Università degli Studi di Padova

MUR 20228WHTYC

Wallions: Dark Matter from the Boundaries of Field Space

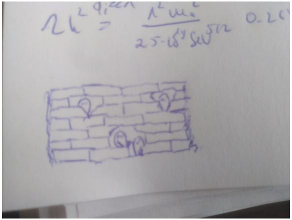
based on 2511.09622 (PRL 136, 2026)

in collaboration with Francesco D'Eramo and Ville Vaskonen

Mathias Becker

IDM Zaragoza, June 2026

Why “wallions”?



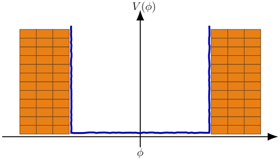
potentials with field space boundaries

(“walls”)

UNIVERSITÀ DEGLI STUDI DI PADOVA

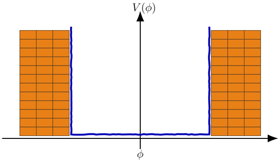
MUR 20228WHTYC

Why "wallions"?



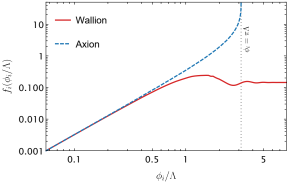
potentials with field space boundaries ("walls")

Why “wallions”?



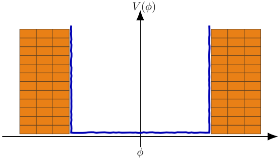
potentials with field space boundaries (“walls”)

Why (ultra)light Dark Matter?



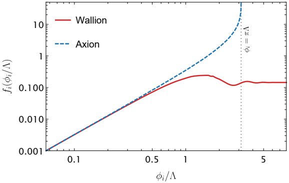
exponentially suppressed masses.

Why “wallions”?



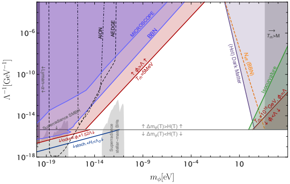
potentials with field space boundaries (“walls”)

Why (ultra)light Dark Matter?



exponentially suppressed masses.

How to realize them?



non-perturbative dynamics, quadratic SM coupling

Field Space Boundaries

A **field space boundary** [Cheung, Rothstein (2024)]: dynamics restricted to $|\phi| < \bar{\phi}$

Naive model: infinite wall potential

$$V_{\text{wall}} = \begin{cases} 0, & |\phi| < \bar{\phi}, \\ \infty, & |\phi| > \bar{\phi}, \end{cases} .$$

Field Space Boundaries

A **field space boundary** [Cheung, Rothstein (2024)]: dynamics restricted to $|\phi| < \bar{\phi}$

Naive model: infinite wall potential

$$V_{\text{wall}} = \begin{cases} 0, & |\phi| < \bar{\phi}, \\ \infty, & |\phi| > \bar{\phi}, \end{cases} .$$

Regulated version (finite wall width Λ)

$$V_B = \Lambda^4 \exp\left(\frac{\phi^2 - \bar{\phi}^2}{\Lambda^2}\right) = \exp\left(-\frac{\bar{\phi}^2}{\Lambda^2}\right) \Lambda^4 \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{\phi}{\Lambda}\right)^{2n}$$

Field Space Boundaries

A **field space boundary** [Cheung, Rothstein (2024)]: dynamics restricted to $|\phi| < \bar{\phi}$

Naive model: infinite wall potential

$$V_{\text{wall}} = \begin{cases} 0, & |\phi| < \bar{\phi}, \\ \infty, & |\phi| > \bar{\phi}, \end{cases} .$$

Regulated version (finite wall width Λ)

$$V_B = \Lambda^4 \exp\left(\frac{\phi^2 - \bar{\phi}^2}{\Lambda^2}\right) = \exp\left(-\frac{\bar{\phi}^2}{\Lambda^2}\right) \Lambda^4 \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{\phi}{\Lambda}\right)^{2n}$$

Field Space Boundaries

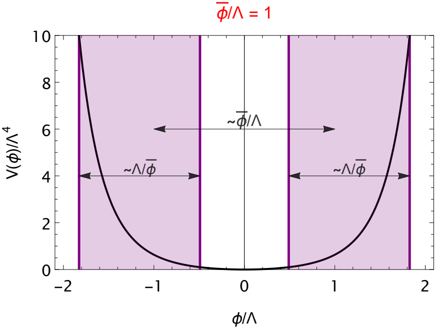
Regulated Version vs. Infinite wall potential

$$V_B = \Lambda^4 \exp\left(\frac{\phi^2 - \bar{\phi}^2}{\Lambda^2}\right) \quad \xrightarrow{\bar{\phi} \gg \Lambda} \quad V_{\rm wall} = \begin{cases} 0, & |\phi| < \bar{\phi}, \\ \infty, & |\phi| > \bar{\phi}, \end{cases} .$$

Field Space Boundaries

Regulated Version vs. Infinite wall potential

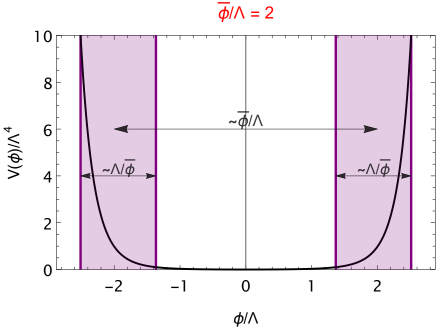
$$V_B = \Lambda^4 \exp\left(\frac{\phi^2 - \bar{\phi}^2}{\Lambda^2}\right) \quad \xrightarrow{\bar{\phi} \gg \Lambda} \quad V_{\text{wall}} = \begin{cases} 0, & |\phi| < \bar{\phi}, \\ \infty, & |\phi| > \bar{\phi}, \end{cases} .$$



Field Space Boundaries

Regulated Version vs. Infinite wall potential

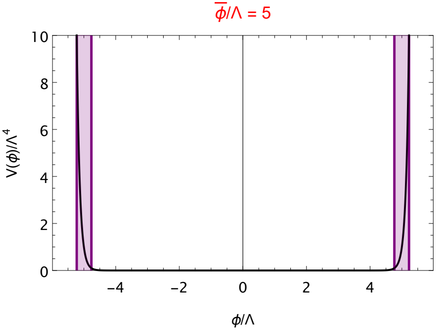
$$V_B = \Lambda^4 \exp\left(\frac{\phi^2 - \bar{\phi}^2}{\Lambda^2}\right) \xrightarrow{\bar{\phi} \gg \Lambda} V_{\mathrm{wall}} = \begin{cases} 0, & |\phi| < \bar{\phi}, \\ \infty, & |\phi| > \bar{\phi}, \end{cases} .$$



Field Space Boundaries

Regulated Version vs. Infinite wall potential

$$V_B = \Lambda^4 \exp\left(\frac{\phi^2 - \bar{\phi}^2}{\Lambda^2}\right) \quad \xrightarrow{\bar{\phi}\gg\Lambda} \quad V_{\text{wall}} = \begin{cases} 0, & |\phi| < \bar{\phi}, \\ \infty, & |\phi| > \bar{\phi}, \end{cases} .$$



We define as a wallion the scalar field excitation associated with the regulated infinite wall potential!

Wallion mass

$$m_\phi^2 = 2\Lambda^2 \exp(-\frac{\bar{\phi}^2}{\Lambda^2}) \quad \xrightarrow{\bar{\phi}\gg\Lambda} \quad 0\,.$$

UNIVERSITÀ DEGLI STUDI DI PADOVA

MUR 20228WHTYC PRIN

INFN PADOVA

We define as a wallion the scalar field excitation associated with the regulated infinite wall potential!

Wallion mass

$$m_\phi^2 = 2\Lambda^2 \exp\left(-\frac{\bar{\phi}^2}{\Lambda^2}\right) \quad \xrightarrow{\bar{\phi} \gg \Lambda} \quad 0\,.$$

potential radiatively stable if $\bar{\phi} \gg \Lambda$ see [Cheung, Rothstein (2024)]

$\delta m_\phi^2 \sim$  ϕ^4 $+$  ϕ^6 $+$  ϕ^8 $+ \cdots$

vertex $\sim e^{-\bar{\phi}^2/\Lambda^2}$ $\Rightarrow$ only 1-vertex insertion contributes $\rightarrow$ corrections $\sim e^{-\bar{\phi}^2/\Lambda^2}\Lambda^2$

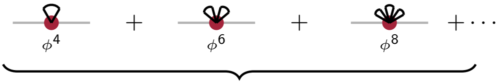
We define as a wallion the scalar field excitation associated with the regulated infinite wall potential!

Wallion mass

$$m_\phi^2 = 2\Lambda^2 \exp\left(-\frac{\bar{\phi}^2}{\Lambda^2}\right) \xrightarrow{\bar{\phi} \gg \Lambda} 0\,.$$

potential radiatively stable if $\bar{\phi} \gg \Lambda$ see [Cheung, Rothstein (2024)]

$\delta m_\phi^2 \sim$



vertex $\sim e^{-\bar{\phi}^2/\Lambda^2}$ $\Rightarrow$ only 1-vertex insertion contributes $\rightarrow$ corrections $\sim e^{-\bar{\phi}^2/\Lambda^2}\Lambda^2$

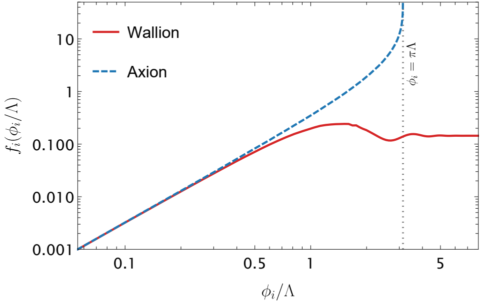
$\rightarrow$ a good (ultralight) dark matter candidate.

Wallion Cosmology I

Scalar equation of motion $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$ implies

Present-Day Scalar abundance depends on the post-inflation value ϕ_i

$$\Omega_\phi h^2 \sim \Lambda^2 \, m_\phi^{1/2} \, f\left(\frac{\phi_i}{\Lambda}\right)$$



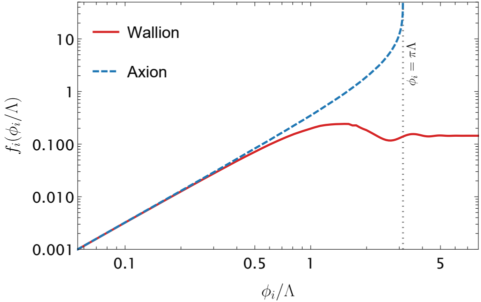
Independent of initial conditions above a threshold misalignment

Wallion Cosmology I

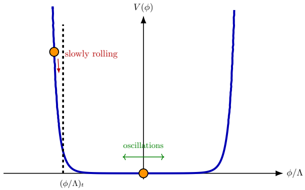
Scalar equation of motion $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$ implies

Present-Day Scalar abundance depends on the post-inflation value ϕ_i

$$\Omega_\phi h^2 \sim \Lambda^2 \, m_\phi^{1/2} \, f\left(\frac{\phi_i}{\Lambda}\right)$$



Independent of initial conditions above a threshold misalignment



Wallion Cosmology: Isocurvature

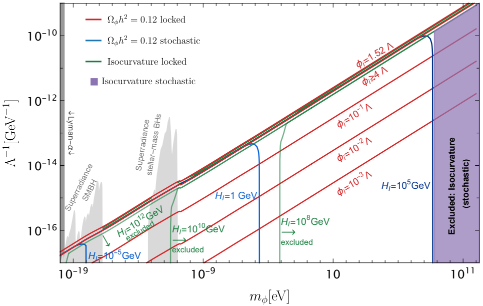
Isocurvature perturbations (oversimplified)

$$P_S \simeq \left(\frac{\delta\rho_\phi}{\rho_\phi}\right)^2 \sim \left(\frac{f'(\phi_i/\Lambda)}{f(\phi_i/\Lambda)}\frac{H_I}{2\pi\Lambda}\right)^2 \overset{\text{CMB}}{\lesssim} 10^{-10}$$

Wallion Cosmology: Isocurvature

Isocurvature perturbations (oversimplified)

$$P_S \simeq \left(\frac{\delta\rho_\phi}{\rho_\phi}\right)^2 \sim \left(\frac{f'(\phi_i/\Lambda)}{f(\phi_i/\Lambda)}\frac{H_I}{2\pi\Lambda}\right)^2 \overset{\text{CMB}}{\lesssim} 10^{-10}$$



Non-Perturbatively Generated Field Space Boundaries

Couple a scalar quadratically to a dark $SU(N)$ gauge kinetic term:

$$\frac{\phi^2}{4M^2}\, G^{\mu\nu} G_{\mu\nu} \quad \longrightarrow \quad \frac{1}{g_{\text{eff}}^2(\mu,\phi)} = \frac{1}{g_d^2(\mu)} - \frac{\phi^2}{M^2}.$$

Non-Perturbatively Generated Field Space Boundaries

Couple a scalar quadratically to a dark $SU(N)$ gauge kinetic term:

$$\frac{\phi^2}{4M^2}\,G^{\mu\nu}G_{\mu\nu} \quad \longrightarrow \quad \frac{1}{g_{\rm eff}^2(\mu,\phi)} = \frac{1}{g_d^2(\mu)} - \frac{\phi^2}{M^2}\,.$$

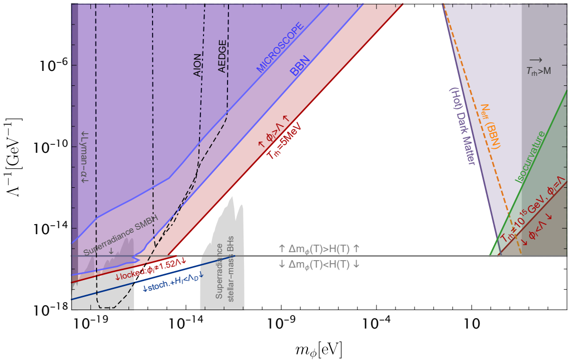
One-instanton amplitude then suggests

$$V(\phi) \;\simeq\; \exp\left(-\frac{8\pi^2}{g_{\rm eff}^2(\mu,\phi)}\right) \;\simeq\; \exp\left(\frac{\phi^2-\bar{\phi}^2}{\Lambda^2}\right)\,.$$

This mechanism may induce interactions with SM gauge bosons

$$\frac{\phi^2}{4M^2}\,F^{\mu\nu}F_{\mu\nu}\,.$$

Experimental Constraints



Constraints from: variations of fundamental constants (BBN, atom interferometers), Weak Equivalence Principle, Isocurvature, Superradiance, . . .

However: large SM interactions can destabilize the potential.

Future Directions

1. **Broader wallion phenomenology**
   - dark energy see [Borghetto et. al (2025)] , inflation?, repulsive/superfluid DM? [Berezhiani et. al (2025)], scalar fragmentation? see e.g. [Chatrchyan et. al (2023)]
2. **Refining the current framework**
   - Radiative stability with large SM interactions see e.g. [Delaunay et. al (2025)], different wallion potentials, refined early-universe observables
3. **Alternative origins of wallion-type potentials**
   - Goldstones from non-compact internal symmetries for inspiration see [Burgess et. al (2014)], metric-affine/Einstein–Cartan gravity, axiverse/multi-axion

Summary

Wallions

Non-periodic scalars **restricted in field space** ($|\phi| < \bar{\phi}$) with an **exponentially suppressed, radiatively stable** mass:

$$m_\phi^2 = 2\Lambda^2 \, e^{-\bar{\phi}^2/\Lambda^2} \ll \Lambda^2 \qquad (\bar{\phi} \gg \Lambda)$$

As ultralight dark matter:

- Relic abundance *independent of initial conditions* for sufficiently large misalignment
- Isocurvature suppressed in the same regime $\rightarrow$ easier realizable for large H_I than e.g. pre-inflationary axions.

Our realization: $\phi^2 G^{\mu\nu} G_{\mu\nu} / M^2$ generates the wallion potential *non-perturbatively*; experimental reach via variations of fundamental constants, WEP tests or superradiance