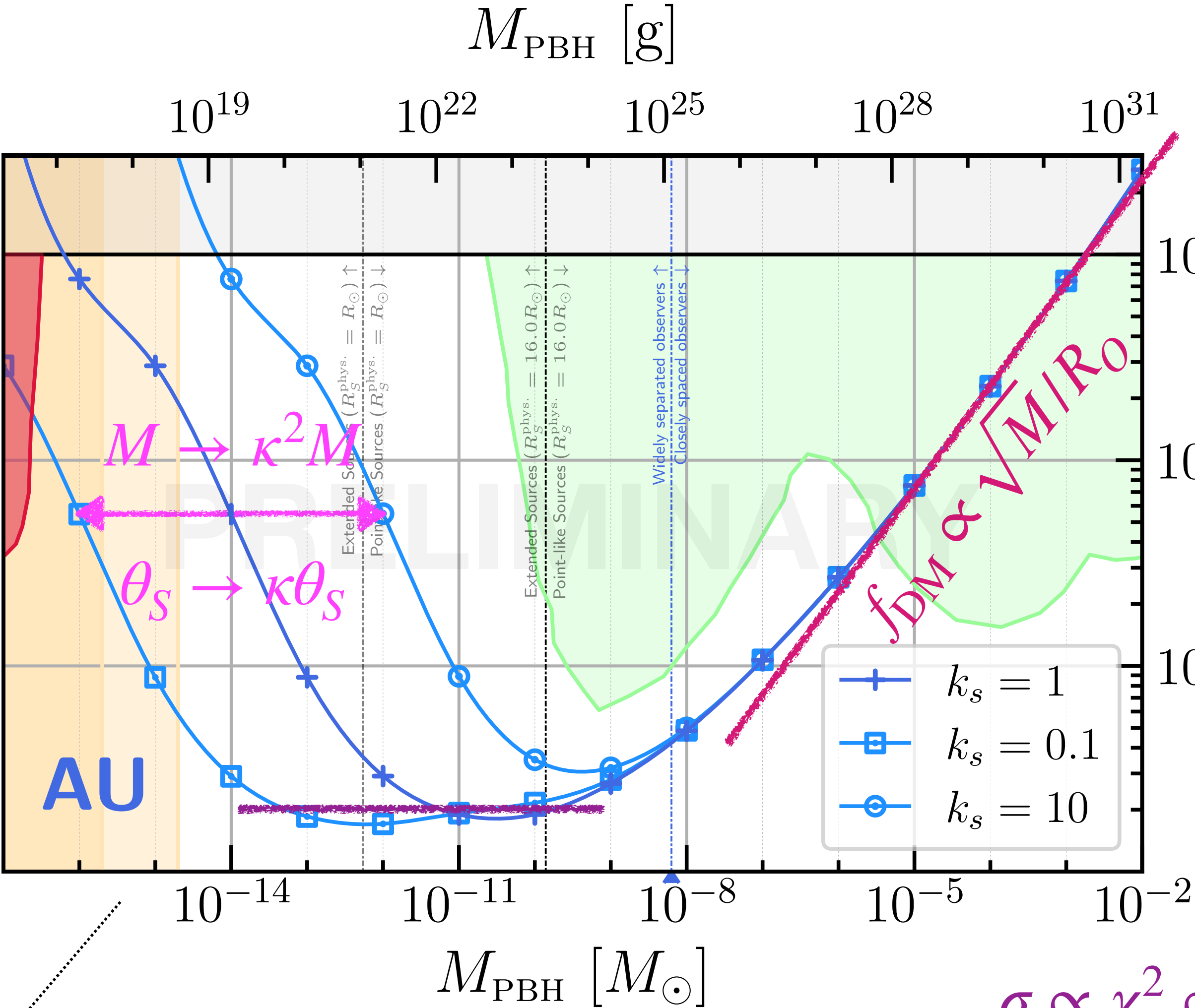
$$\sigma \propto \chi_E^2 \propto G_N M \chi_L$$

$$\mathcal{V} \propto G_N M \chi_S^2 \quad f_{\text{DM}}^{\text{limit}} \propto \frac{M}{\mathcal{V}} \propto M^0$$

Scalings

EXTENDED
SOURCE, WIDELY
SEPARATED
OBSERVERS

POINT SOURCES,
CLOSELY SPACED
OBSERVERS



$$\sigma \propto R_O \chi_E$$

$$\propto R_O \sqrt{G_N M \chi_L}$$

$$\mathcal{V} \propto R_O \sqrt{G_N M \chi_S^3}$$

$$f_{\text{DM}}^{\text{limit}} \propto \frac{\sqrt{M}}{R_O}$$

$$\sigma \propto \chi_E^2 \propto G_N M \chi_L$$

$$\mathcal{V} \propto G_N M \chi_S^2$$

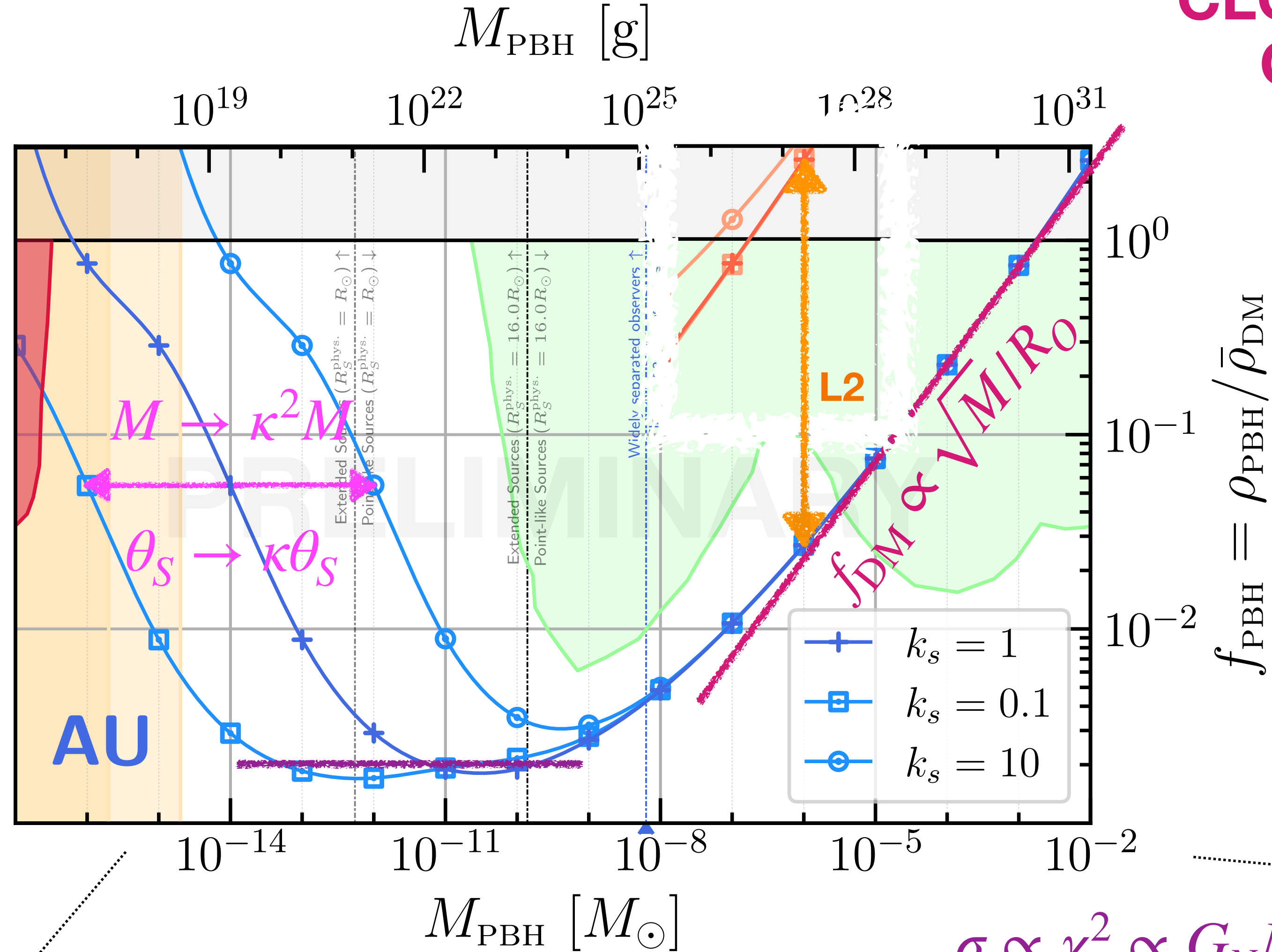
$$f_{\text{DM}}^{\text{limit}} \propto \frac{M}{\bar{\mathcal{V}}} \propto M^0$$

~POINT SOURCES, WIDELY
SEPARATED OBSERVERS

**EXTENDED
SOURCE, WIDELY
SEPARATED
OBSERVERS**

$$R_{L2} \sim 10^{-2} \text{AU}$$

POINT SOURCES, CLOSELY SPACED OBSERVERS



$$\begin{aligned}\sigma &\propto R_O \chi_E \\ &\propto R_O \sqrt{G_N M \chi_L}\end{aligned}$$

$$\mathcal{V} \propto R_O \sqrt{G_N M \chi_S^3}$$

$$f_{\text{DM}}^{\text{limit}} \propto \frac{\sqrt{M}}{R_o}$$

$$\sigma \propto \chi_E^2 \propto G_N M \chi_L$$

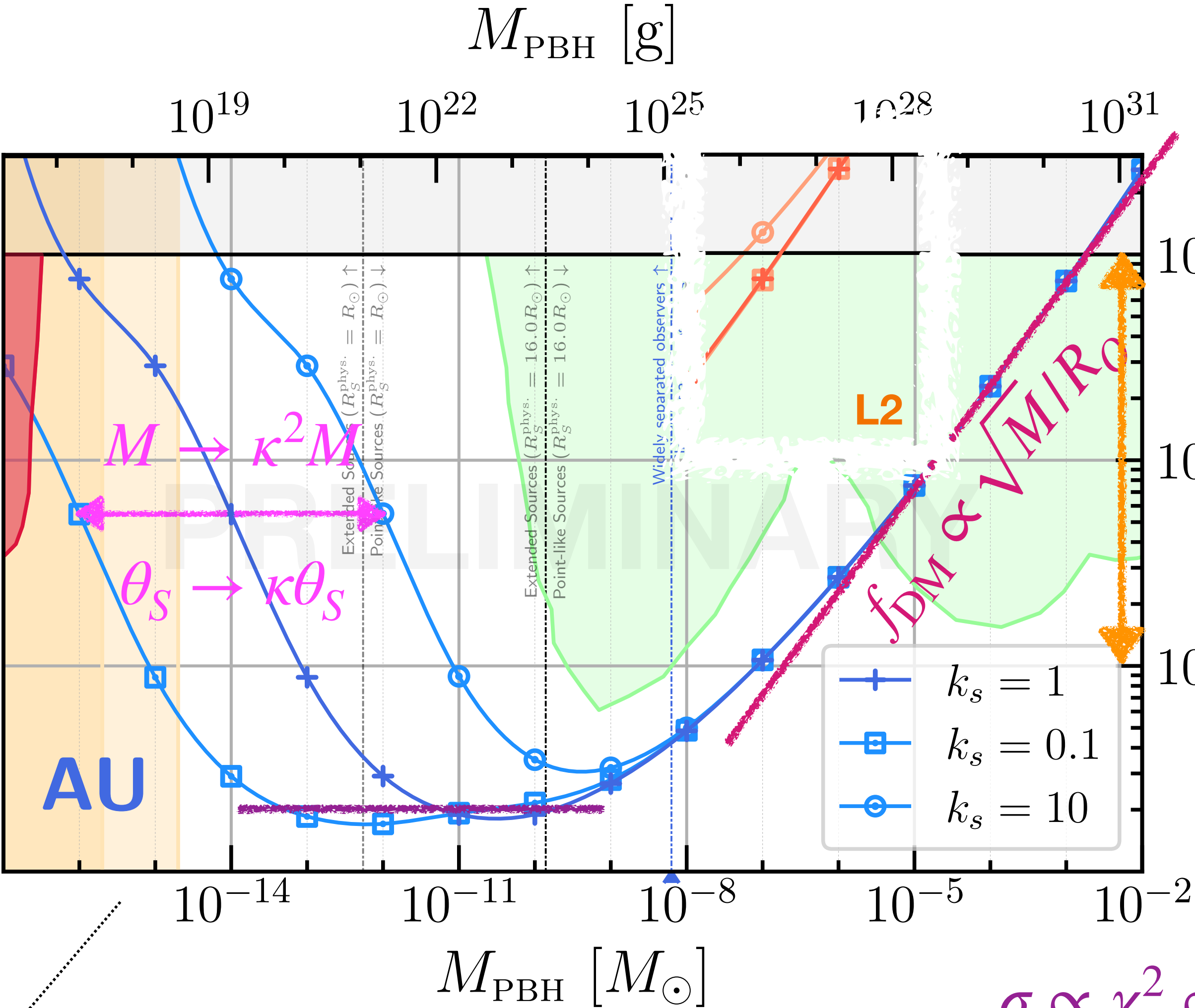
$$\mathcal{V} \propto G_N M \chi_S^2$$

$$f_{\text{DM}}^{\text{limit}} \propto \frac{M}{\bar{\mathcal{V}}} \propto M^0$$

~POINT SOURCES, WIDELY SEPARATED OBSERVERS

Scalings

EXTENDED
SOURCE, WIDELY
SEPARATED
OBSERVERS



$$R_{L2} \sim 10^{-2} \text{AU}$$

POINT SOURCES,
CLOSELY SPACED
OBSERVERS

$$\sigma \propto R_O \chi_E$$

$$\propto R_O \sqrt{G_N M \chi_L}$$

$$\mathcal{V} \propto R_O \sqrt{G_N M \chi_S^3}$$

$$f_{\text{DM}}^{\text{limit}} \propto \frac{\sqrt{M}}{R_O}$$

~POINT SOURCES, WIDELY
SEPARATED OBSERVERS

$$\sigma \propto \chi_E^2 \propto G_N M \chi_L$$

$$\mathcal{V} \propto G_N M \chi_S^2$$

$$f_{\text{DM}}^{\text{limit}} \propto \frac{M}{\bar{\mathcal{V}}} \propto M^0$$

Scalings

**EXTENDED
SOURCE, WIDELY
SEPARATED
OBSERVERS**

$$\mathcal{V} \propto \frac{(G_N M)^3}{\theta_S^4}$$

M -dependence
of $f_{\text{DM}}^{\text{limit}}$
dominated by
population
distribution of θ_S

$$f_{\text{DM}}^{\text{limit}} \propto \frac{(\theta_S^{\text{min}})^4}{M^2 \times N_S^>}$$

$$\bar{\mathcal{V}} \sim (G_N M)^3 \sum_{i=1}^N (\theta_S^i)^{-4} \sim (G_N M)^3 (\theta_S^{\text{min}})^{-4} \sum_{i=1}^N (\theta_S^{\text{min}} / \theta_S^i)^4$$

**POINT SOURCES,
CLOSELY SPACED
OBSERVERS**

$$\sigma \propto R_O \chi_E$$

$$\propto R_O \sqrt{G_N M \chi_L}$$

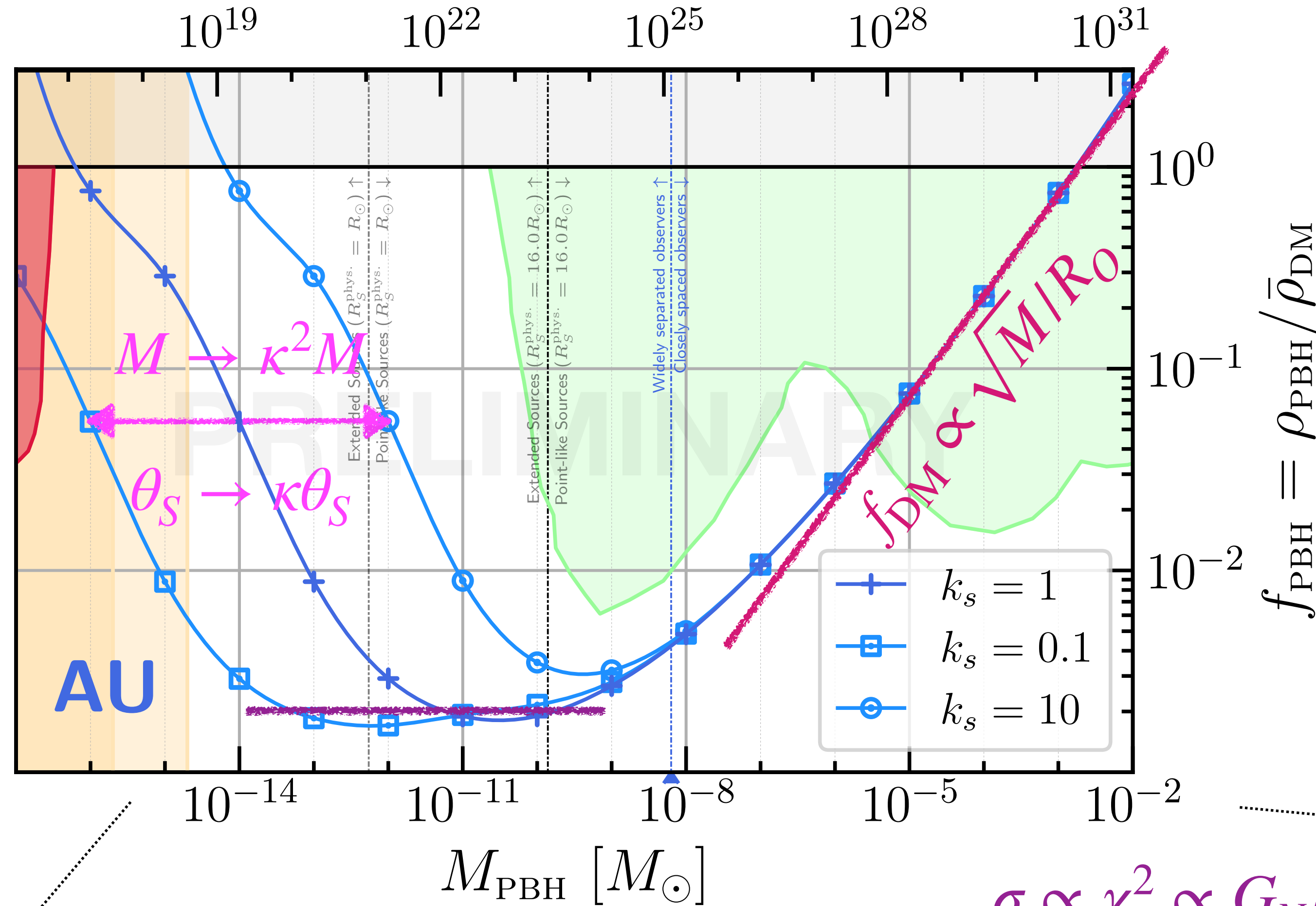
$$\mathcal{V} \propto R_O \sqrt{G_N M \chi_S^3}$$

$$f_{\text{DM}}^{\text{limit}} \propto \frac{\sqrt{M}}{R_O}$$

$$\sigma \propto \chi_E^2 \propto G_N M \chi_L$$

$$\mathcal{V} \propto G_N M \chi_S^2$$

$$f_{\text{DM}}^{\text{limit}} \propto \frac{M}{\bar{\mathcal{V}}} \propto M^0$$



**~POINT SOURCES, WIDELY
SEPARATED OBSERVERS**

Minimum Variability Timescale

To find the observed minimum variability time scale for each GRB, we have used the method utilized by [Bhat et al. \(2012\)](#); [Bhat \(2013a,b\)](#), which searches for a characteristic time scale at which the variance ratios per bin width is minimum. The characteristic time scale is interpreted as an upper limit on the minimum variability time scale. This method incorporates the following steps: first, the time interval of the prompt emission is selected based on T_{90} ; then, a background time interval of an equal duration is selected. Both the signal and the background intervals are used to derive differentials, which, in the next step, are used to calculate variances of the signal and the background. The ratios of the variances are calculated for different binnings in the range from 10^{-3} s up to $0.1 \times T_{90}$ using ten logarithmic bins per decade. The bin width at which the variance ratio divided by the bin width obtains its minimum value is interpreted as a minimum observed variability time scale, t_{var} , (e.g., see Figure 1 in [Bhat 2013a](#)). The resulting minimum variability time scales for the entire sample are listed in Table 1.

Barnacka, Loeb

[1408.1232]

Ratio of (binned signal variance)
to (binned background variance),
normalised to number of bins

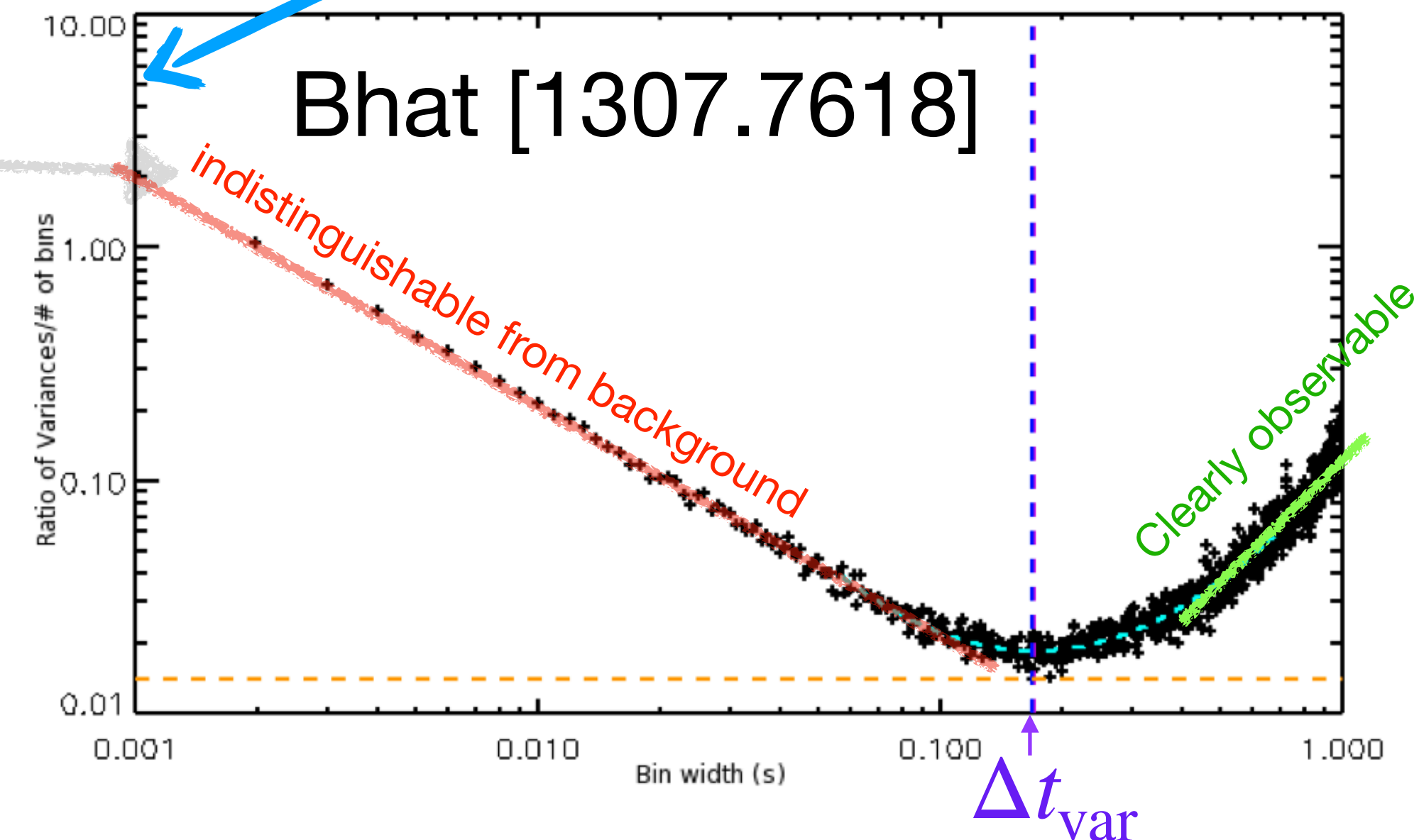


Figure 1: : Variation of the ratio of the variances per bin to the histogram bin-width. At very fine bin-widths the GRB signal is indistinguishable from background fluctuations and hence the ratio decreases monotonically with increasing bin-width. At larger bin-widths the signal is clearly visible from the background and hence the ratio per bin starts increasing. The bin-width at the turn over is defined as the minimum variability time scale where the bin-width is expected to be optimum. Cyan dashed line shows a fitted parabola around the minimum that has a minimum at a bin-width indicated by the vertical dashed line in blue.