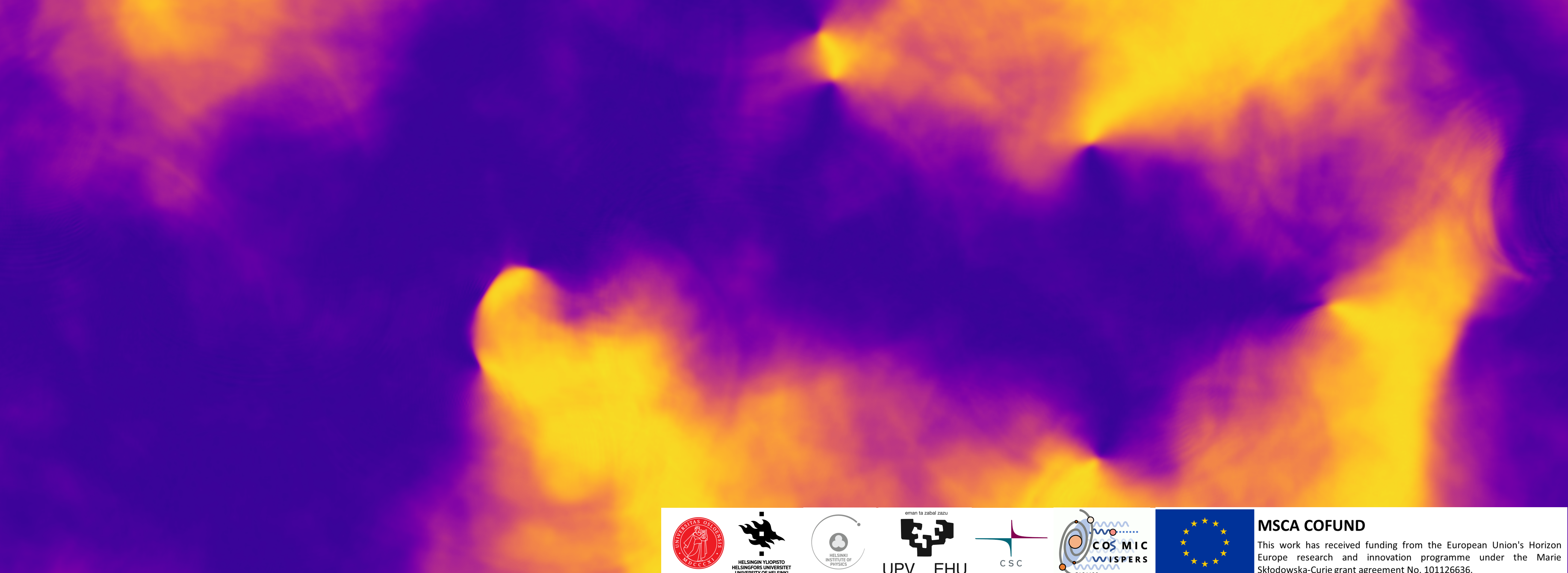


The Spectrum of axions from a scaling string network

J. Correia, M. Hindmarsh, J. Lizarraga,
A. Lopez-Eiguren, K. Rummukainen, J. Urrestilla

Based on arXiv:2512.13653
Resources from CSC Finland: 462000084

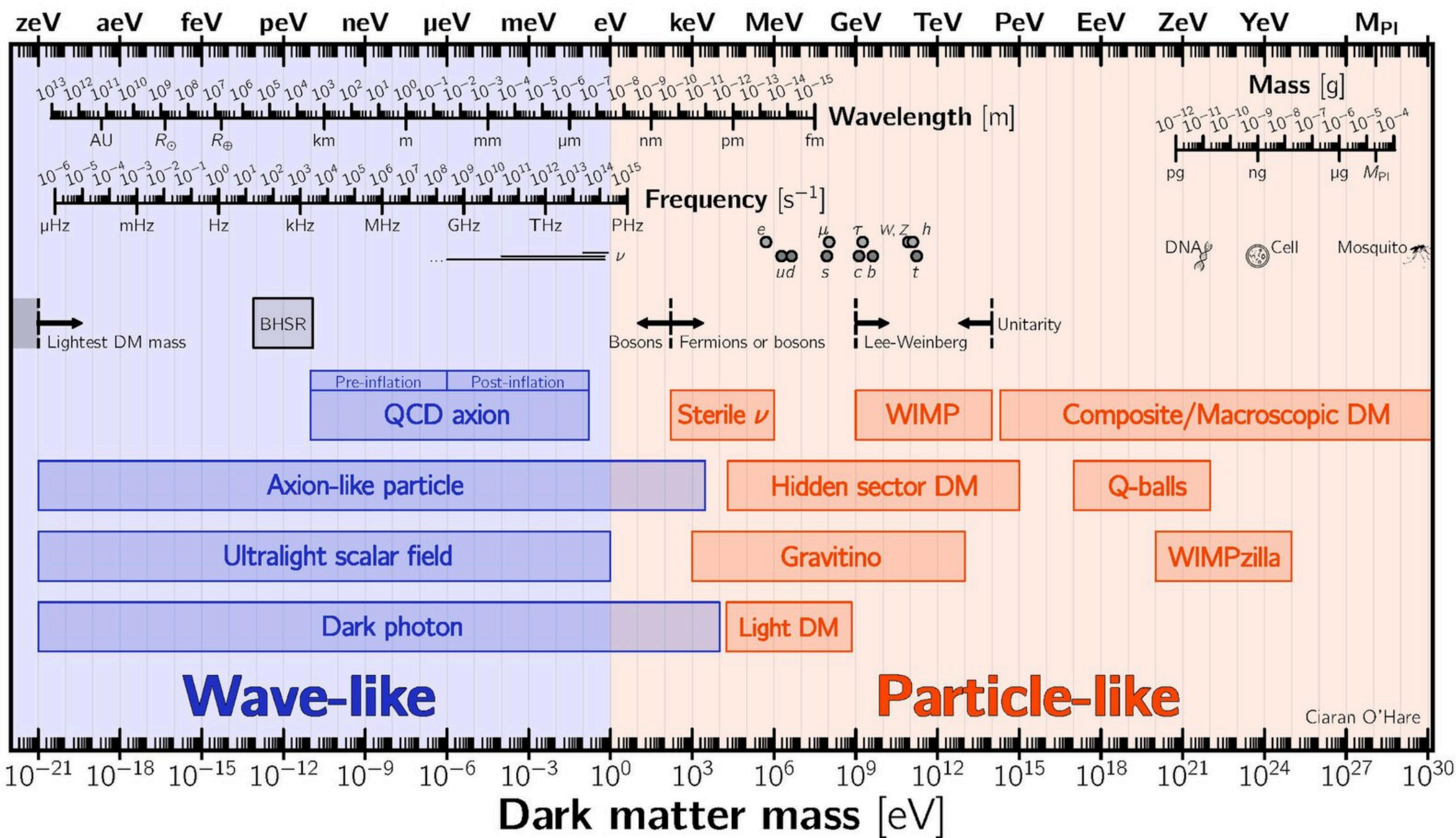
Axions - Two birds with one stone

- Solves Strong CP problem, see [Peccei & Quinn, 1977]), but also a **dark matter candidate**.
- Light, inherently cold
- Weakly interacting and stable
- Can be **abundantly produced** in the **early Universe**
- Myriad experiments are searching for axions (ADMX, MADMAX, CASH, ABRACADABRA...)
- Models with axions postulate the existence of a global U(1) symmetry being broken [Peccei & Quinn, 1977]
 - Pre-inflation
 - **Post-inflation** (topological defects!)

Neutron electric dipole moment:

$$d_n \leq 1.8 \cdot 10^{-26} e \text{ cm}$$

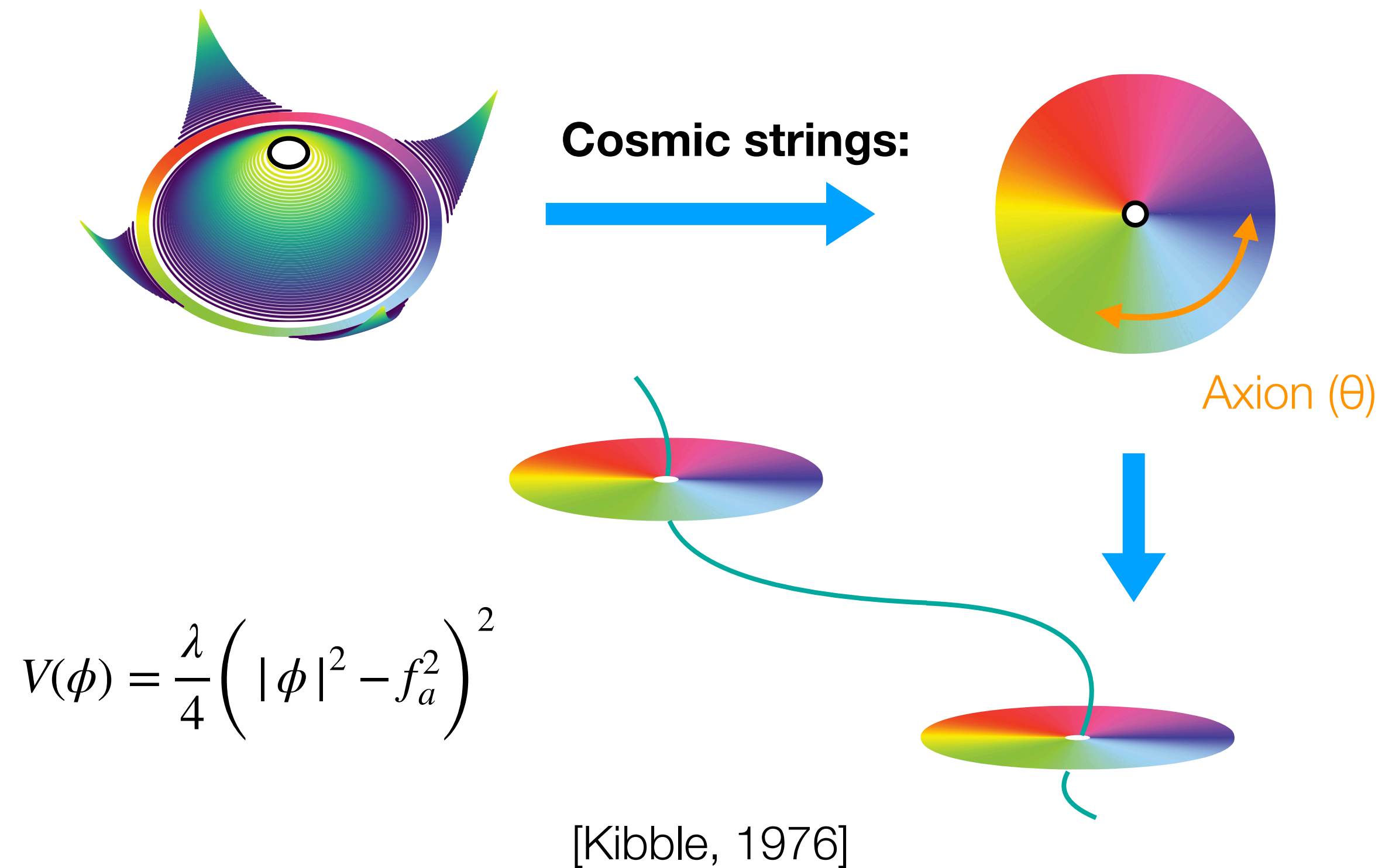
[Abel et al, 2020]



[C. A. J. O' Hare et al, 2024]

Axions in the post-inflationary scenario I

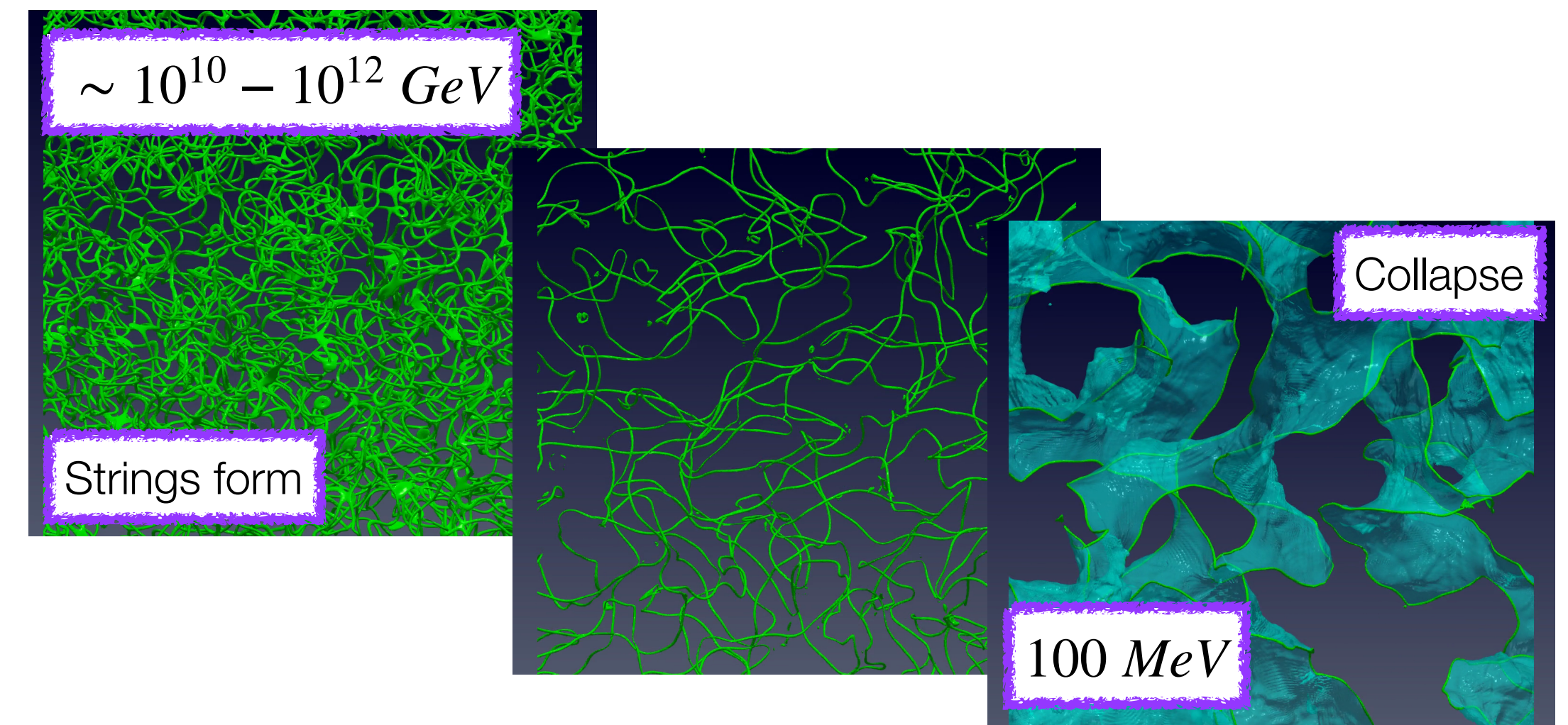
Take a **complex scalar field** $\phi = \psi e^{i\theta}$ invariant under **U(1)** **Global** symmetry, which breaks at phase transition:



- Breaking global U(1) post-inflation → string network [Davis & Shellard, 1986]...

...and the **string network** will emit **axion waves** continuously until the QCD phase transition

- At QCD PT, a system of **domain walls** + **strings** forms → **collapses**. In this talk, strings only
 - Simulations only probe directly a small portion of the necessary dynamic range (**extrapolations**)



Axions in the post-inflationary scenario II

- Goal: obtain a **dark matter mass prediction**. Many factors which can greatly impact this, we highlight two:
 - The **number of strings per Hubble volume** at the QCD phase transition ζ :

$$\zeta \approx \text{const.} \quad \text{or} \quad \zeta \propto \log(m_s t)$$

- **Emission** of axions:

Scale-invariant

$\mathcal{P} \propto \ln(k/k^*)$

[Hariri & Sikivie, 1986]

or

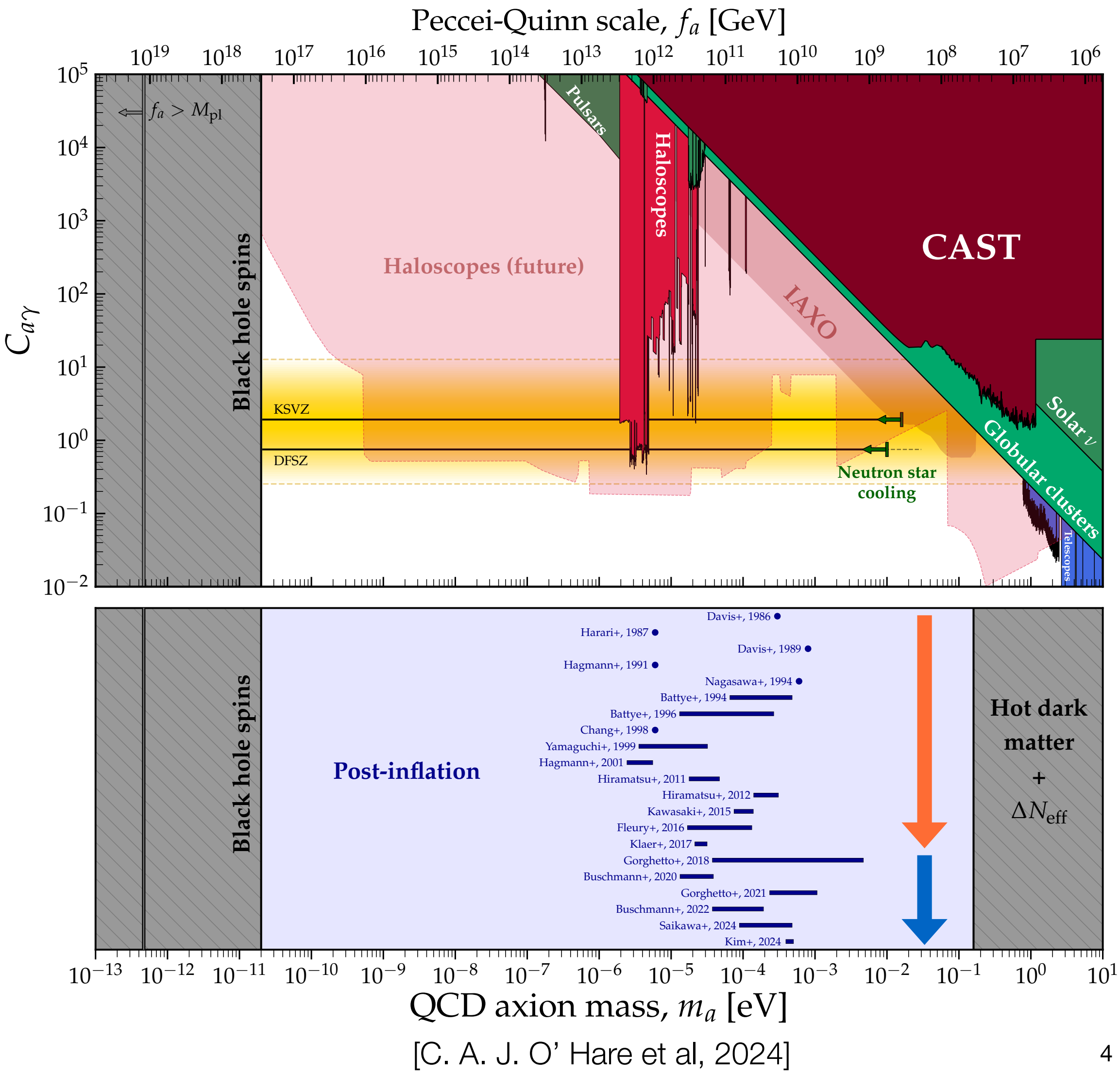
Peaked at horizon

$\mathcal{P} \propto (k/k_*)^{1-q}$

[Davis & Shellard, 1986]

Summary of mass predictions → rather active research area!

See Cosmic WISPer white paper (arXiv:2603.03433)



Simulations setup: initial conditions

- We report on **terasite** fixed grid simulations performed using HILA (<https://github.com/CFT-HY/HILA>), ran at **LUMI** (9th at TOP500),

$$\ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} - \nabla^2\phi = -a^2\lambda(\phi^*\phi - \eta^2)\phi$$

- “Initial conditions don’t matter** - string networks will be driven towards an attractor”. Yes, but **dynamic range is not infinite**. It is always possible one never quite reaches this attractor exactly.
- Gaussian random field with different initial correlation lengths l_ϕ at conformal time $\tau = 50$, diffusion until $\tau_{diff} = 70$
- “core growth” (also known as conformal strings) until τ_{cg} , after that radiation epoch evolution until half-light crossing time

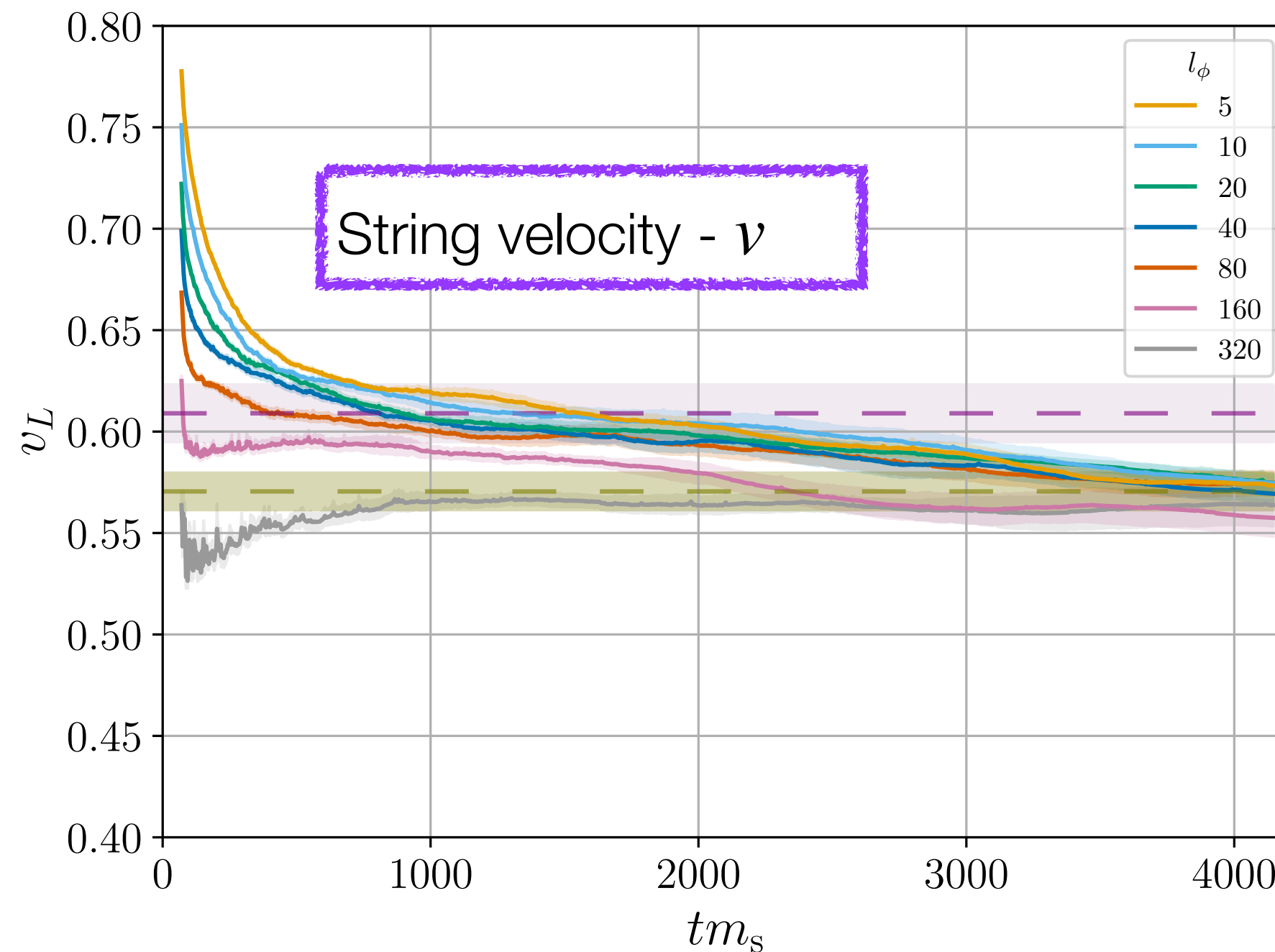
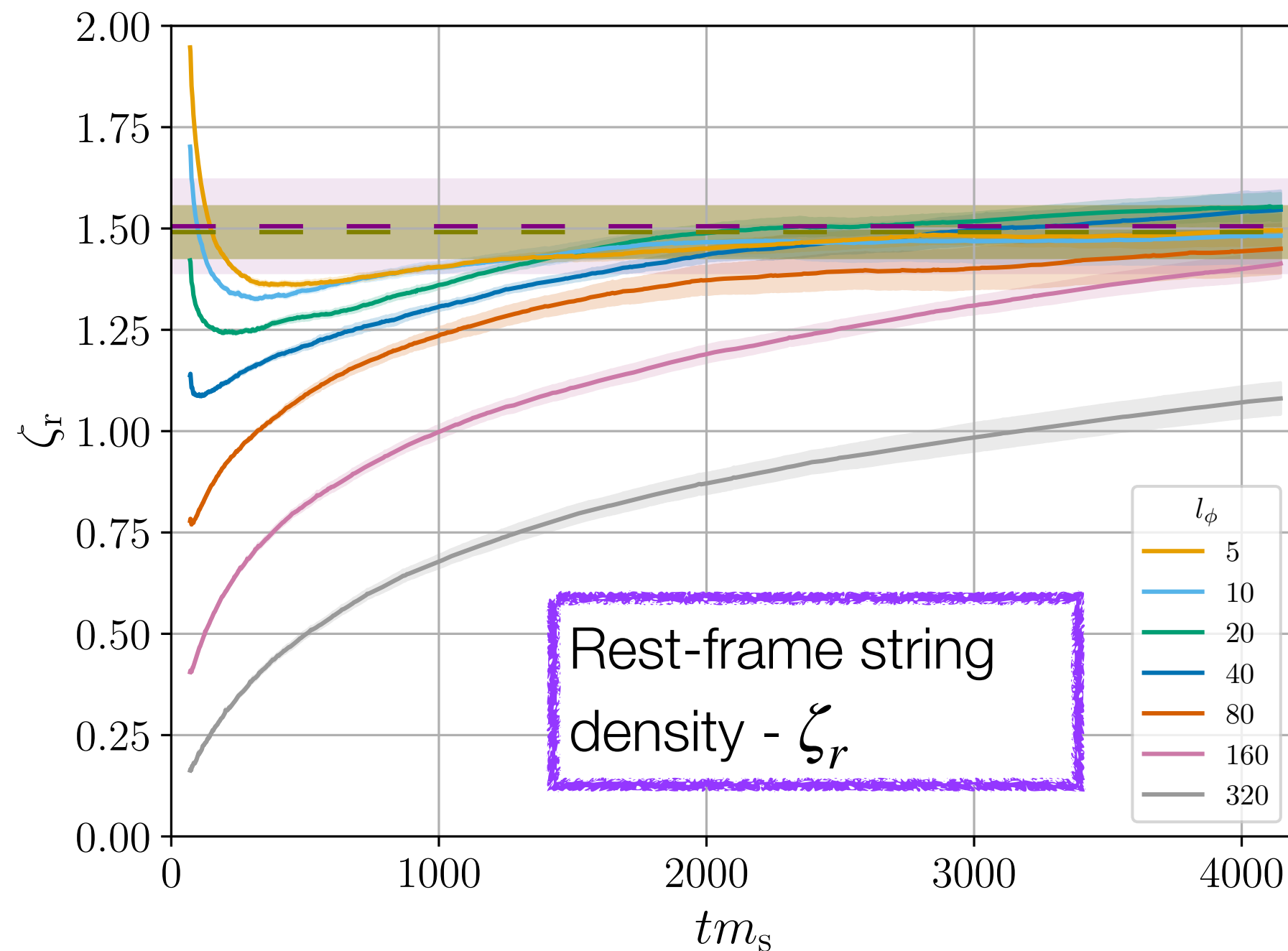


Box size	Seeds	Runs	l_ϕ
4k	9	45	5,10,20,40,80
8k	9	45	5,10,20,40,80
12k	6	30	5,10,20,40,80
16k	8	56	5,10,20,40,80,160,320

Scaling density of axion strings from terasite simulations

- Quick-review of results from arXiv:2410.18064 / PhysRevD 111 063532. **Convergence** of **rest-frame** string density parameter ζ_r to a constant, for higher initial density.
- Velocity convergence** for low density initial conditions. Evolution phase space leads to evidence for a fixed point.
 - We therefore find it justified to extrapolate from standard scaling: $\hat{\zeta}_{r,*} = 1.491(93)$ and $\hat{v}_{L,*} = 0.5705(93)$

$$\hat{\zeta}_{w,*} = 1.220(57)$$



Universe-frame
string density - ζ_w

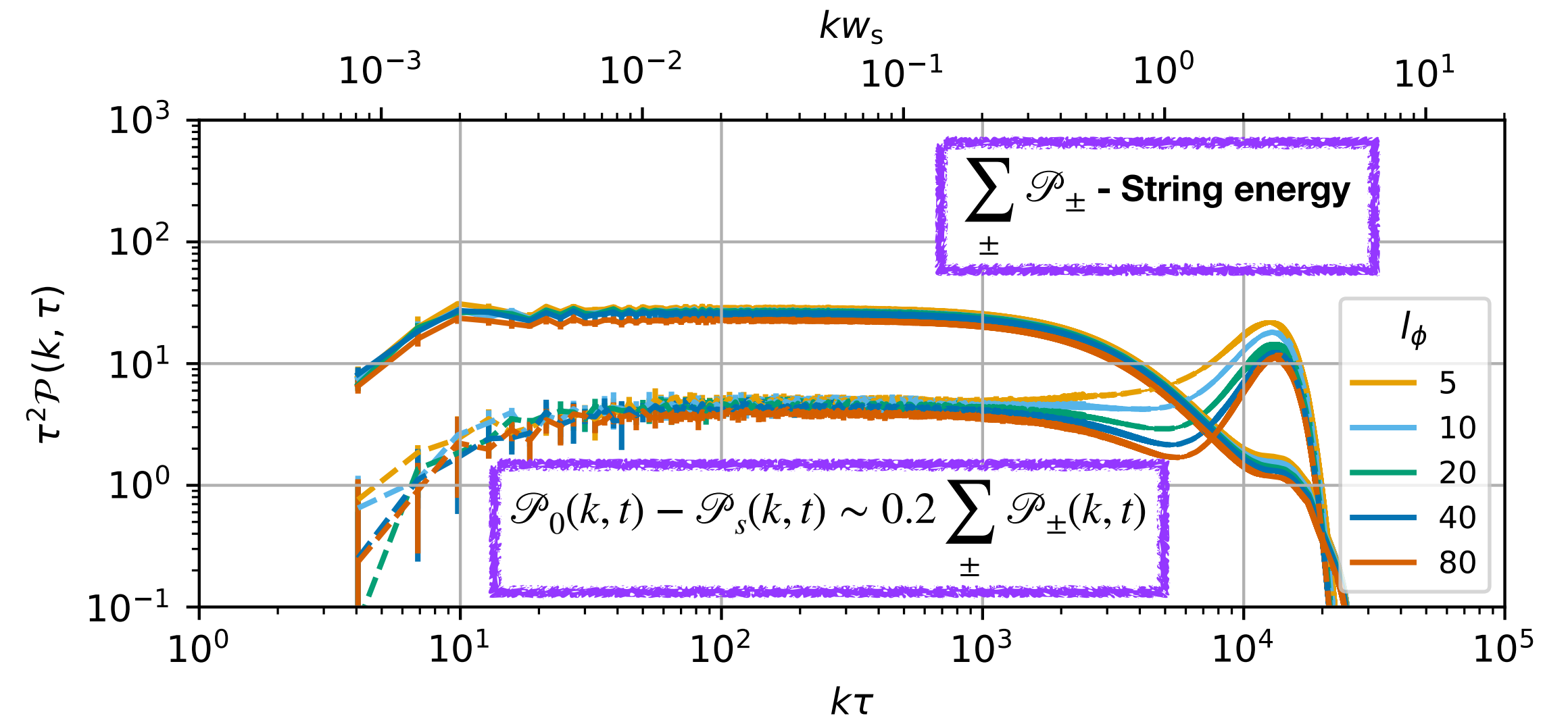
NOTE:

$\hat{\zeta}_w \gamma(\hat{v}_L)$ **coincides** with $\hat{\zeta}_r$.
Convergence after boost!

Observables - excising the self-field

- **Currents:** $T_{\mu\nu} = \partial_\mu \psi \partial_\nu \psi + J_\mu J_\nu - g_{\mu\nu} \mathcal{L}$
 - $J_0 = \text{Im}(\phi^* \Pi)$ - used by most groups
 - $J_i = \text{Im}(\phi^* \partial_i \phi)$ - should be less “polluted” ($\rightarrow$)
- Fourier quantities denoted by $\sim$. Let’s define some projected observables,
 - $\tilde{J}_s = \tilde{J}_i e_s^i$ where $e_s^i = k^i$
 - $\tilde{J}_\pm = \tilde{J}_i e_\pm^i$ where $i \epsilon_{ijk} e_s^i e_\pm^k = \pm e_\pm^j$
 - **Axions** - propagating axions: $|\tilde{J}_0| = |\tilde{J}_s|$. Difference is an estimate of self-field contribution
 - **Strings** only - $\tilde{J}_\pm$ used for self-field estimate ($\rightarrow$)

- [Drew et al, 2022; Battye et al, 2026] also highlight the importance of removing non-propagating modes (self-field)
- Radial projections are used (similar in spirit to ours)
- Affects both q and the axion number density (next)



Unconnected Segment Model

- Used for CMB power spectra is a modeling framework:

- **Unconnected Segment Model** - power spectra from n_s segments with average velocity v , length L (here $L = \xi_w$ - mean string separation). See [Charnok et al, 2016]

- **Predictions:**

$$\cdot \quad \mathcal{P}_{\pm}(k, \tau) \approx \frac{1}{2} \left(1 + \frac{1}{2} \langle \gamma^2 v^2 \rangle_v \right) \mathcal{P}_{str}(k, \tau)$$

$$\cdot \quad \mathcal{P}_0(k, \tau) = \frac{1}{2} \langle \gamma^2 v^2 \rangle_v \mathcal{P}_{str}(k, \tau)$$

$$\cdot \quad \mathcal{P}_s(k, \tau) = \frac{1}{8} \langle (\gamma - 1)^2 \rangle_v \mathcal{P}_{str}(k, \tau)$$

$$\text{where } \mathcal{P}_{str} \approx 2\pi f_a^2 n_s L, \quad 1 \ll kL \ll L/w_{str}$$

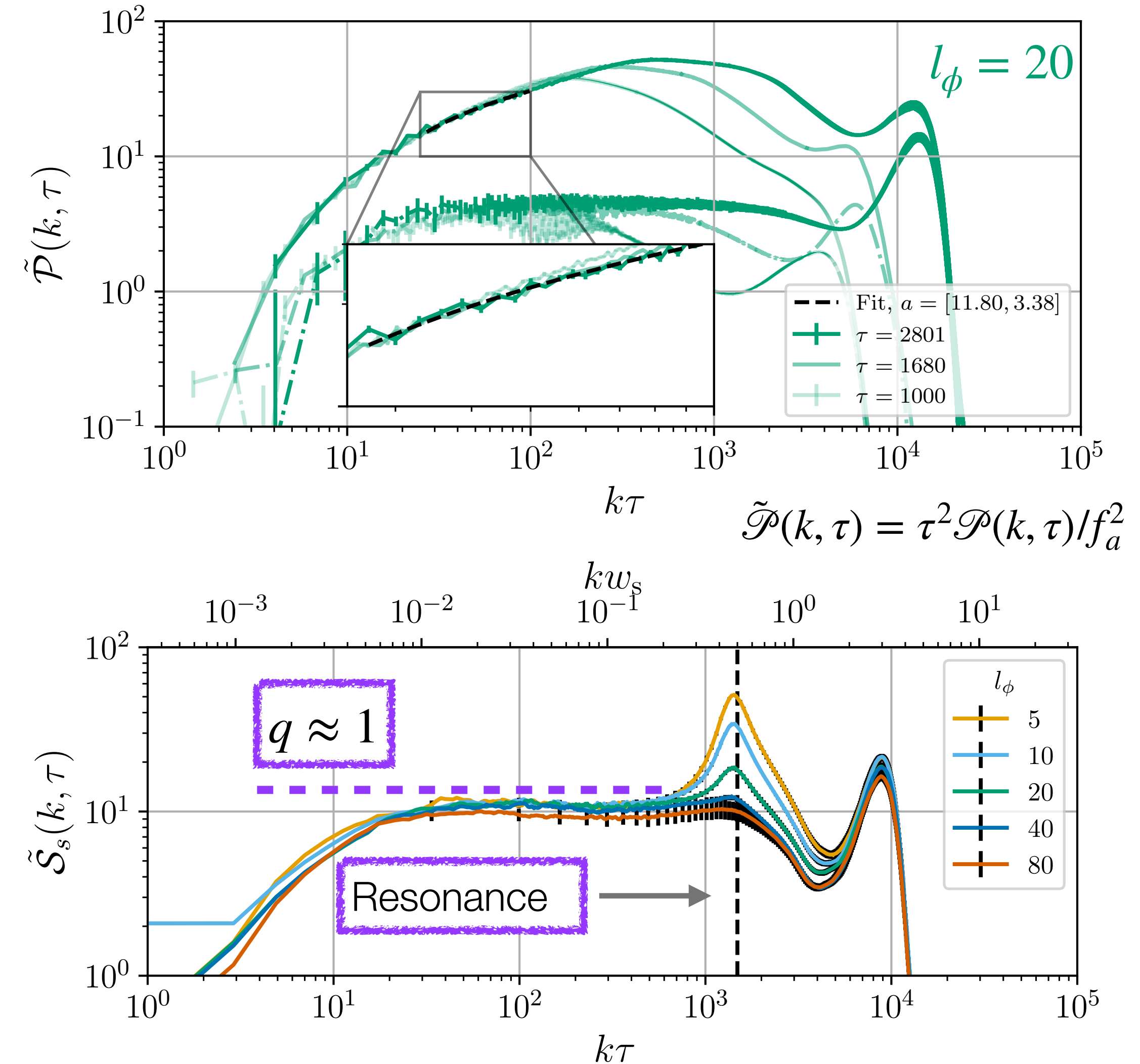
- Self-field contribution to $J_0 \sim \mathcal{O}(v^2)$ and $J_s \sim \mathcal{O}(v^4)$
- $q \leq 1$ (if one measures $q \gtrsim 1 \rightarrow \mathcal{P} \propto \ln(k)$)
- Ratio between magnitudes of $J_{\pm}$ and $J_{0,s}$
- Time decorrelation of $J_{\pm}, J_{0,s}$
- **Some caveats:**
 - Should be extended with subsegments (kinks [Silva, 2023]) and loops (see [Rybak, 2021])
 - See also the model of [Battye et al, 2026]
 - Correlations between segments not present

The spectrum of axions in a scaling axion string network

- **Power Spectra:** $\mathcal{P} = \frac{k^3}{2\pi} \langle |\tilde{J}(k, \tau)|^2 \rangle$ also,
 - Fitting $\mathcal{P}_s = p_{ax} \ln(k\tau) + c_s$ in the range $25 < k\tau < 100$
 - Roughly $p_{ax} \approx 10$. Ratios between spectra estimated ok-ish by USM (within 40% and 15%)

- **Rate of Change of Power Spectra:** $\mathcal{S} = \tau \frac{d}{d\tau} \left(\tau^2 \mathcal{P}(k, \tau) \right)$

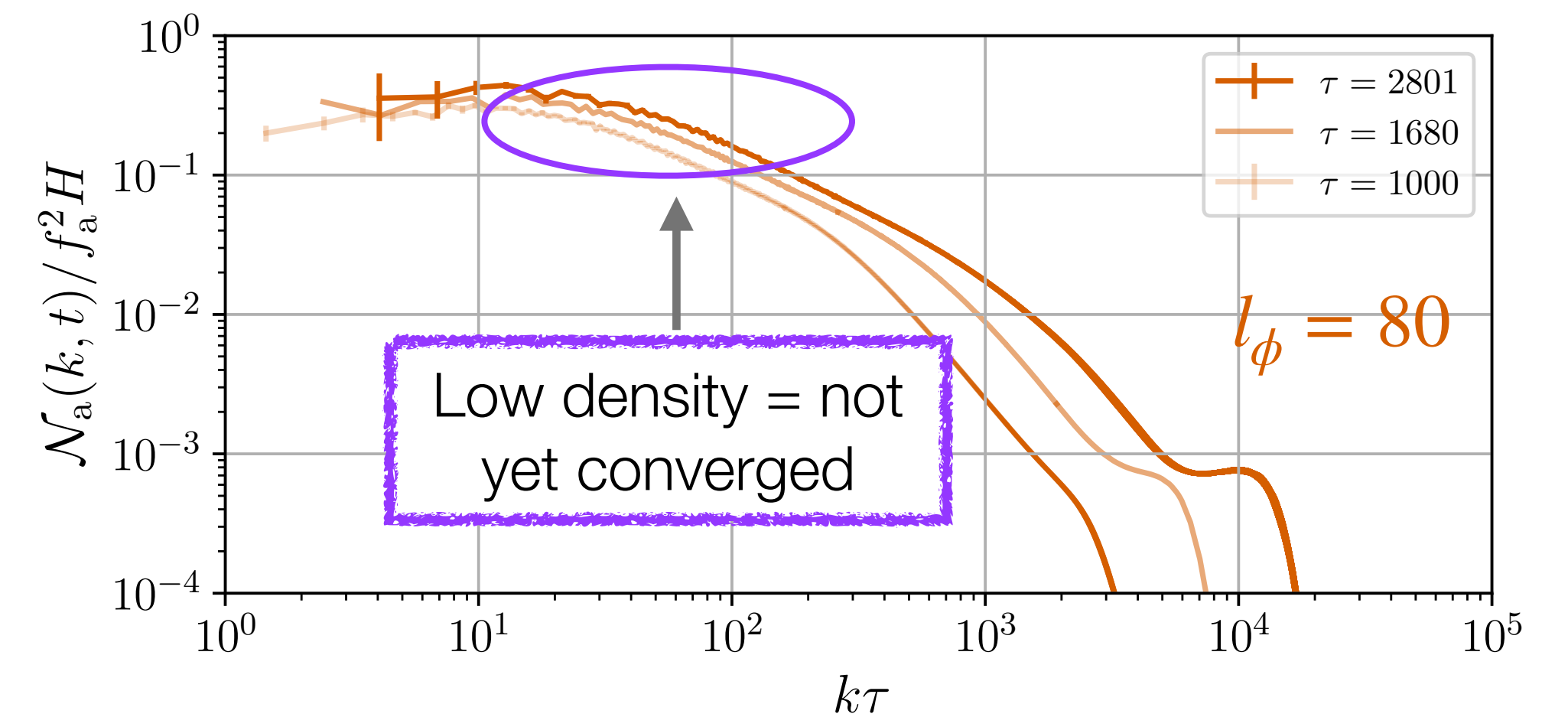
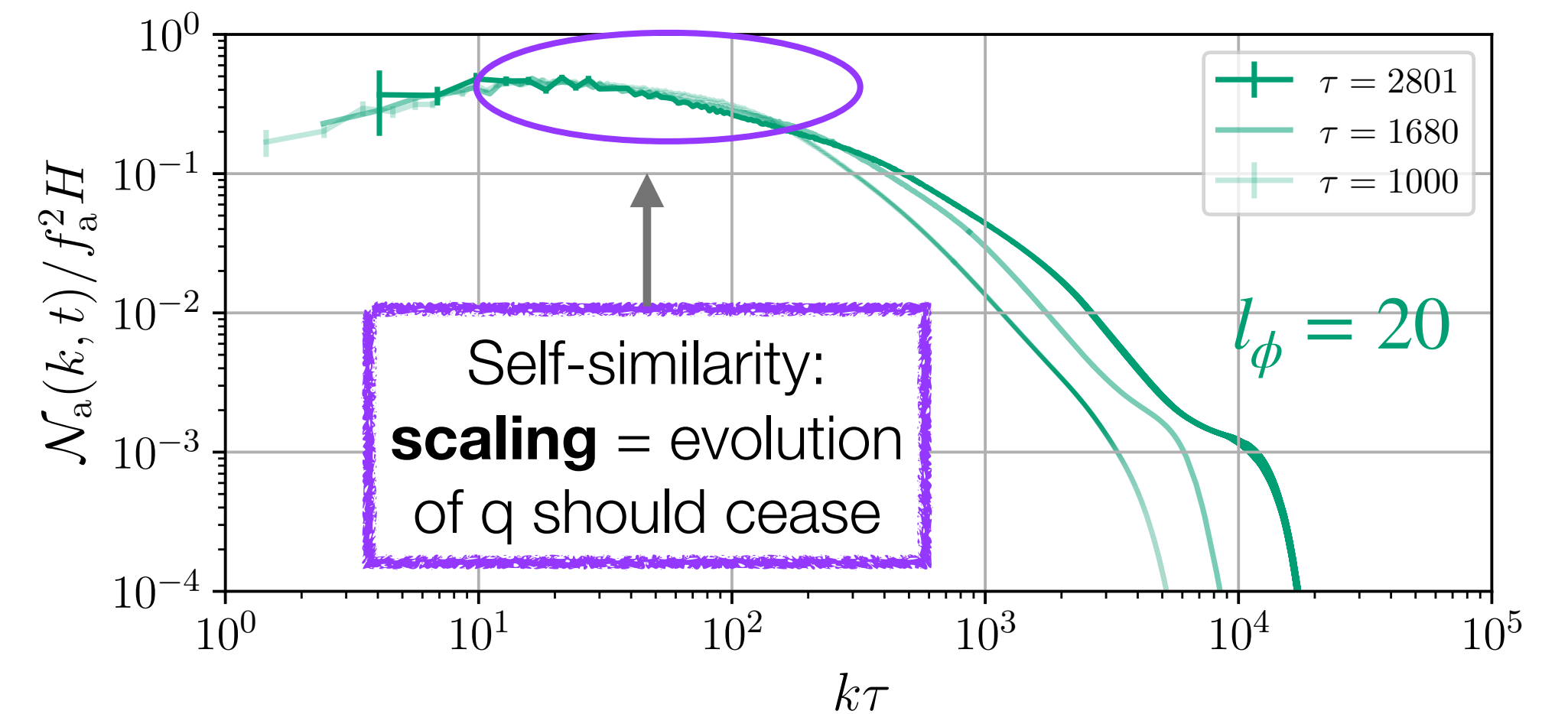
- Fits to power law $\mathcal{S} = (k\tau)^{1-q}$
 - $q \approx 1$ as [Saikawa et al, 2024][Benabou et al, 2024]
 - minute time growth may be related to increasing ζ_w (to cease; USM). Note: other systematics might be at play!
- In conjunction with USM: $q \leq 1 \rightarrow q \approx 1$



Axion number density

$$N_{ax}(k, \tau) = \frac{1}{ak} \mathcal{P}_s(k, \tau)$$

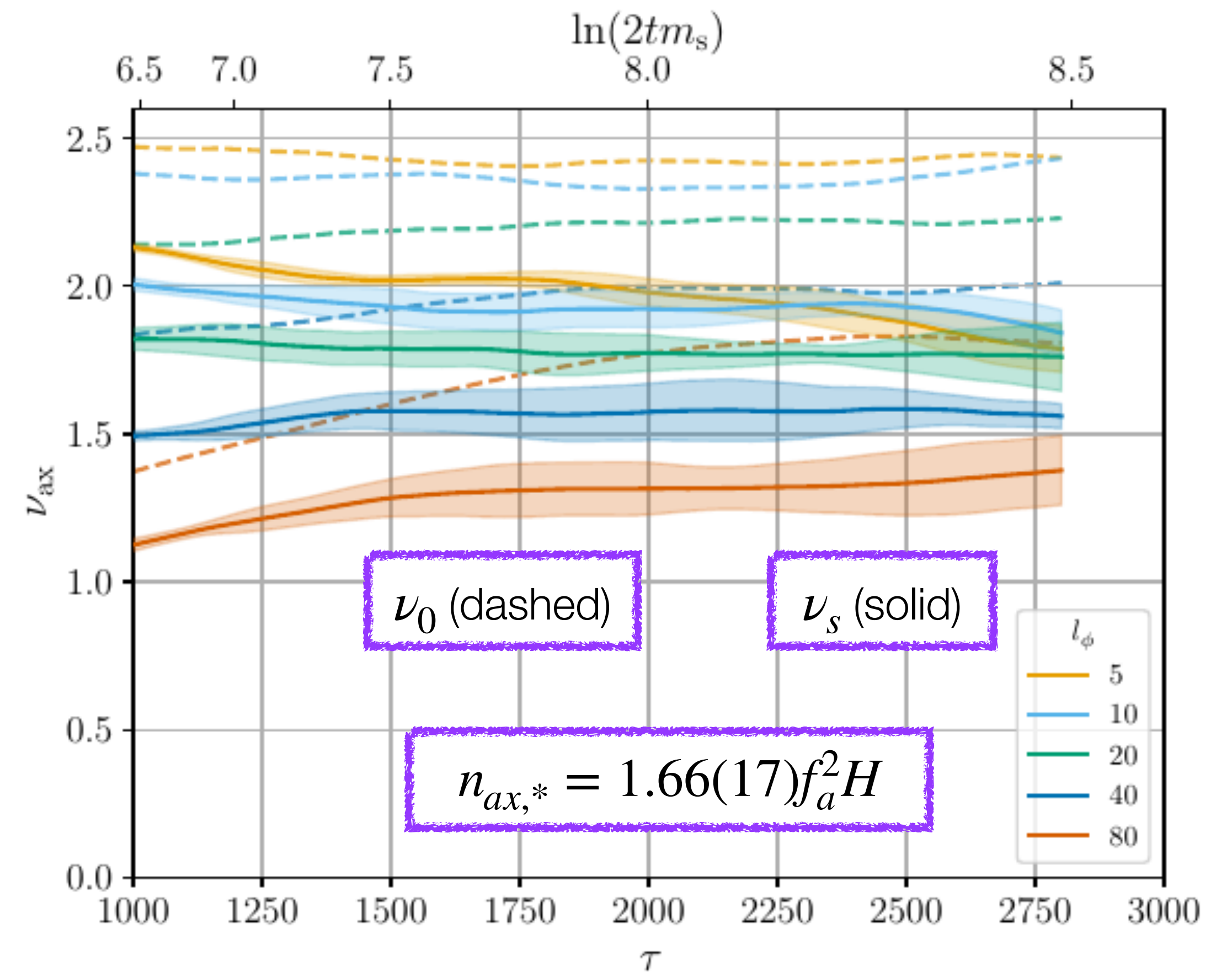
- Network groups tend to use the more polluted current $\tilde{J}_0$ to extrapolate their number density.
- Consistent with a **self-similar** power spectra emerging (**standard scaling**)
- **Self-field** contribution causes an **overestimate** of number density ($\sim 30\%$)
- Number density (based on $\tilde{J}_s$) shows evolution towards a **constant** value, from above and below (**scaling**)
- Given that the string-walls contribution is important, we leave the full mass prediction for the next publication



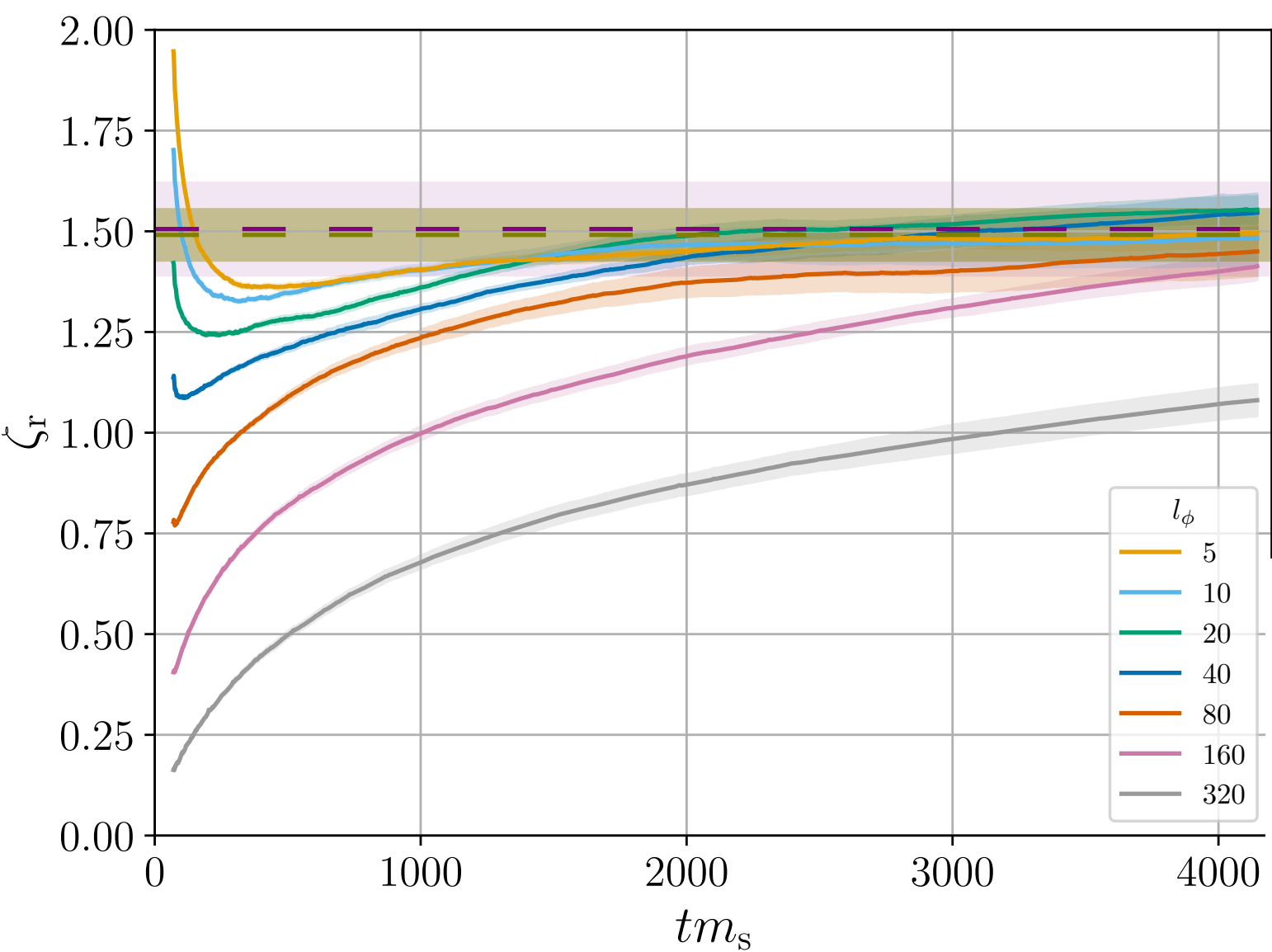
Axion number density

$$n_{ax} = \int \frac{dk}{k} N_{ax}(k, \tau) \rightarrow \nu_a = \frac{n_{ax}}{f_a^2 H}$$

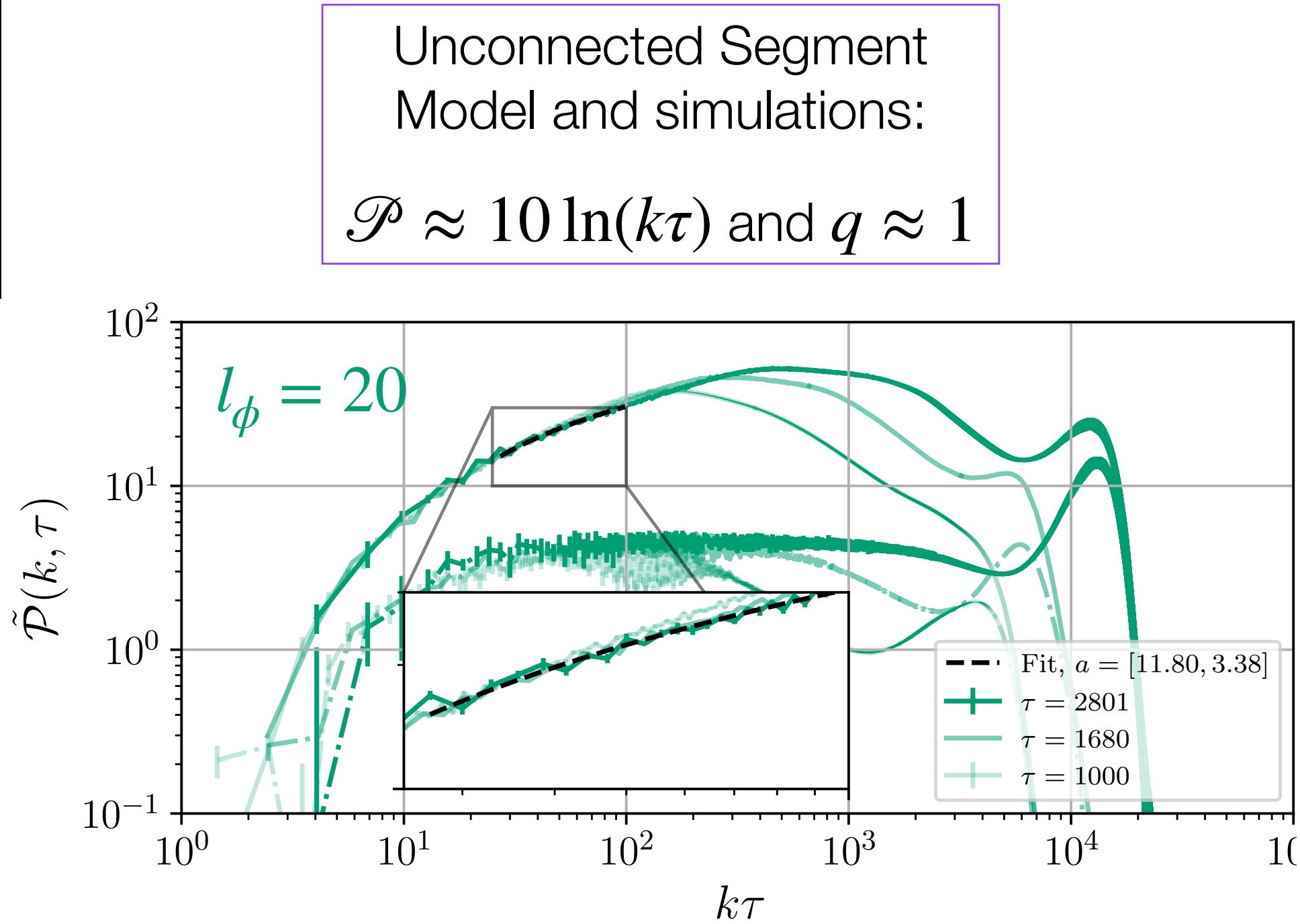
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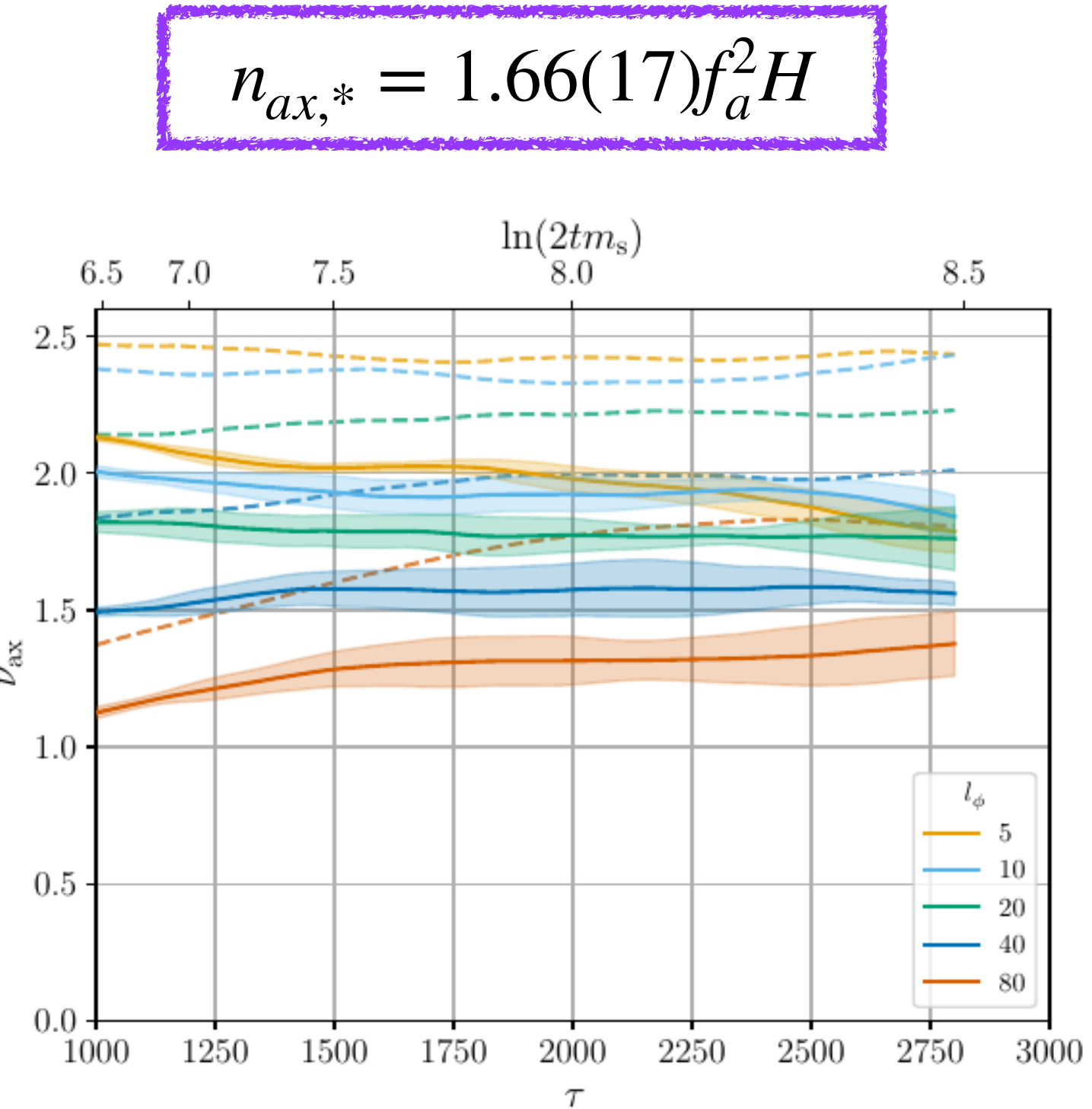
Conclusions (arXiv:2512.13653)



Previous work (arXiv:2410.18064)
Rest-frame string density -
 $\zeta_r \sim \text{const.} \sim \mathcal{O}(1)$
(standard scaling)



self-similar spectra
(standard scaling) =
evolution of q should cease



ν dimensionless number density
convergence to constant **(scaling)**

$$n_{ax,*} = 1.66(17)f_a^2 H$$

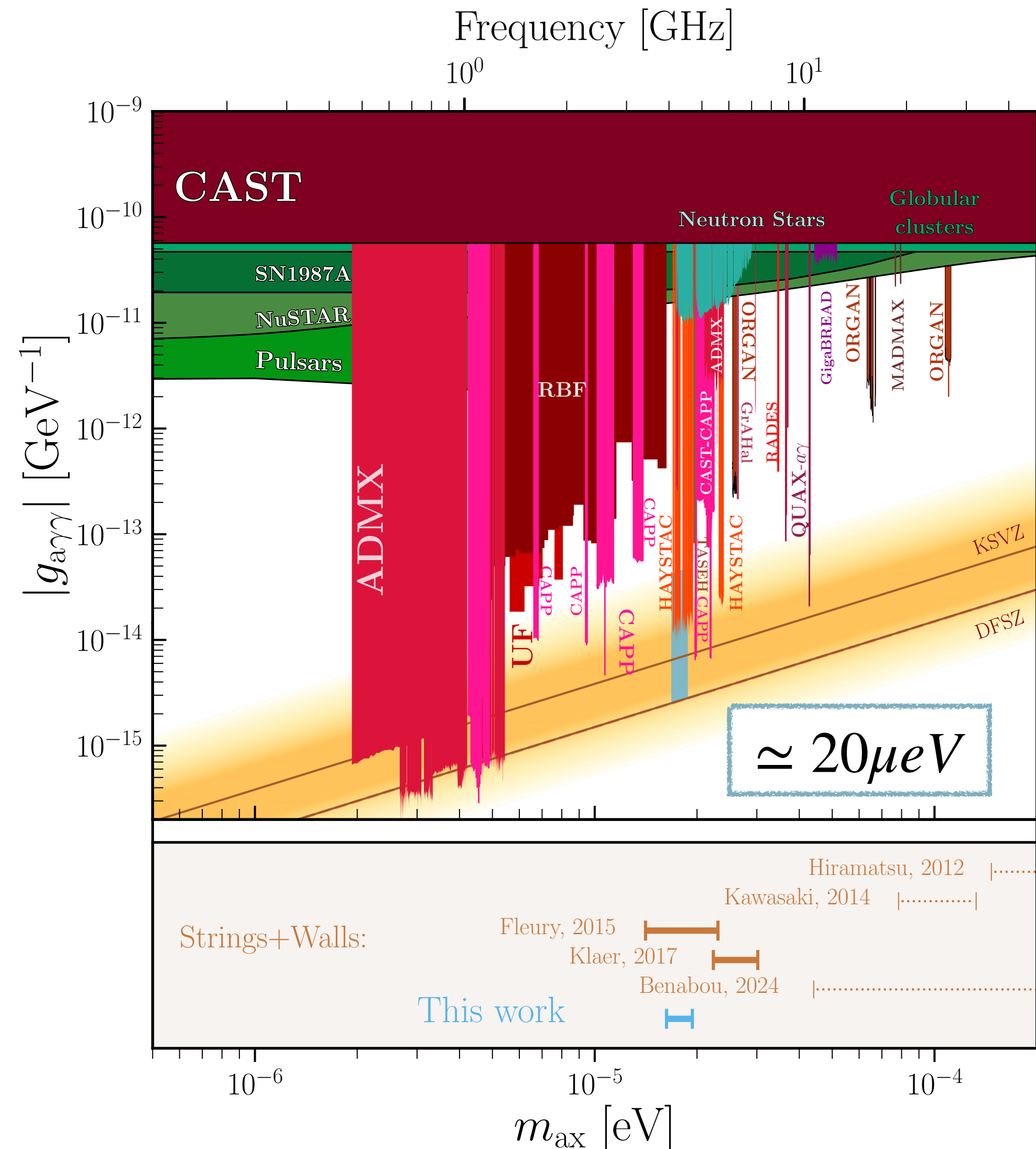
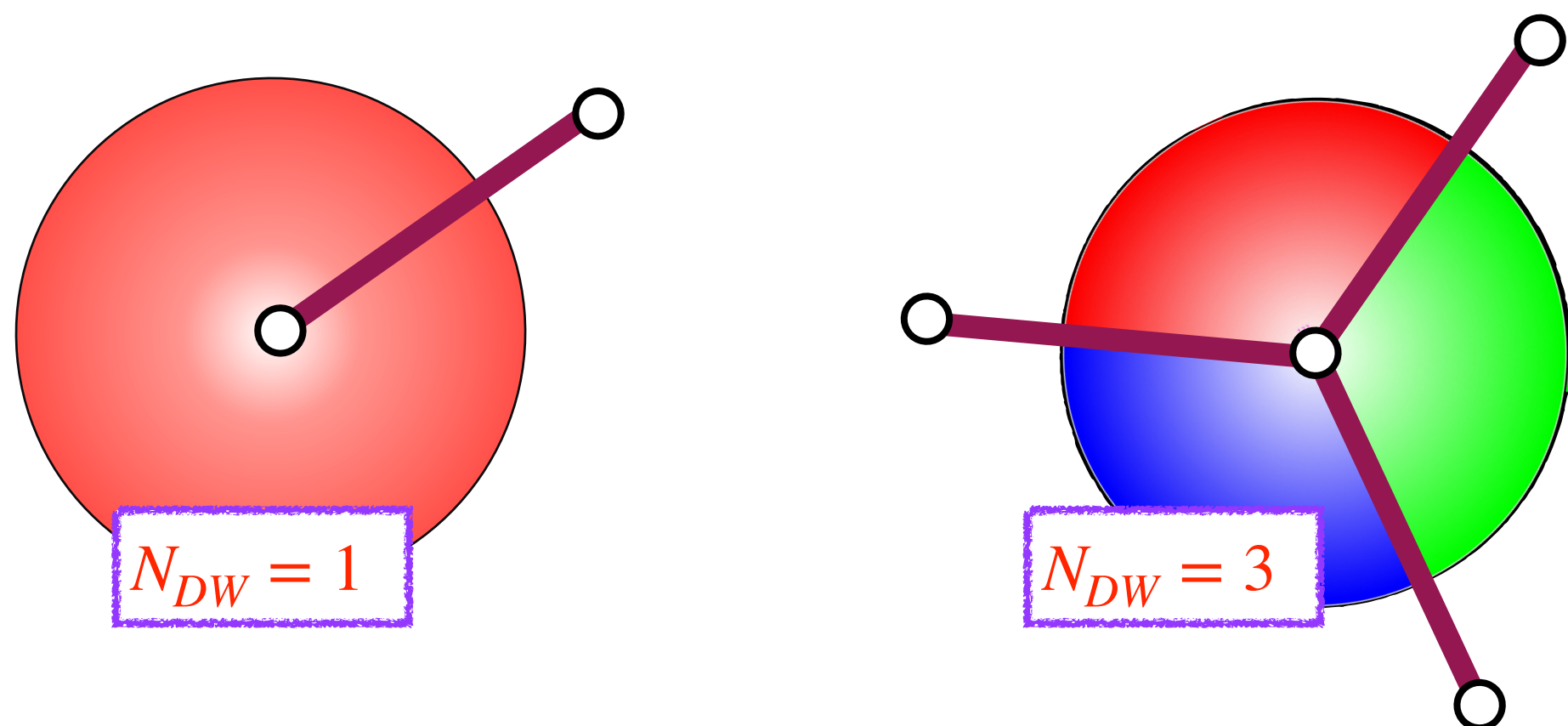
The axion mass in the post-inflationary scenario in a minimal scaling model*

- For the next publication we will address the full axion mass prediction based on field theory networks

- At $T \sim T_{QCD}$ axion becomes massive...

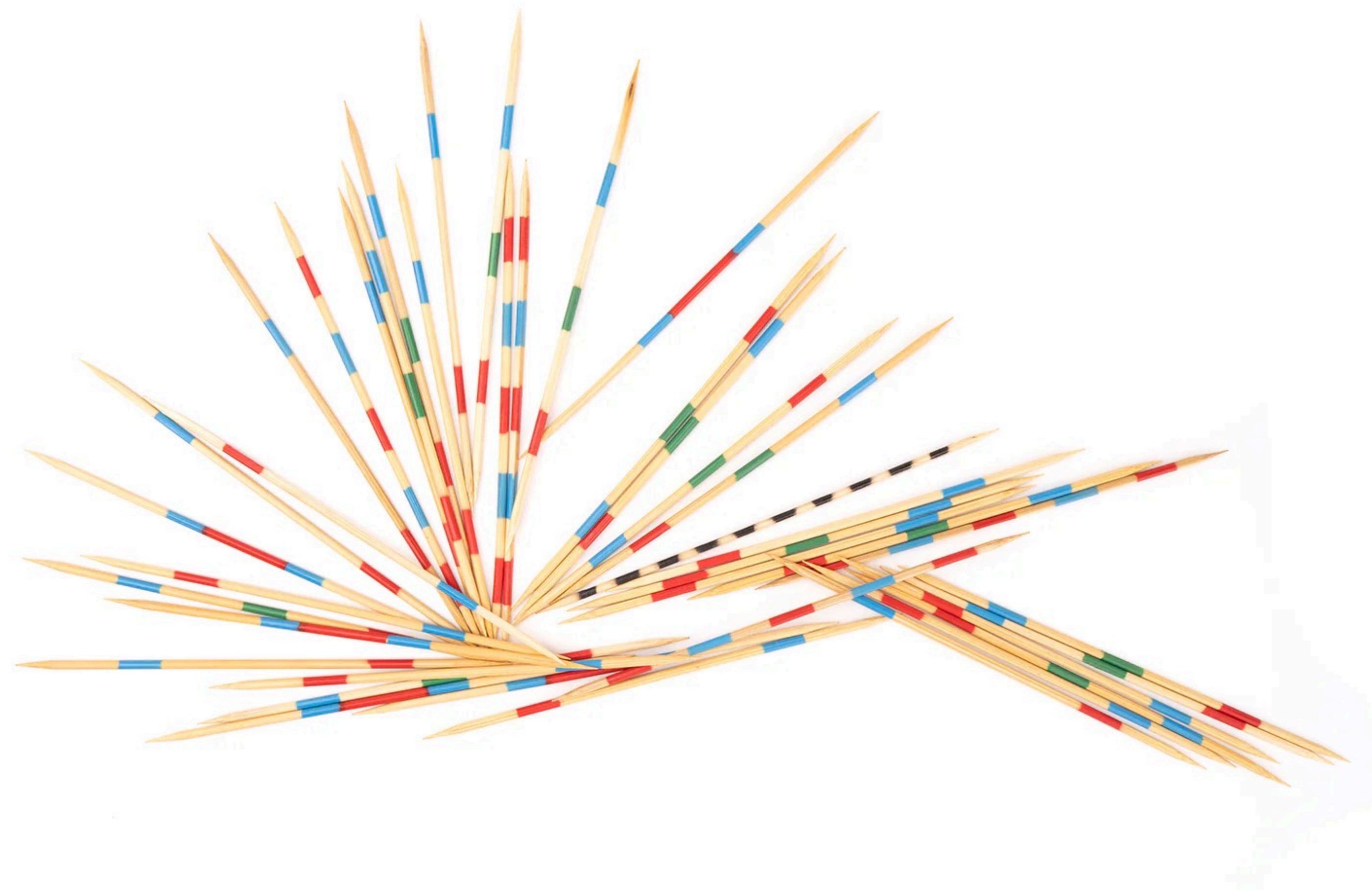
- String-wall collapse ($N_{DW} = 1$)

$$V(\phi) = \frac{\lambda}{4} \left(|\phi|^2 - f_a^2 \right)^2 + \frac{\chi(T)}{f_a^2} |\phi| \left[1 - \cos(\theta) \right]$$



Modified from [C. A. J. O' Hare et al, 2024]

*Correia, et al., 2026 PRELIMINARY



Backup Slides

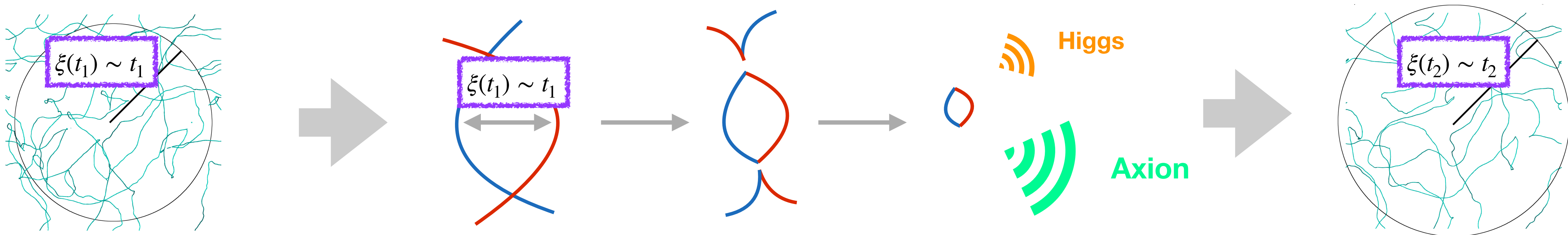
Standard scaling

- Defect networks are expected to evolve towards the **scaling regime / scale-invariant regime** in radiation epoch [Kibble, 1976], [Zeldovich, 1980], [Villenkin, 1981 & 1982], [Martins & Shellard, 1996]

- Mean String **separation**, $\xi \propto t$ and the string **velocity** $v \propto \text{const}$ and therefore $\zeta = \text{const}$
- Scaling given $t \gg t_c; \xi \gg w_s$ (**fixed point attractor** in $\xi/t, v$) and results in:

$$\xi \sim d_H \quad (w_s - \text{string width})$$

- Self-similar evolution sustained via energy loss mechanisms (loop chopping, emission of radiation).

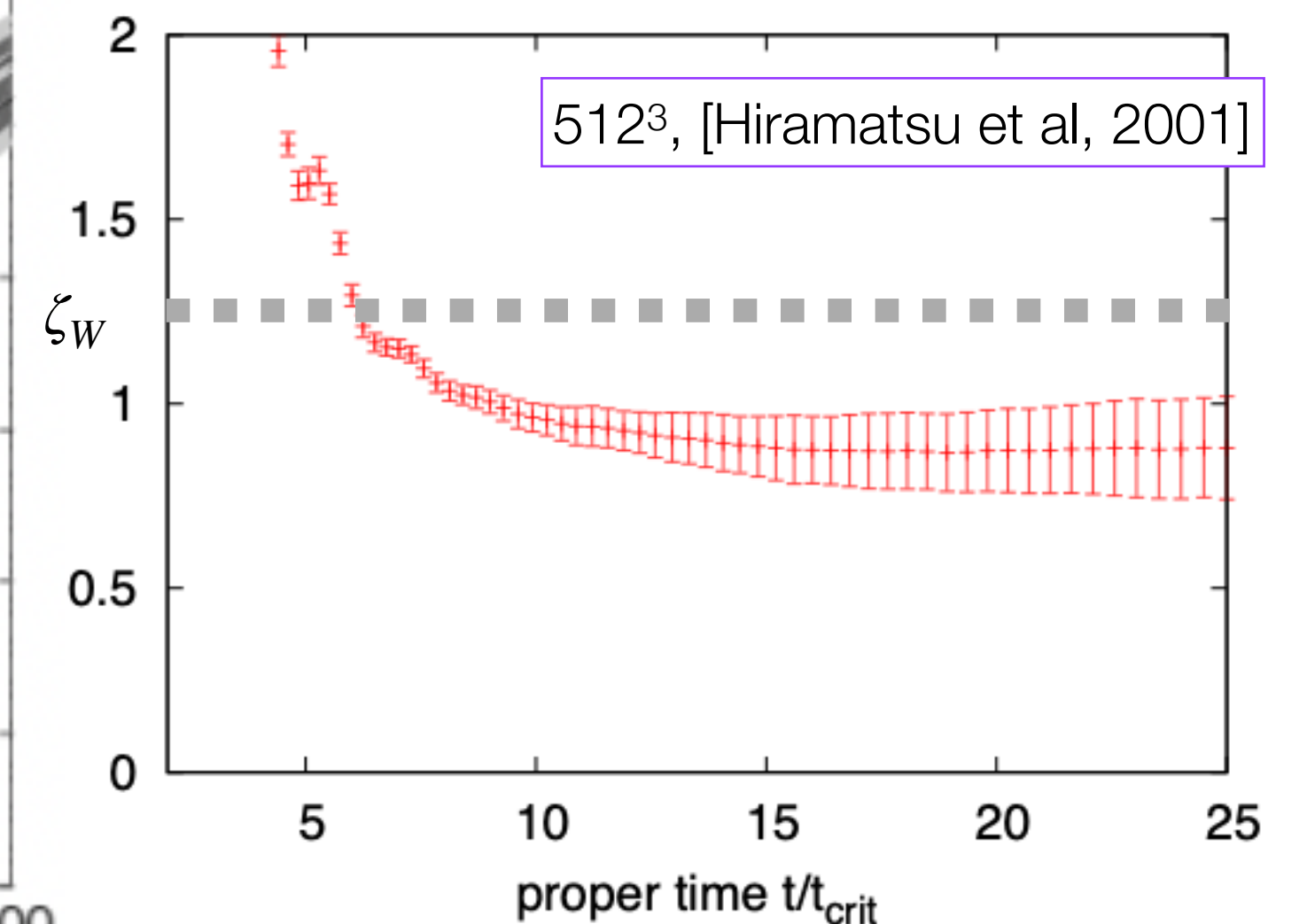
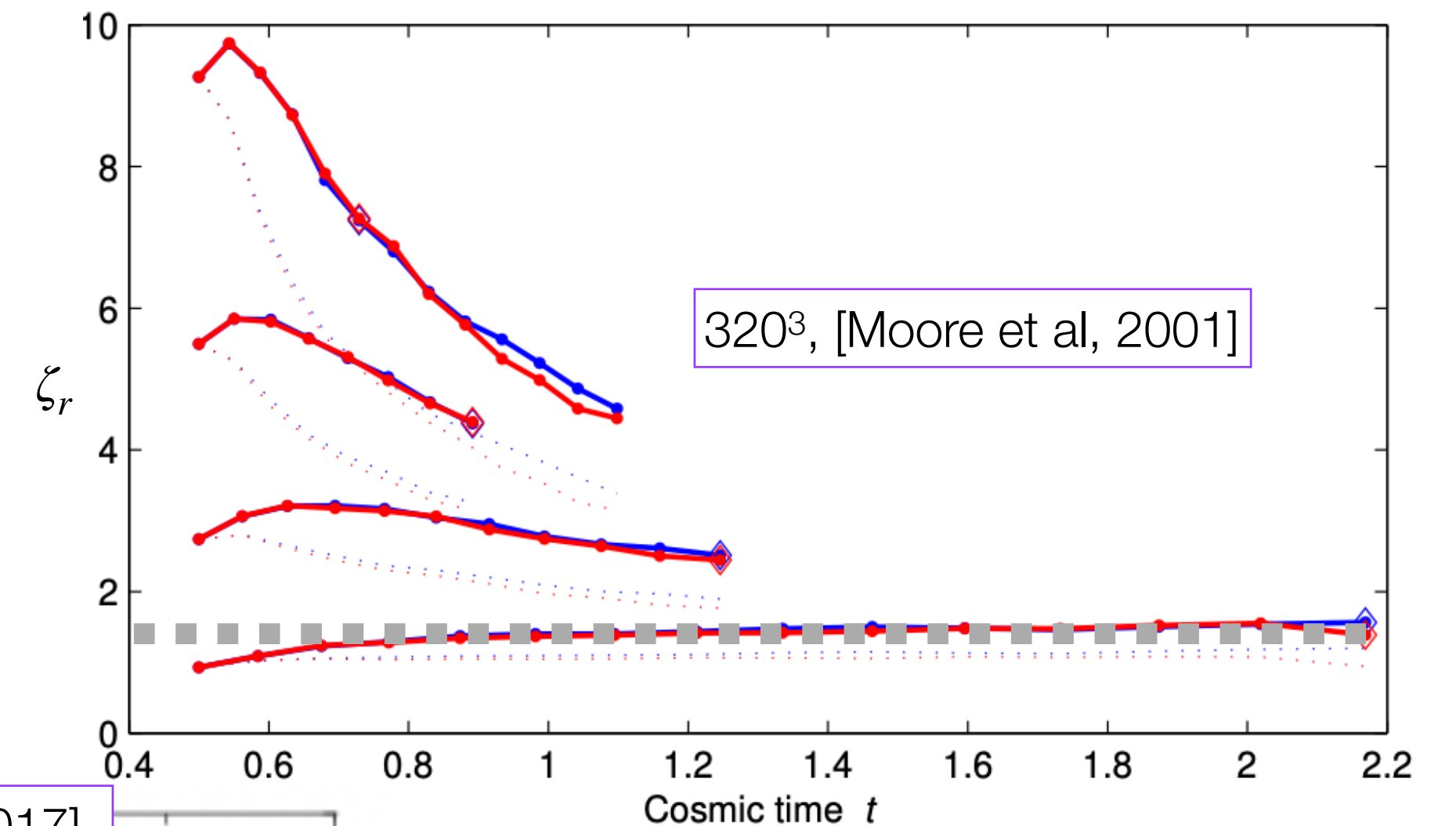
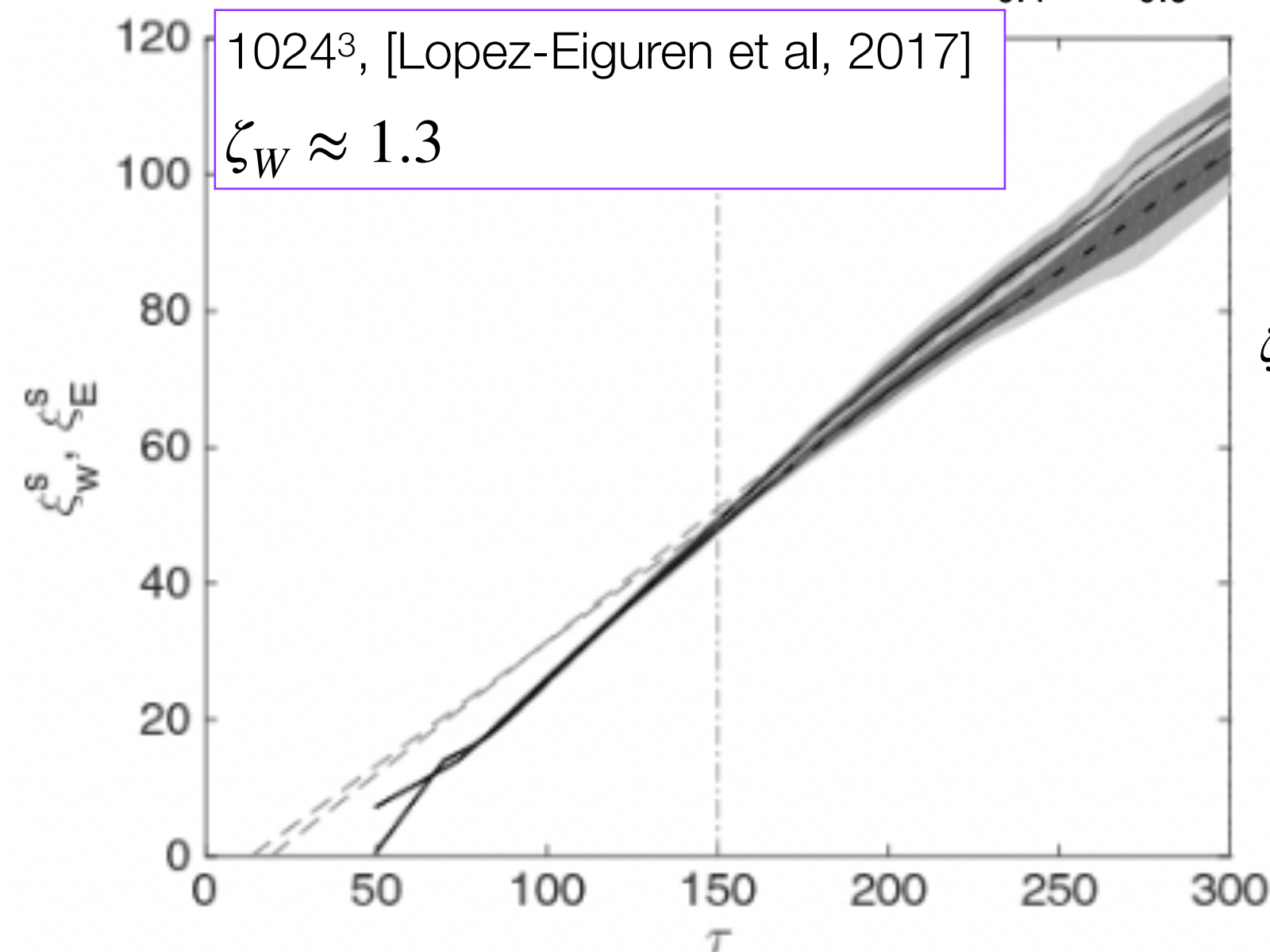
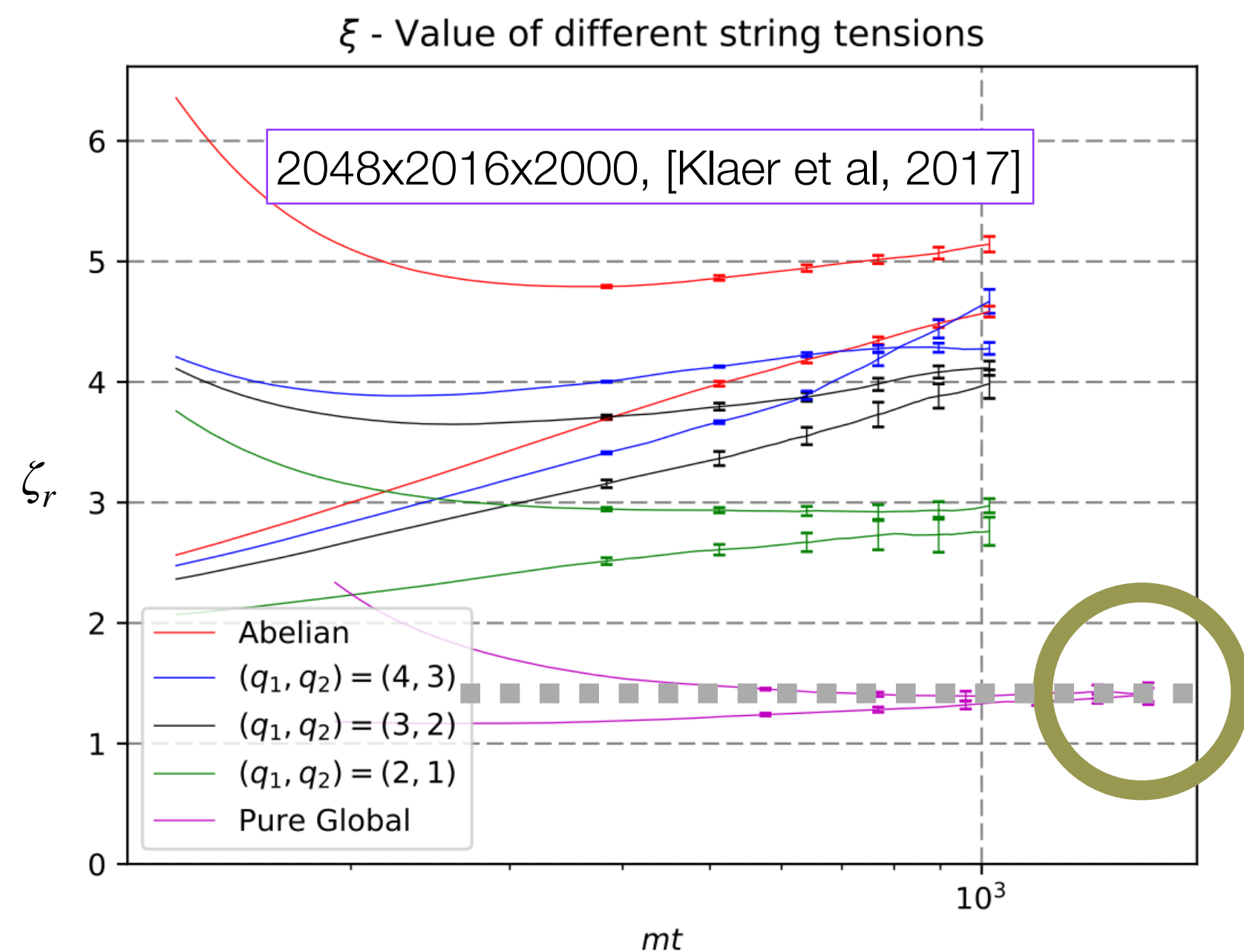


Axion strings - Standard Scaling (pre-2018)

- So far, all consistent with string density parameter / number of strings per Hubble patch $\zeta \approx 1$ (grey dashed line, foreshadows our latest result)
- Some definitions for **rest-frame** and **Universe-frame**:

$$\zeta_r = \gamma \zeta_W$$

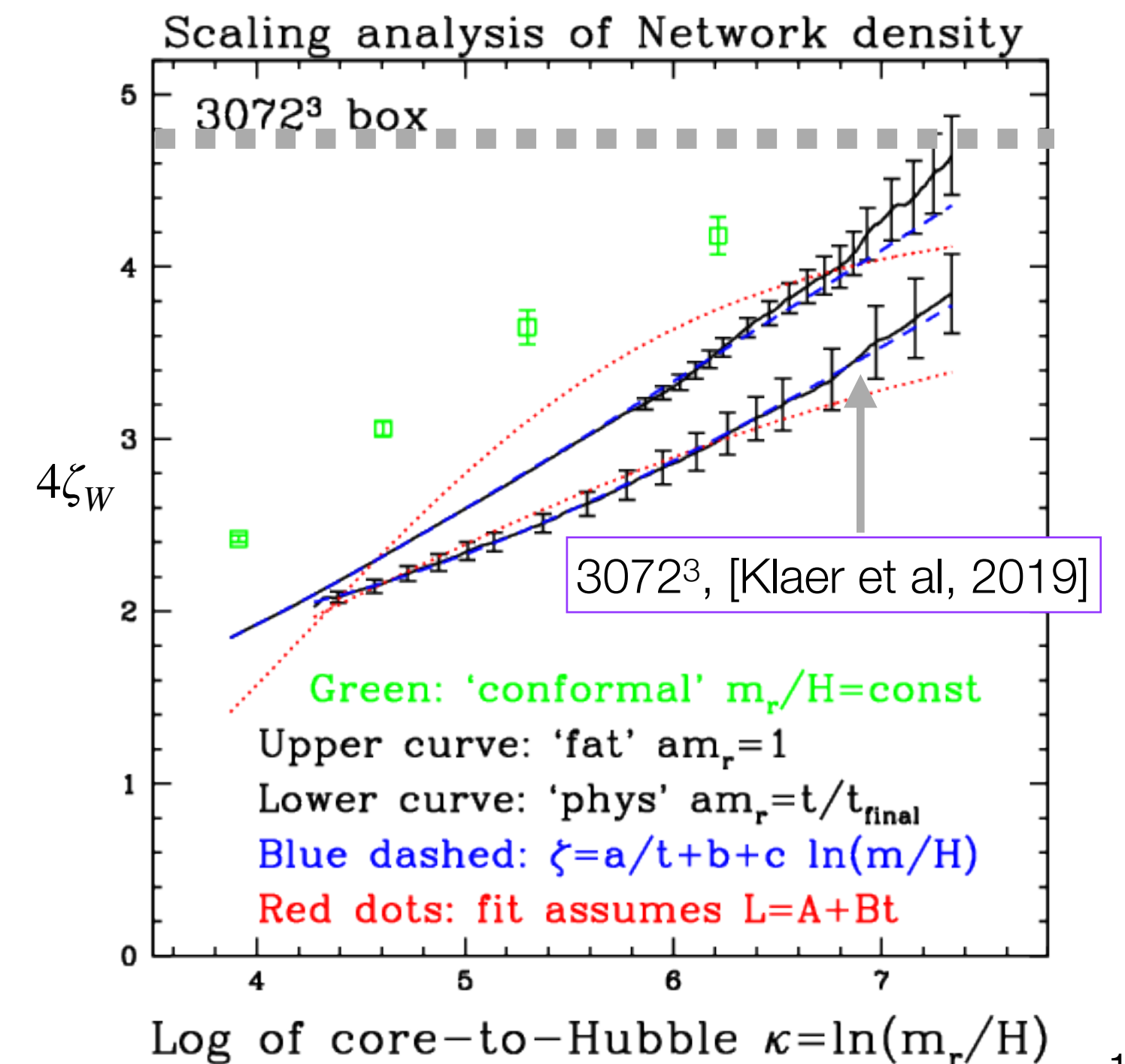
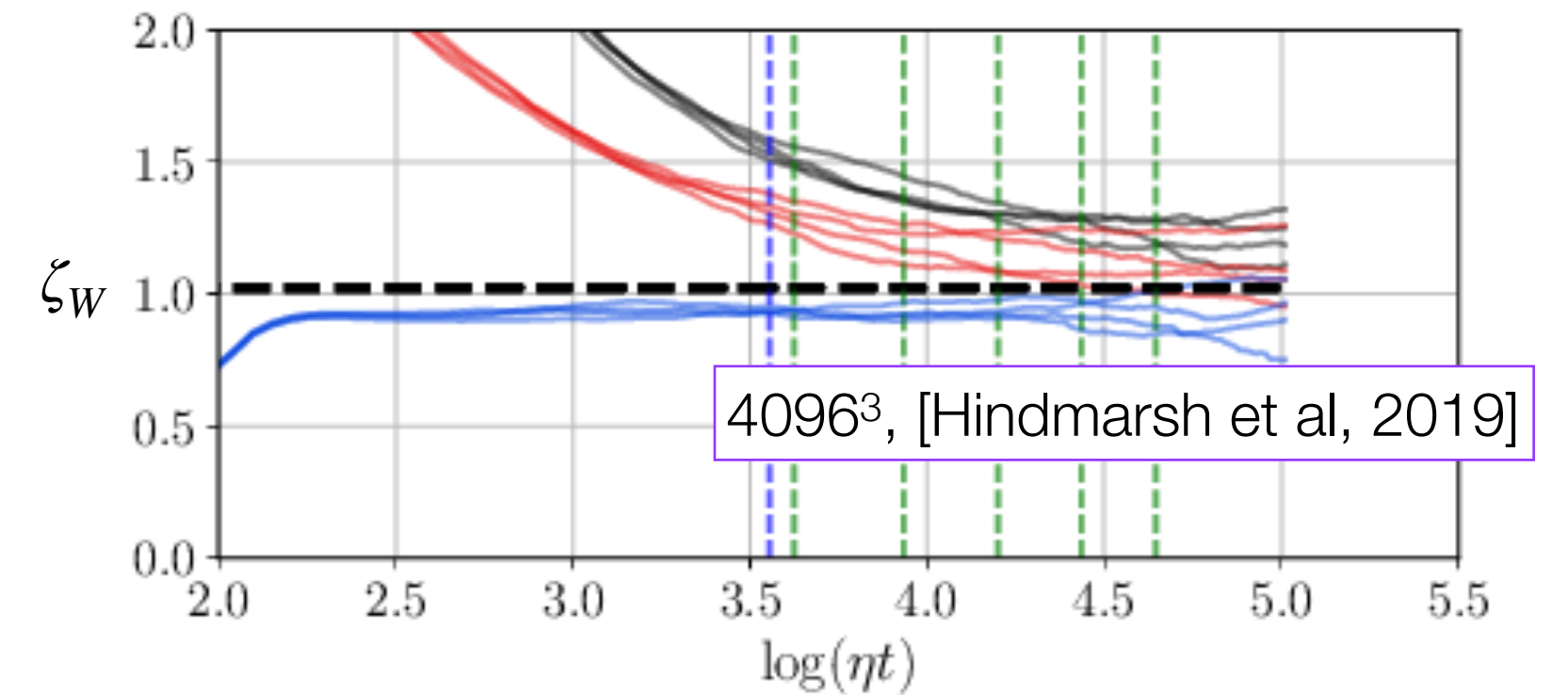
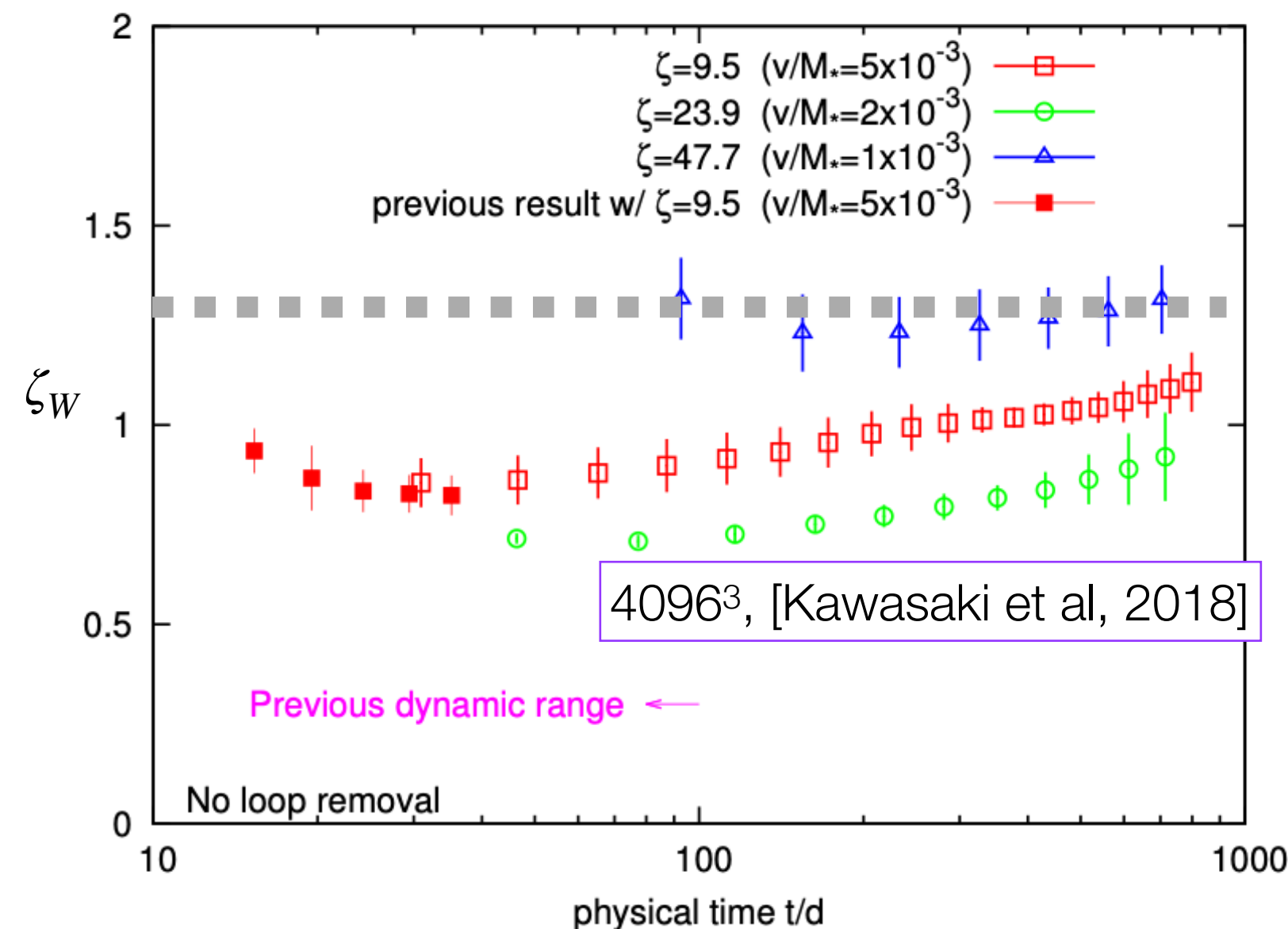
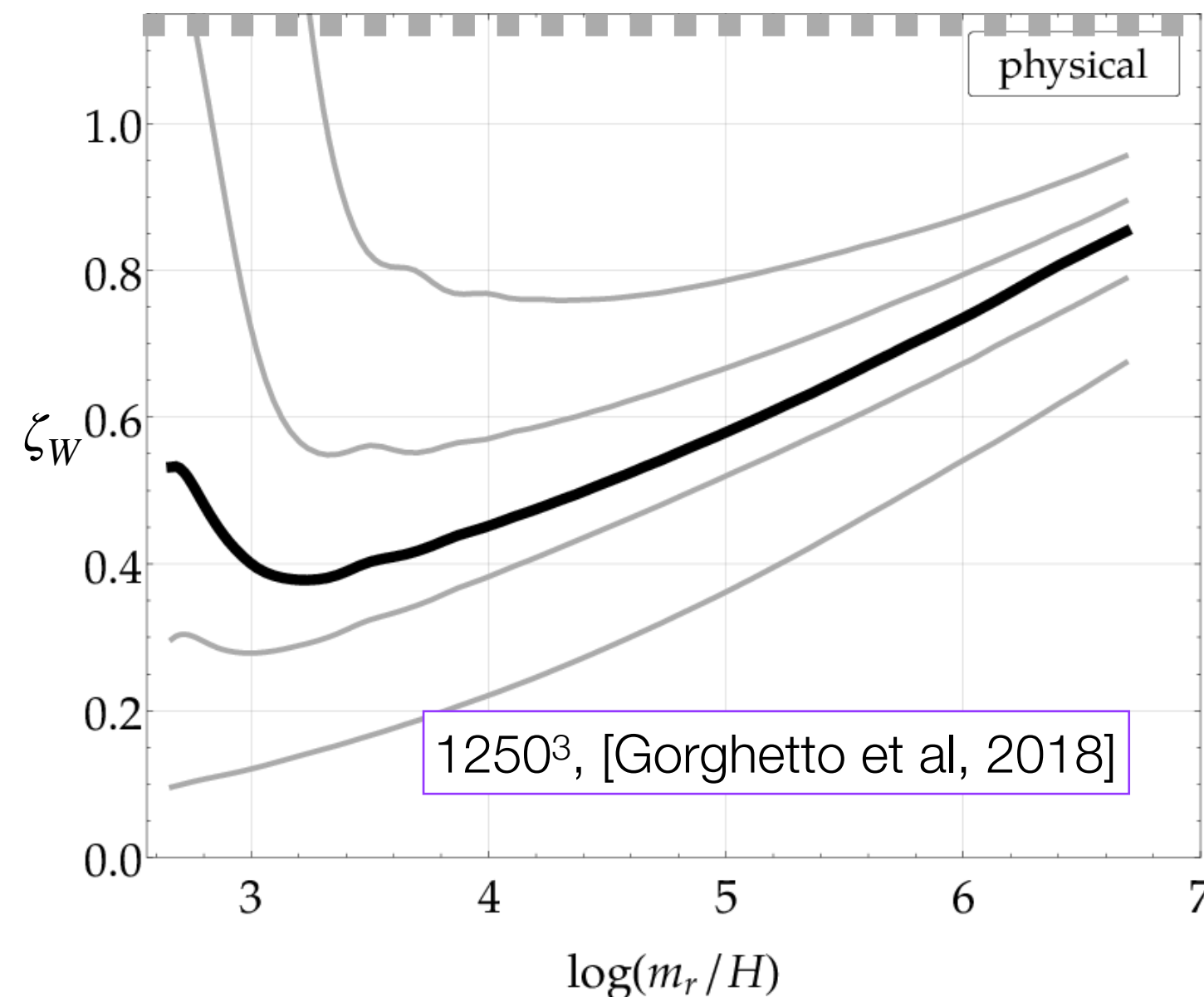
- Root-mean-square velocity $v \approx 0.6$; $\gamma \approx 1.3$ [Klaer et al, 2017]



Axion strings - Scaling Violation? (2018/2019)

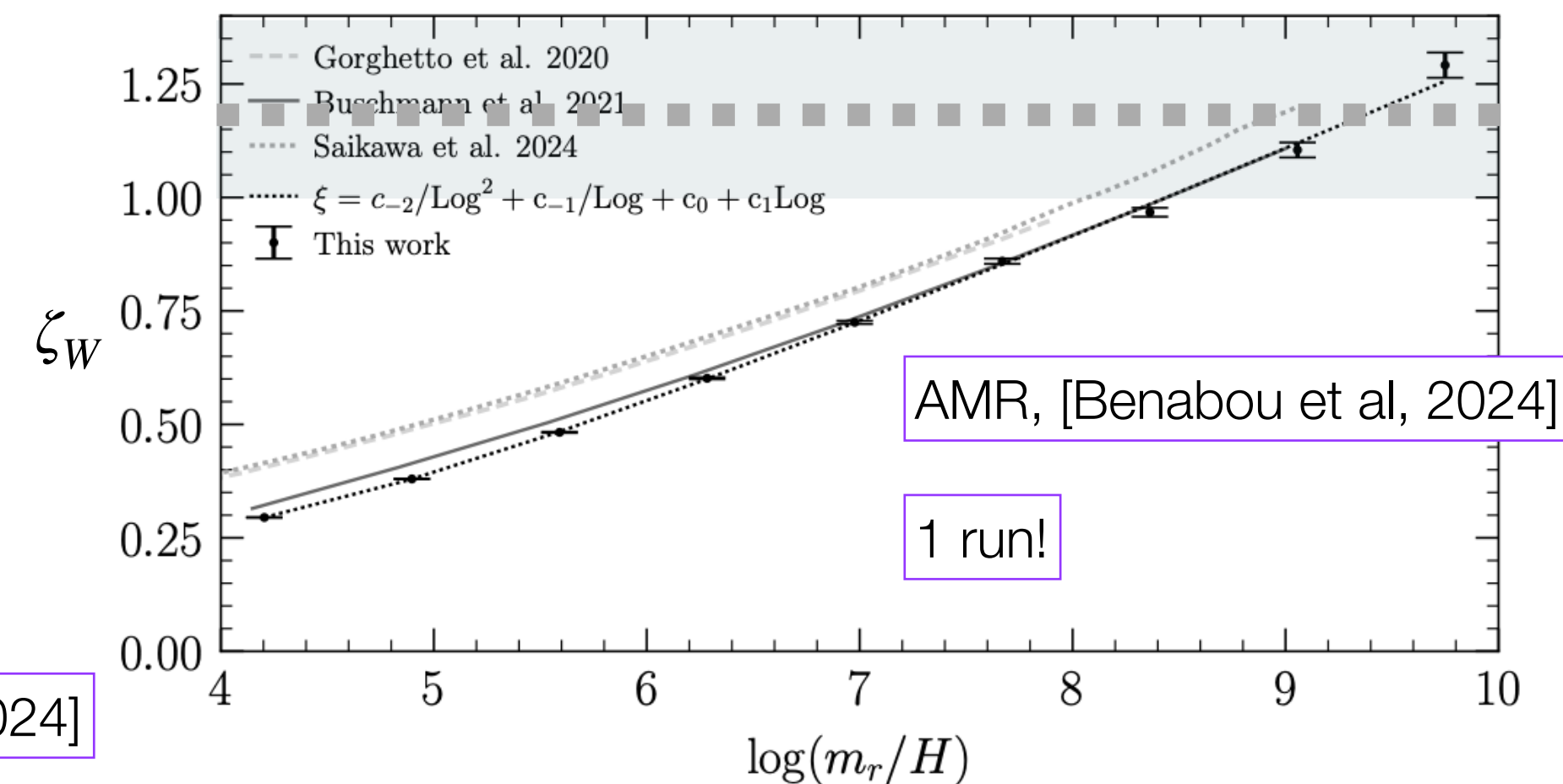
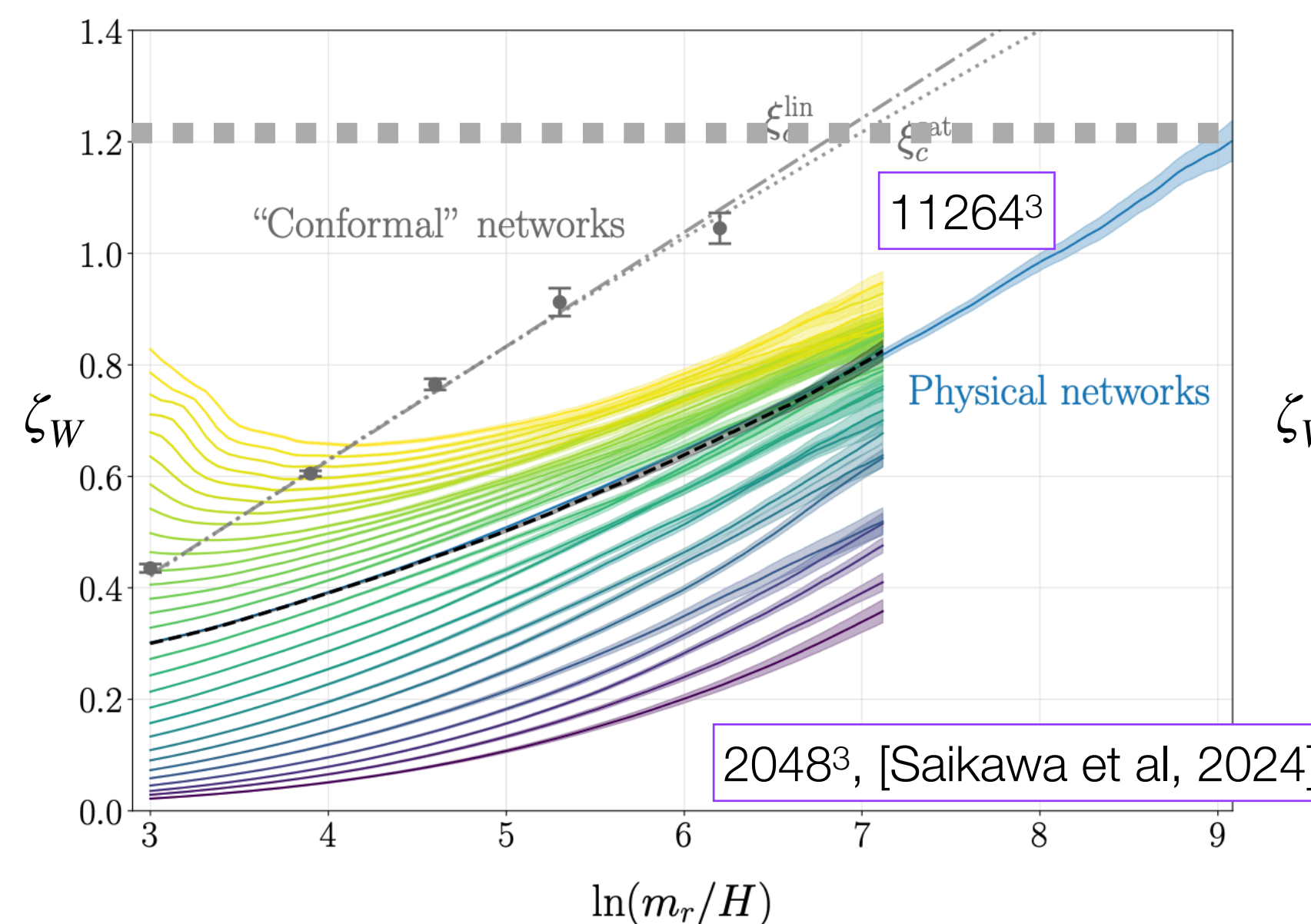
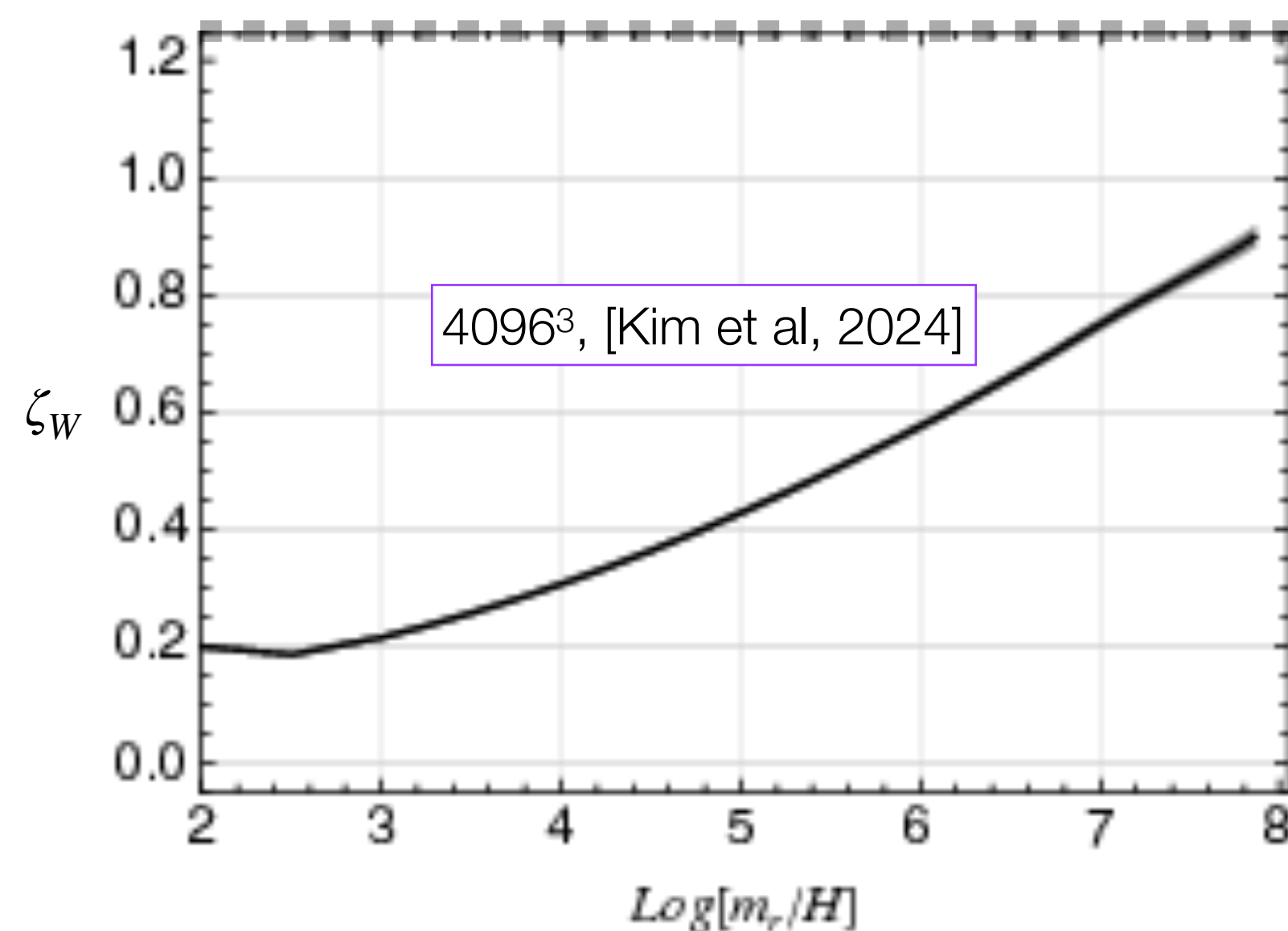
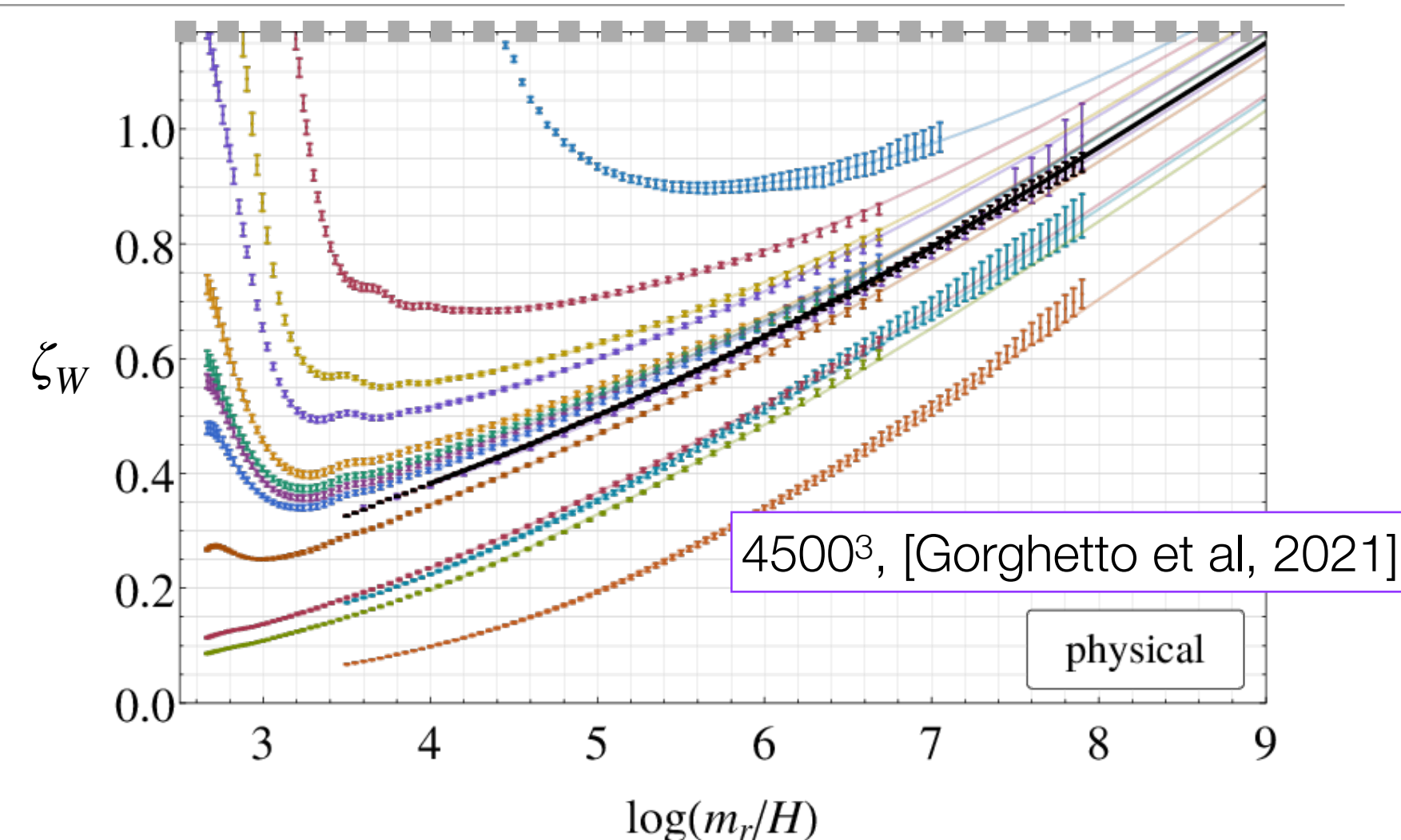
- In [Gorghetto et al, 2018] it is proposed that the density parameter should grow logarithmically with time due to the time-dependent string tension $\mu(t) \propto \log(t)$. **NOTE:** this is **not scaling** in the original sense. Could this be a low-density transient? (See upper right figure)

- Here fits of the form $c_1 \log(t) + c_0$ to $\zeta_W = \frac{t^2 l}{a^2 V}$ are used to argue that this is the true long-term behaviour of the network. However, final values $\zeta_W \sim 1$.



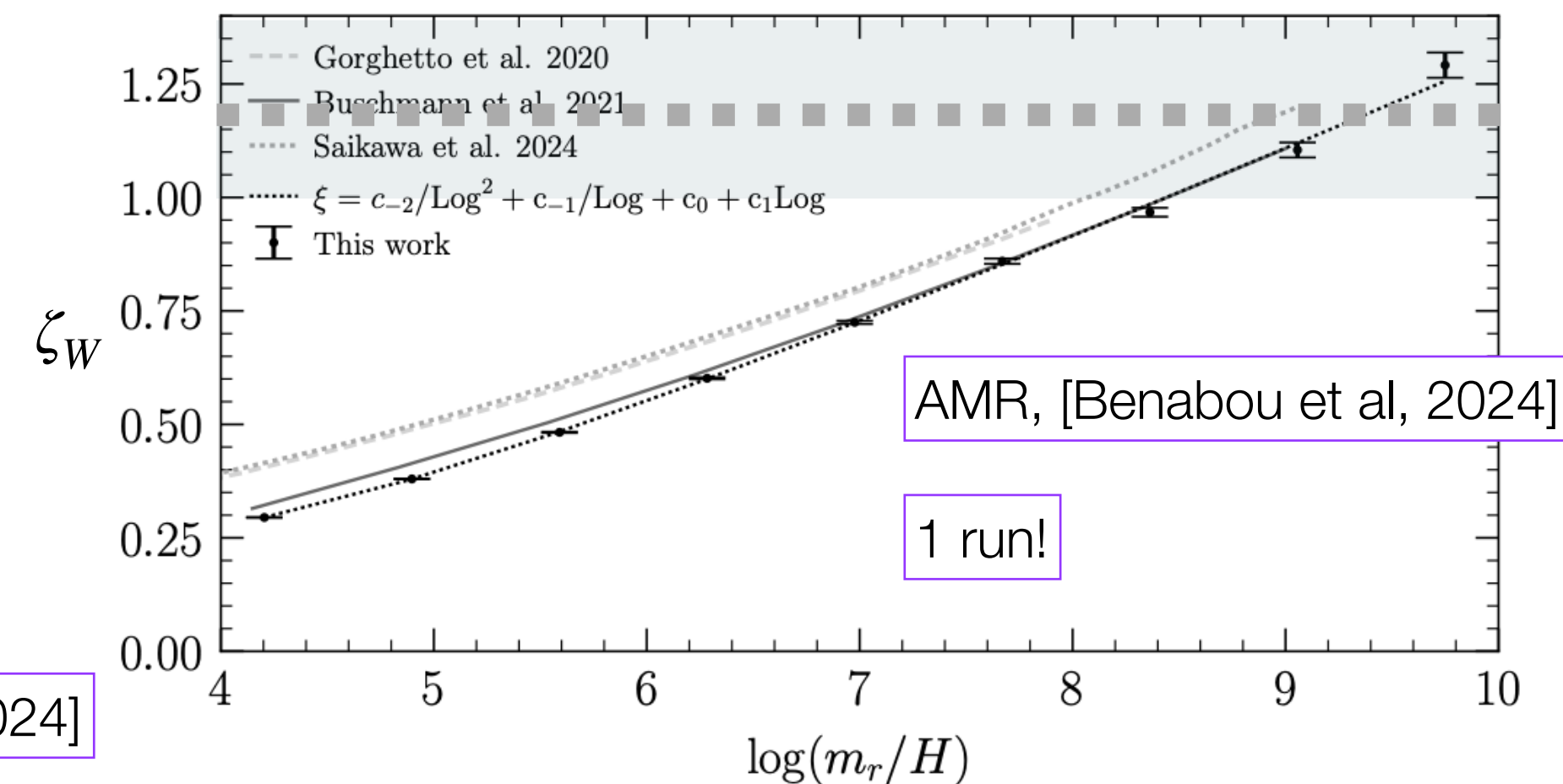
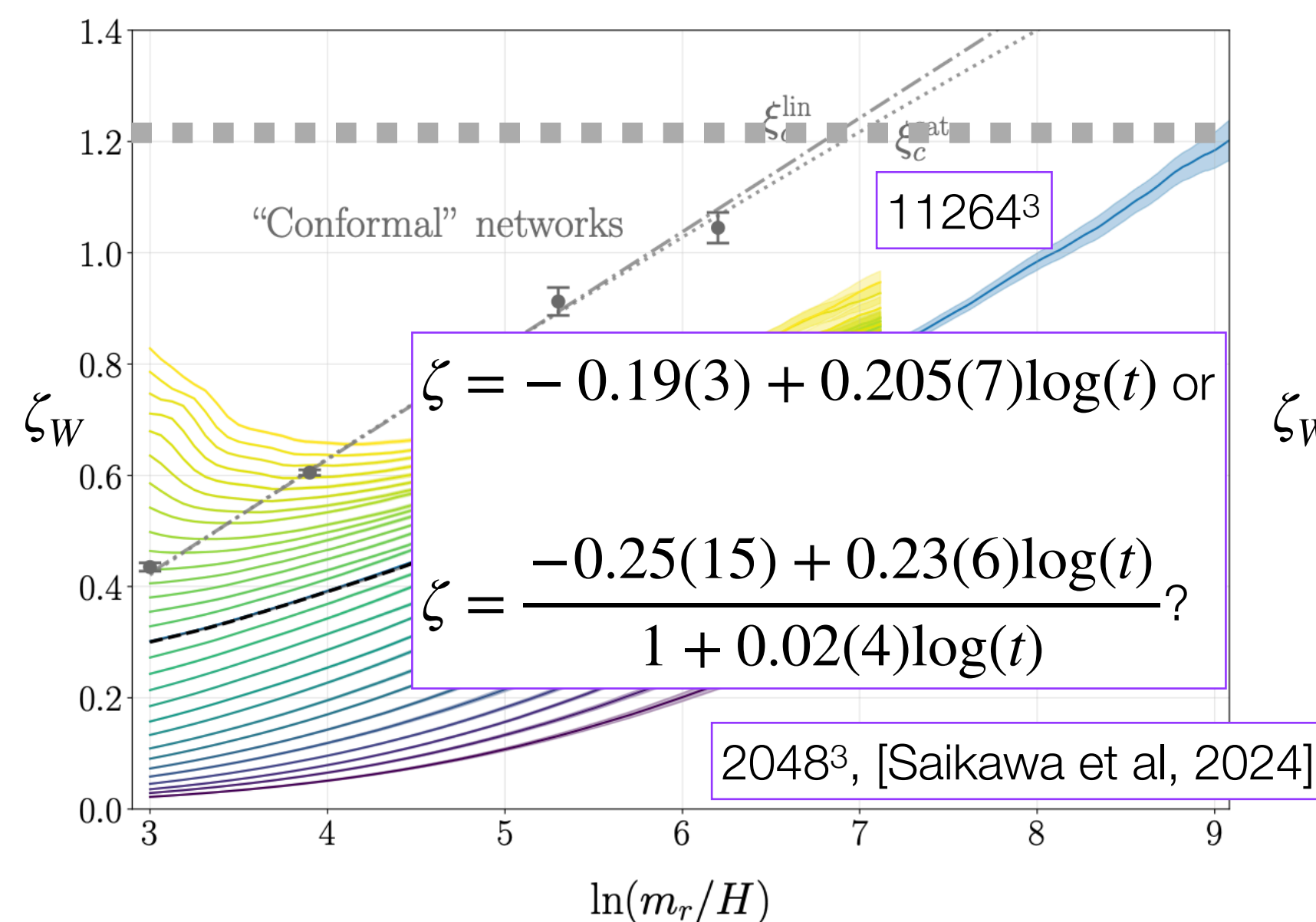
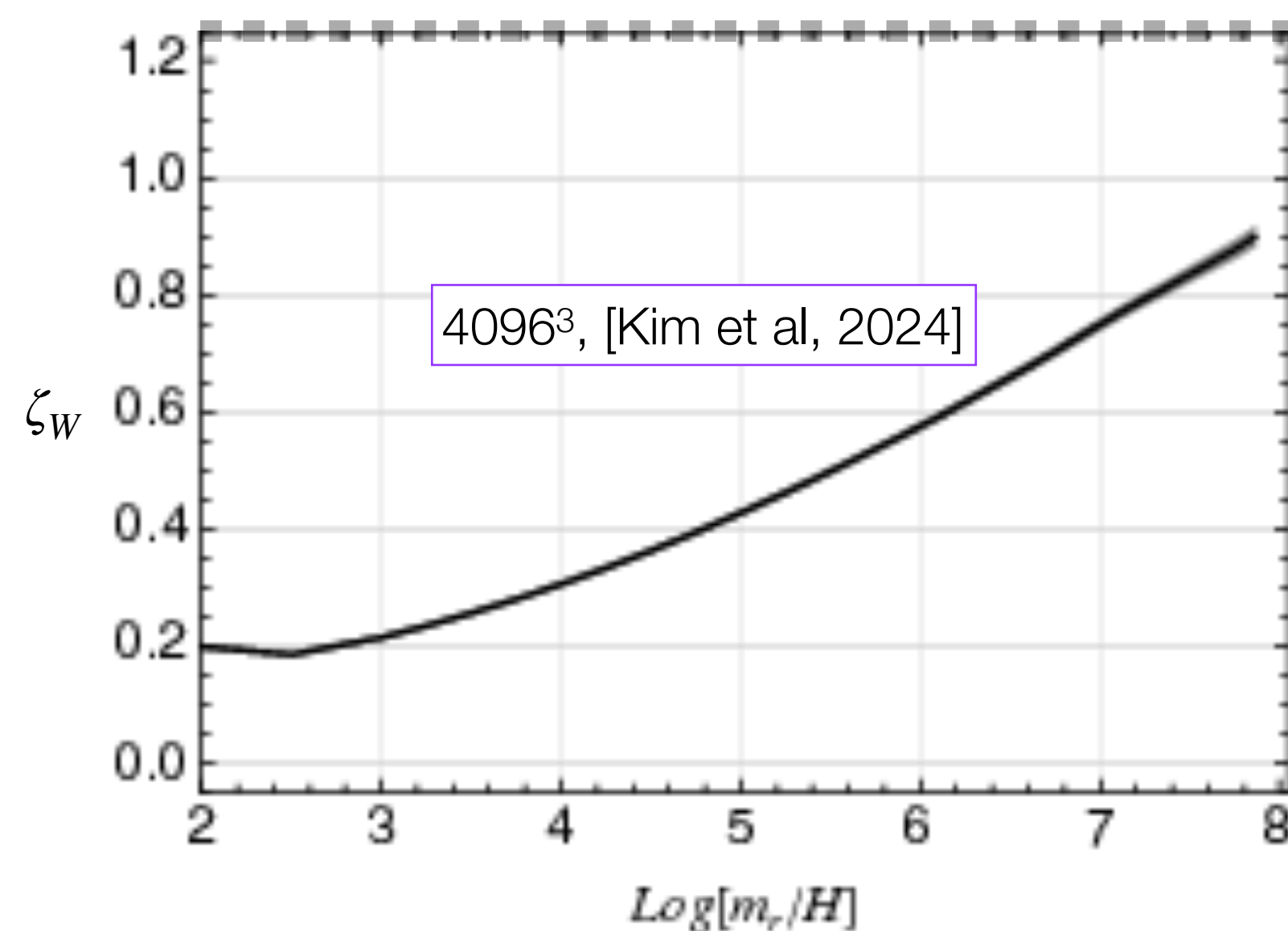
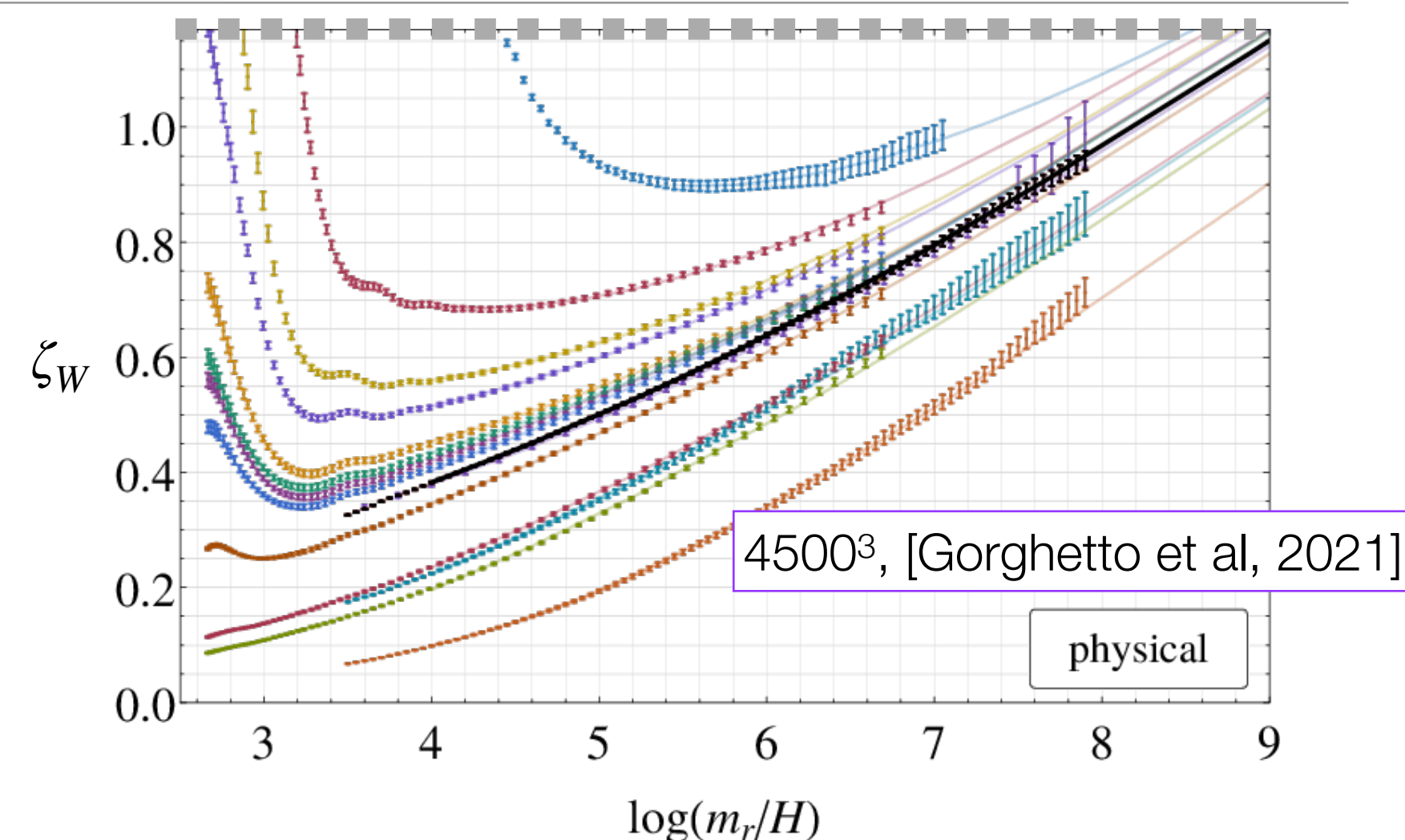
Axion strings - Scaling Violation? (2020s)

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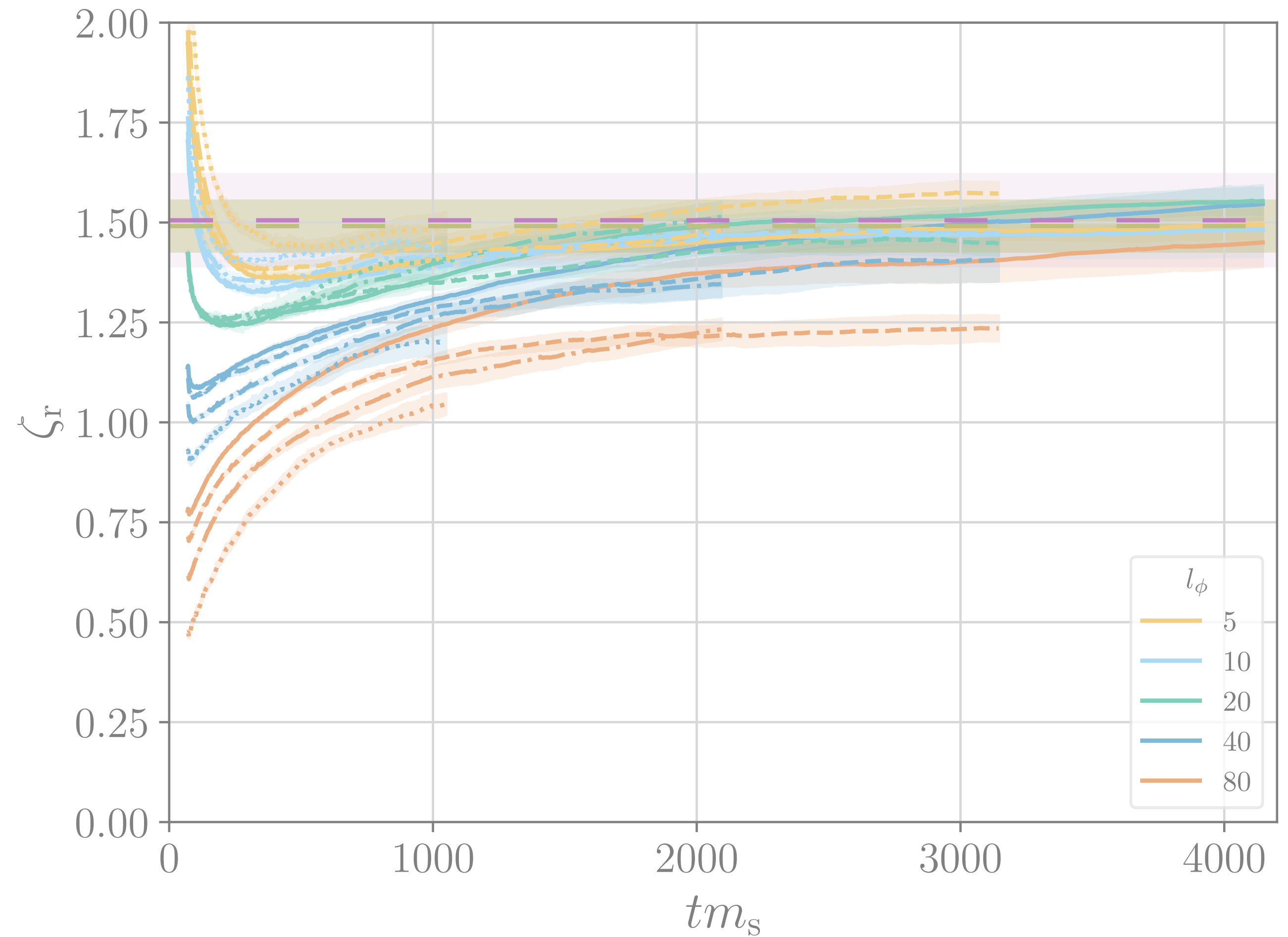
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The Scaling Density of Axion Strings in Terasite Simulations

arXiv:2410.18064, PhysRevD 111 063532
CSC project: 462000084



Simulations setup: observables

- Understanding and interpreting the evolution of this dynamical system, requires measuring both **string length** and **velocity**
- Measure string length, in both **Universe frame** (left, other groups do this only) and **string rest-frame** (right, related to **energy density, what VOS predicts**):

$$\ell_w = \frac{2}{3} n_p \Delta x$$

$n_p \rightarrow$ number of plaquettes pierced by strings

$$\ell_w = \ell_r \langle \gamma^{-1} \rangle$$



$$\ell_r = \frac{E + f_{V,W} L}{\mu_W (1 - f_{V,W})}$$

$\mu_W \rightarrow$ wighted energy per unit length

$f_{V,W} \rightarrow$ fraction contributed by the weighted potential energy density

$E \rightarrow$ Energy

$L \rightarrow$ Lagrangian

- String separation $\xi = \sqrt{\frac{V}{l}}$ and the related $x = \xi/t$

String density parameter $\zeta = \frac{lt^2}{a^2 V} = \frac{1}{x^2}$.

- Some groups measure **velocity**, although not all:

$$v_L^2 = \frac{E - L}{E + f_{V,V} L}$$

Important **consistency check**: $\zeta_r \gamma^{-1} \approx \zeta_W$

Simulations setup: observables

- Understanding and interpreting the evolution of this dynamical system, requires measuring both **string length** and **velocity**
- To explain our estimators: take then straight static string boosted with some rms velocity $v^2 = \langle \dot{X}^2 \rangle$, oriented along z-axis,

$$\Pi(x, t) = \gamma \dot{X} \cdot \partial \phi(x_s)$$

$$\partial \phi(x, t) = \gamma \hat{v} (\hat{v} \cdot \partial \phi_s) + \partial^\perp \phi(x, t)$$

where $\gamma = 1/\sqrt{1 - \dot{X}^2}$, $\hat{v}$ is a velocity unit vector and $D^\perp \phi(x, t) = (\delta_{ij} - \hat{v}_i \hat{v}_j) \partial_j \phi(x, t)$

$$E_\pi = \frac{1}{2} \int dx dy |D(\phi)|^2 \int d\sigma \dot{X}^2$$

$$E_D = \frac{1}{2} \int dx dy |D(\phi)|^2 \int d\sigma (1 + 1/\gamma^2)$$

$$E_V = \frac{1}{2} \int dx dy V(\phi) \int d\sigma (1/\gamma^2)$$

(Where $d\mathcal{L} = |X'| d\sigma = \gamma^{-1} d\sigma$ was used)

- The field energy will be proportional to rest frame string length

$$E = E_\pi + E_D + E_V = \mu_W (1 - f_{V,W} v^2) l_r$$

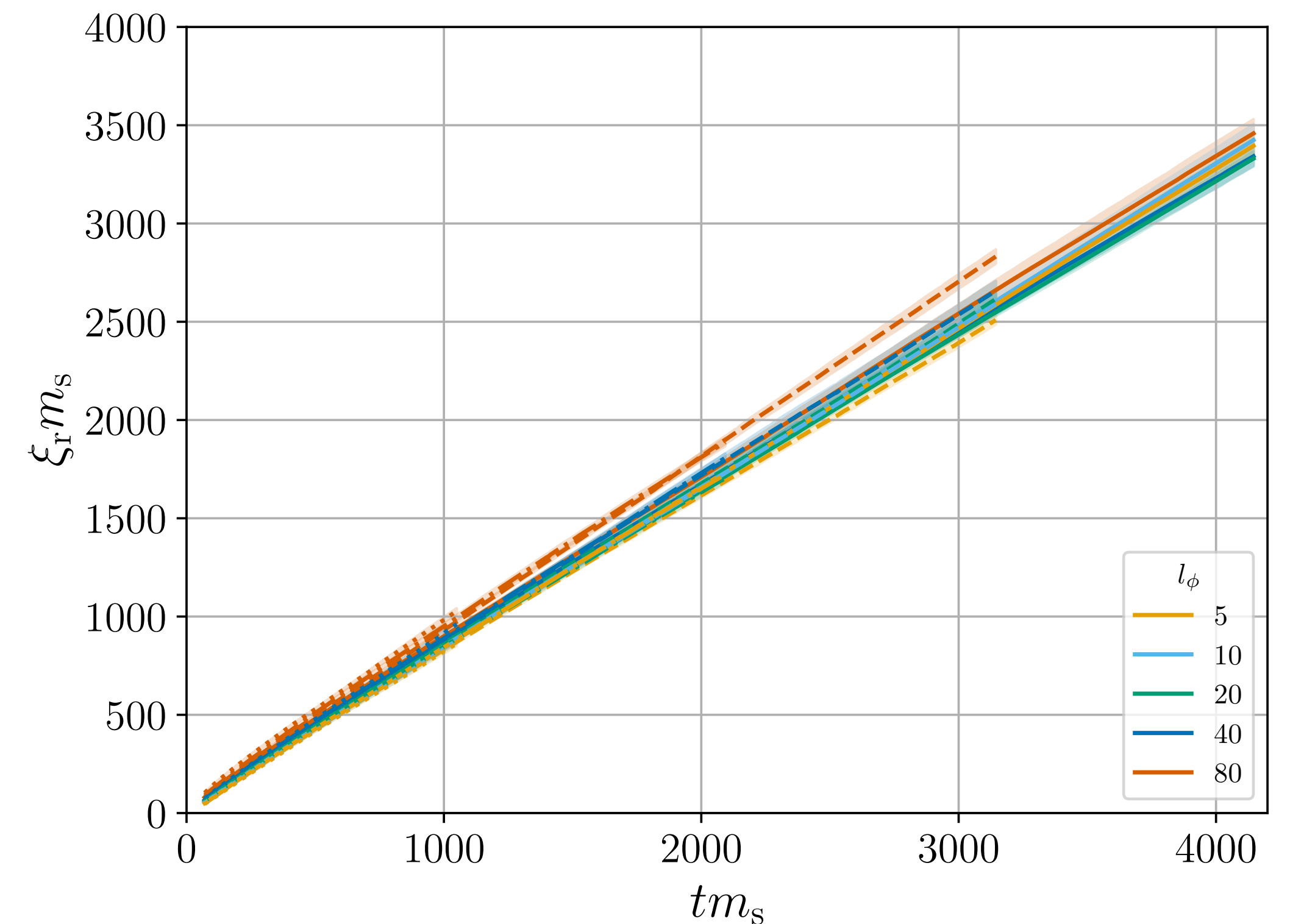
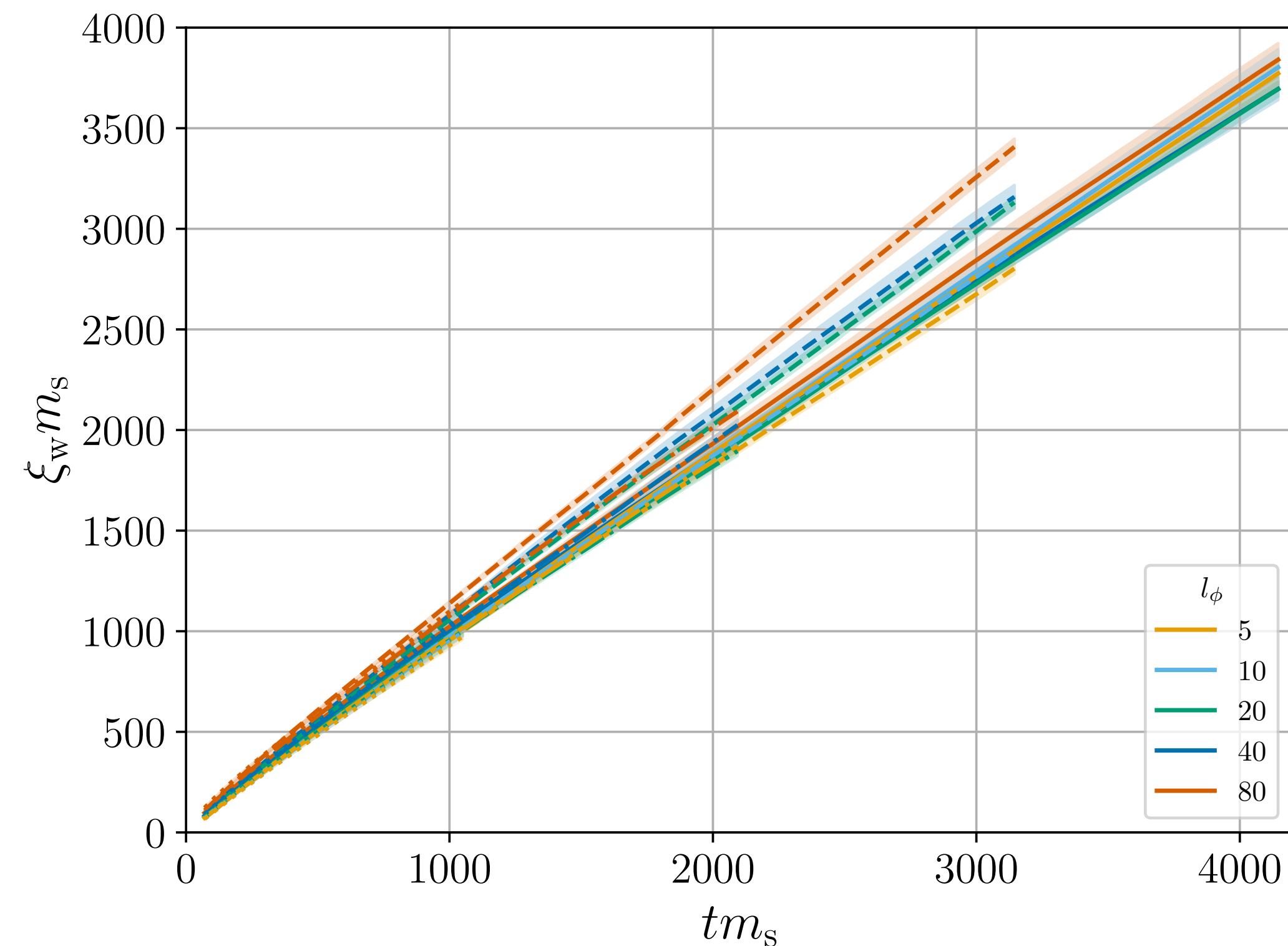
$f_{V,W} \rightarrow$ fraction contributed by the weighted potential energy density

$$L = E_\pi - E_D - E_V = -\mu_W (1 - v^2) l_r$$

$\mu_W \rightarrow$ wighted energy per unit length

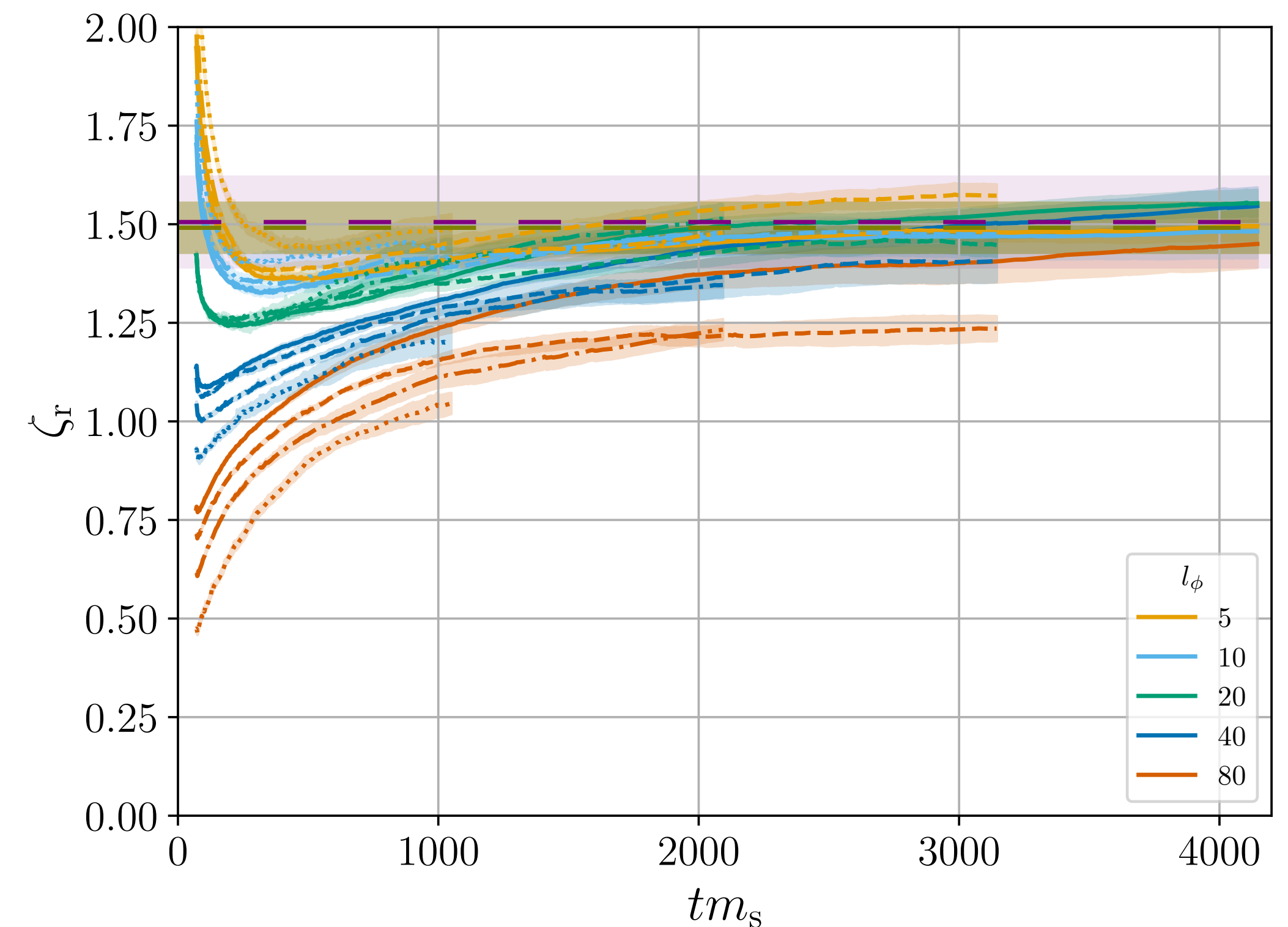
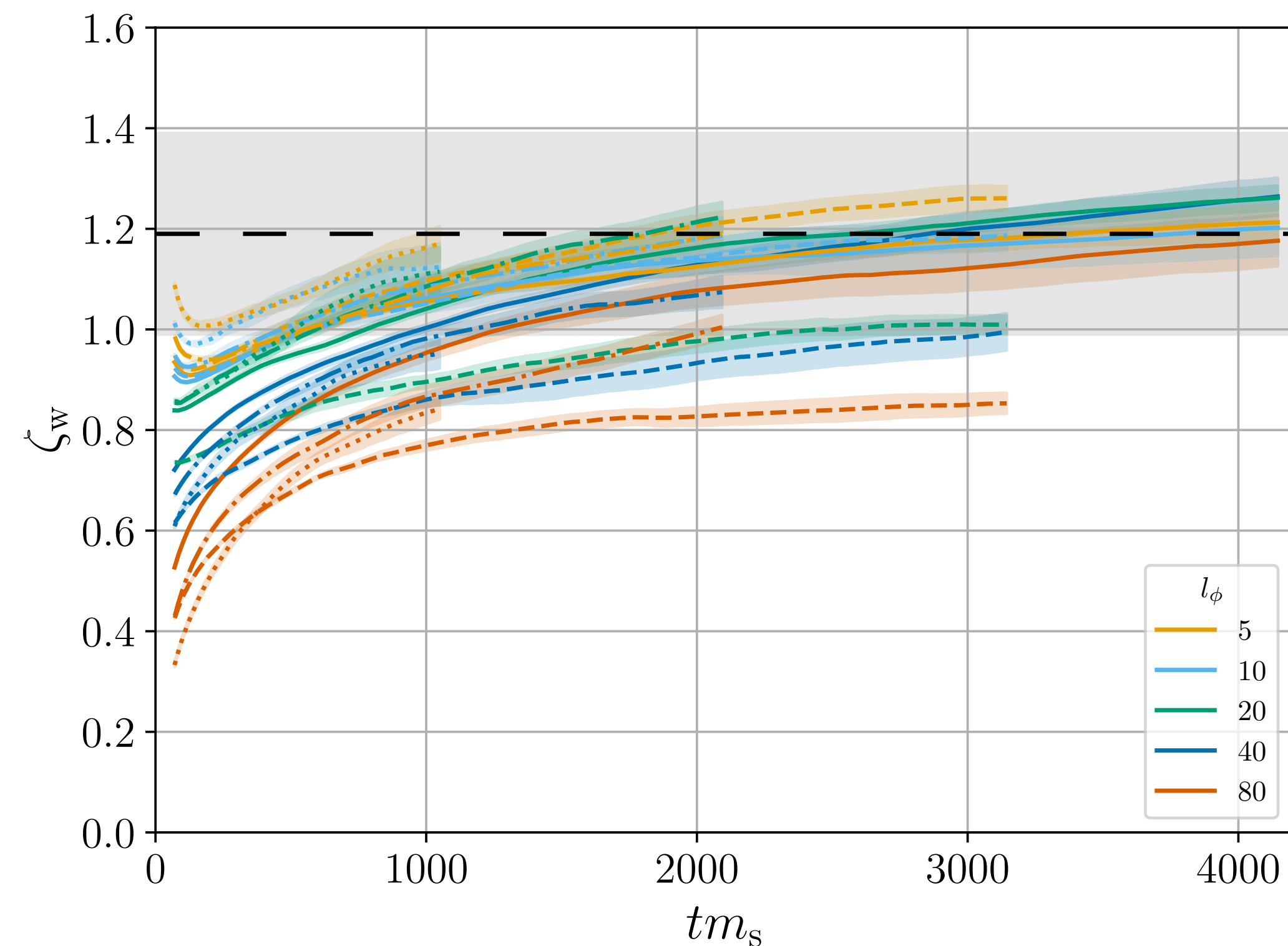
Standard Scaling Analysis: Time Series Data

- From looking at string separation, one does **not** see any **evidence** of **deviations** from **standard scaling**
- Here it looks like using either **Universe frame** or **rest-frame** measures of length is 100% **inconsequential**...



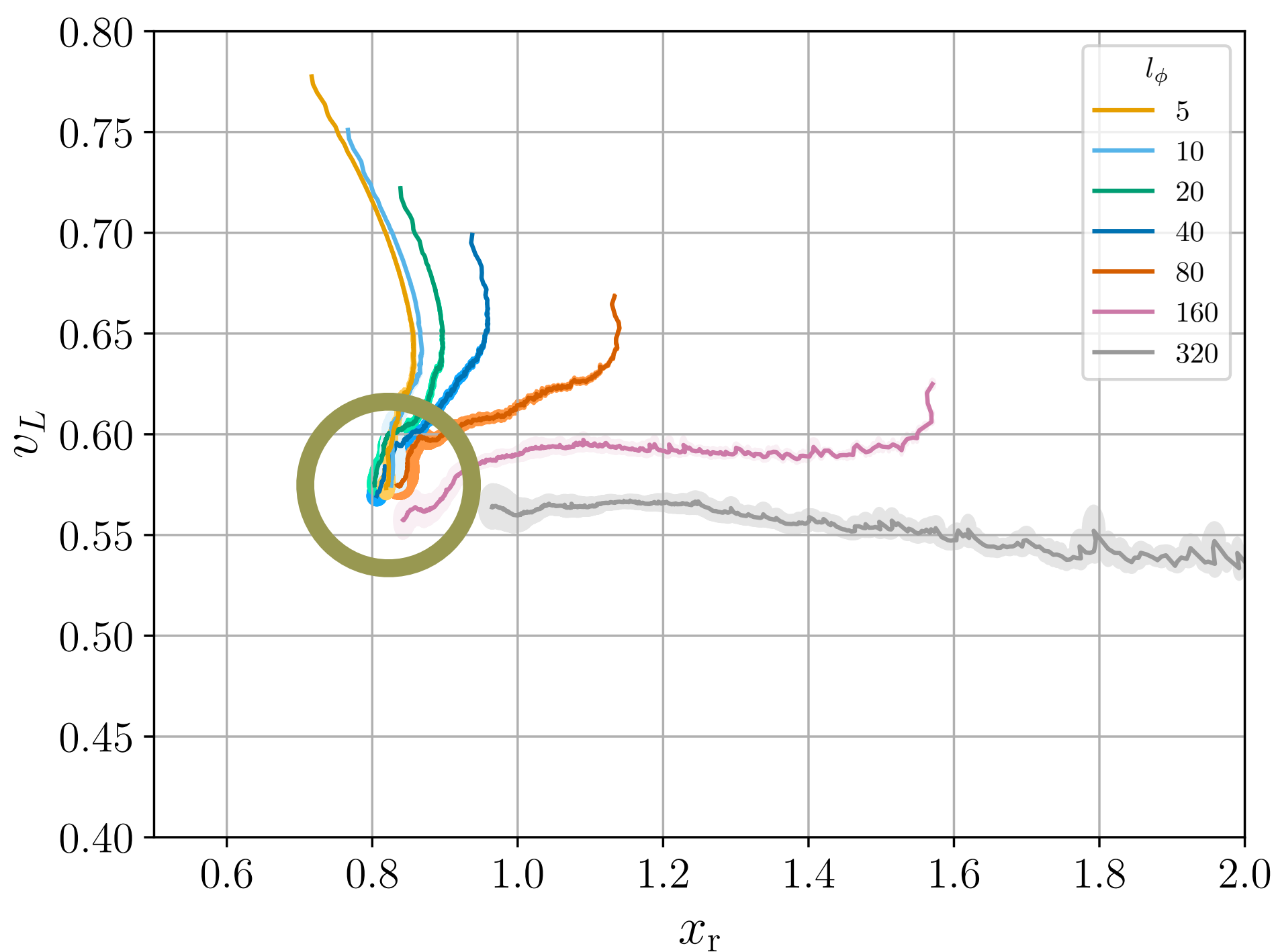
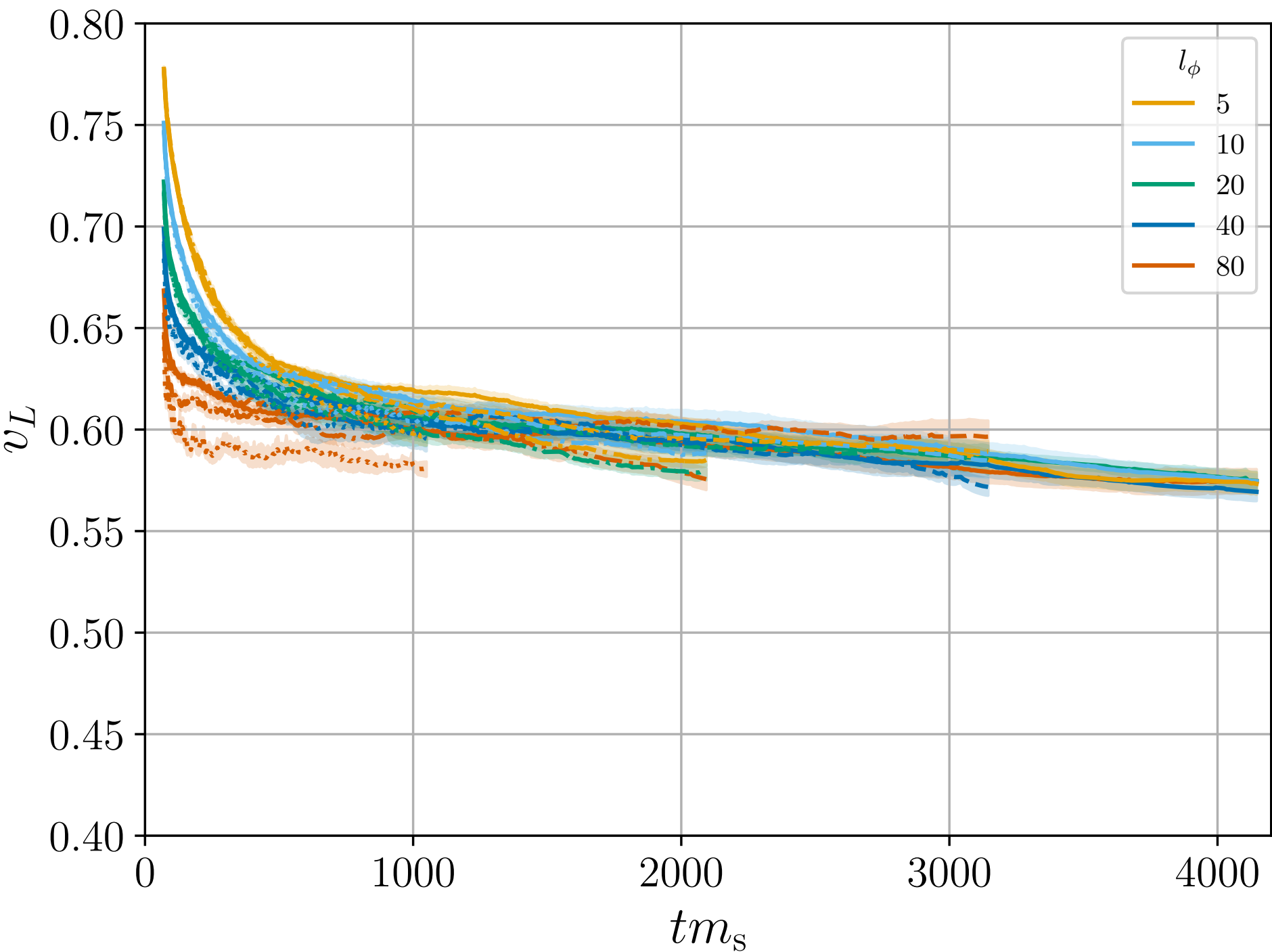
Standard Scaling Analysis: Time Series Data

- We remind the audience that other groups only measure ζ_W , but not ζ_r (and most of the times not v)
- It seems there is a difference between **Universe frame** and **rest-frame** string density parameters. Now we must look at **velocities**. **Rest-frame converges** to constant - depending on initial condition and dynamic range.



Standard Scaling Analysis: Time Series Data

- **Velocity** is still **evolving**. Although **timid**, it is sufficient to take make the length contracted $\zeta_W = \gamma^{-1}\zeta_r$ evolve as well
- The phase space plot (showing only 16k) is quite informative too:
 - **Null-clines** at $dv/dt = 0$ and $dx_r/dt = 0$ explain how to interpret evolution and their crossing leads to a fixed point



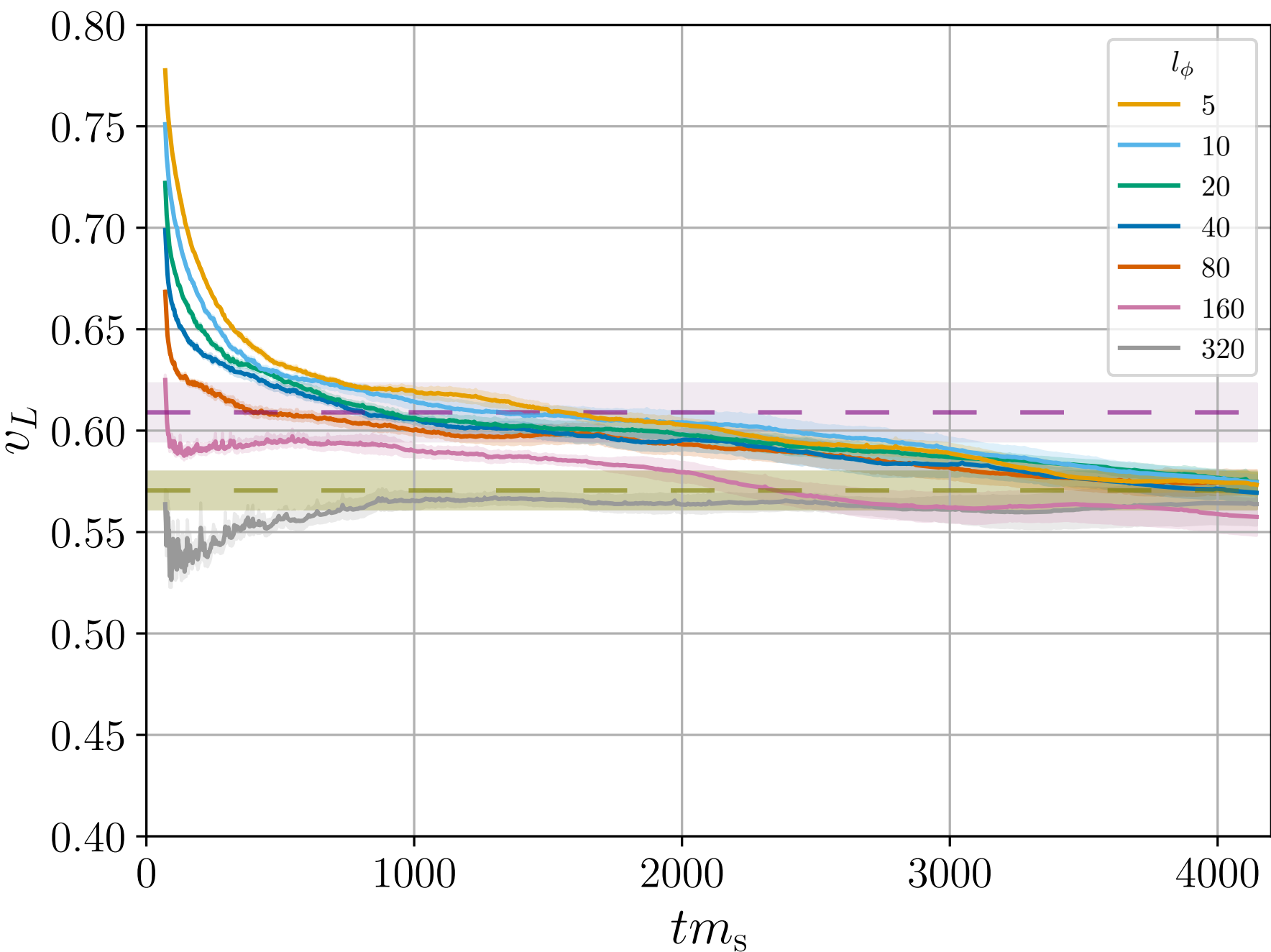
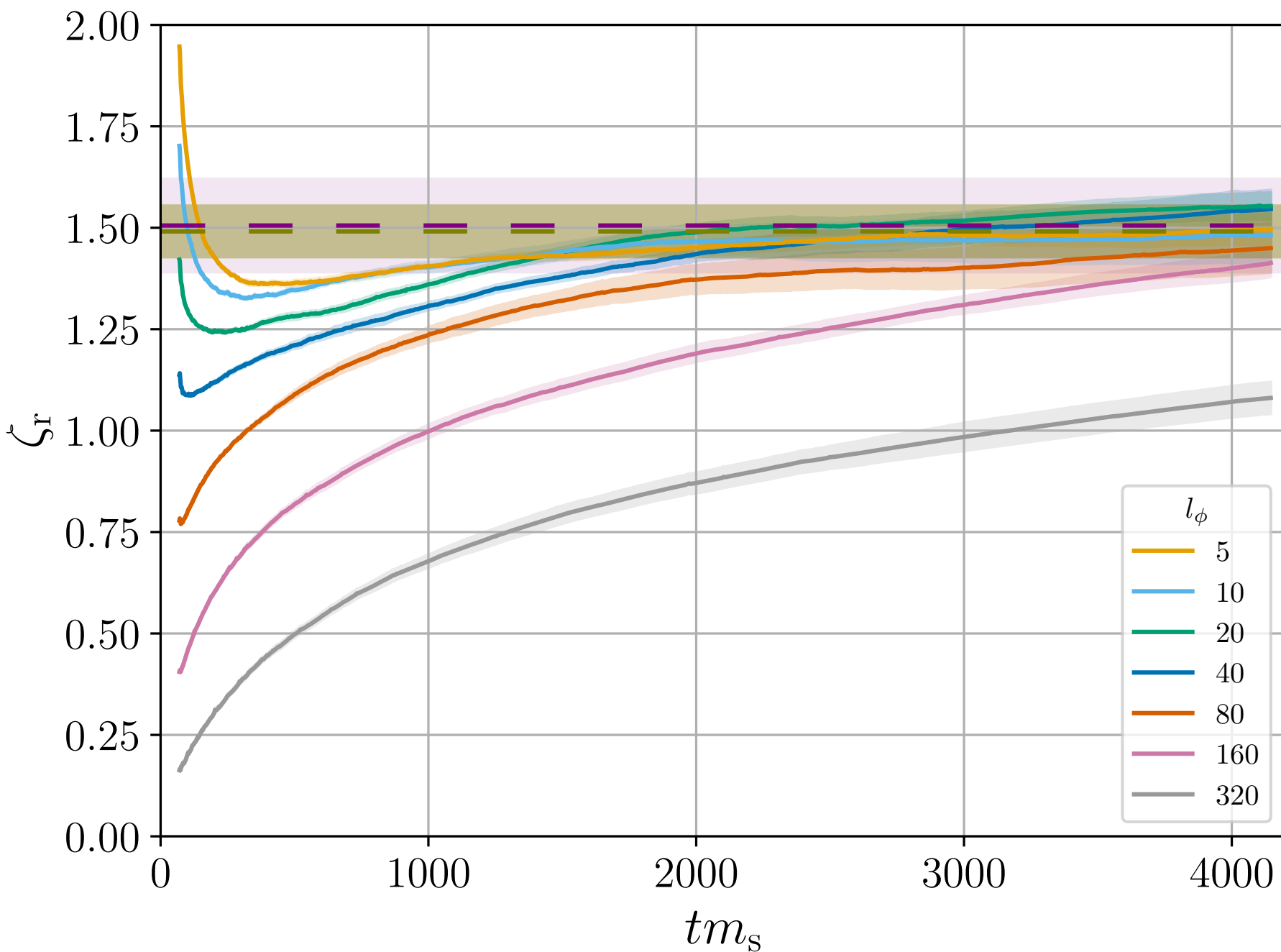
NOTE:

$\hat{\zeta}_w \gamma(\hat{v}_L)$ **coincides** with $\hat{\zeta}_r$.
Convergence after boost!

l_ϕ	$\hat{\zeta}_w$	$\hat{\zeta}_r \gamma^{-1}(\hat{v}_L)$
5	1.211(34)	1.214(24)
10	1.200(56)	1.202(46)
20	1.258(24)	1.259(21)
40	1.259(37)	1.257(30)
80	1.172(51)	1.174(42)
160	1.156(34)	1.156(24)
320	0.882(32)	0.880(34)

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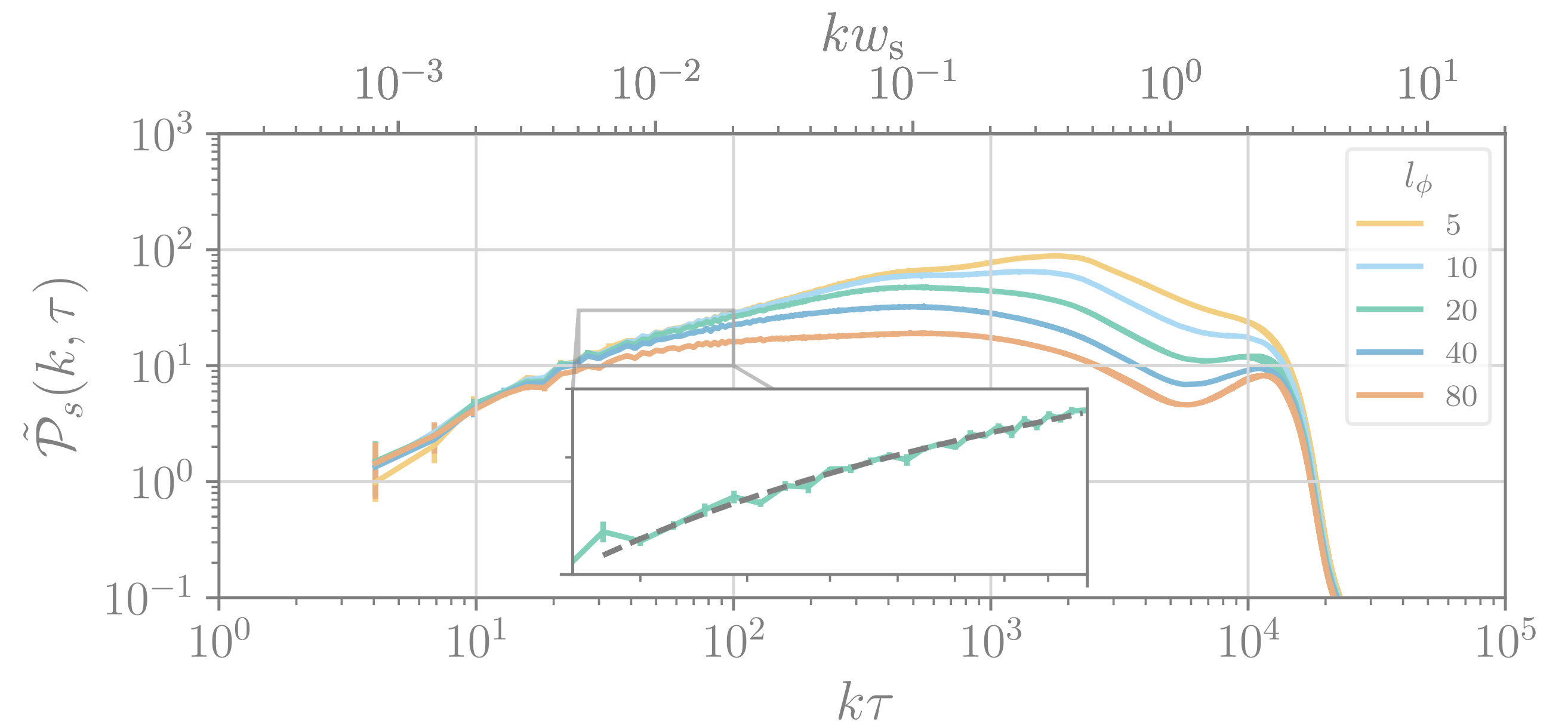
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The spectrum of axions in a scaling string network

arXiv:2512.13653
CSC project: 462000084



The spectrum of axions

- Axion energy spectrum $\rho_a = \int \frac{dk}{k} \mathcal{P}_a(k, t)$

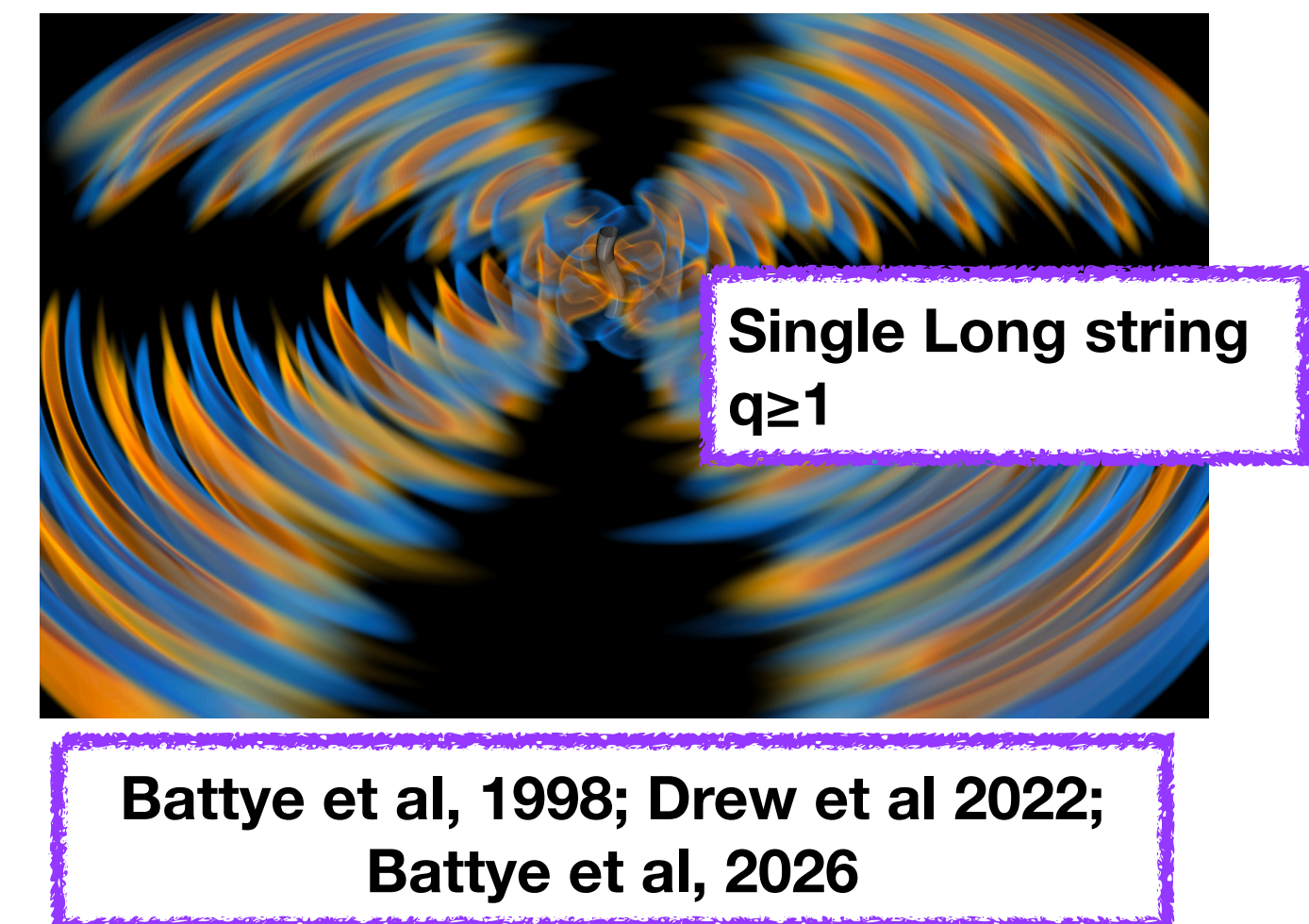
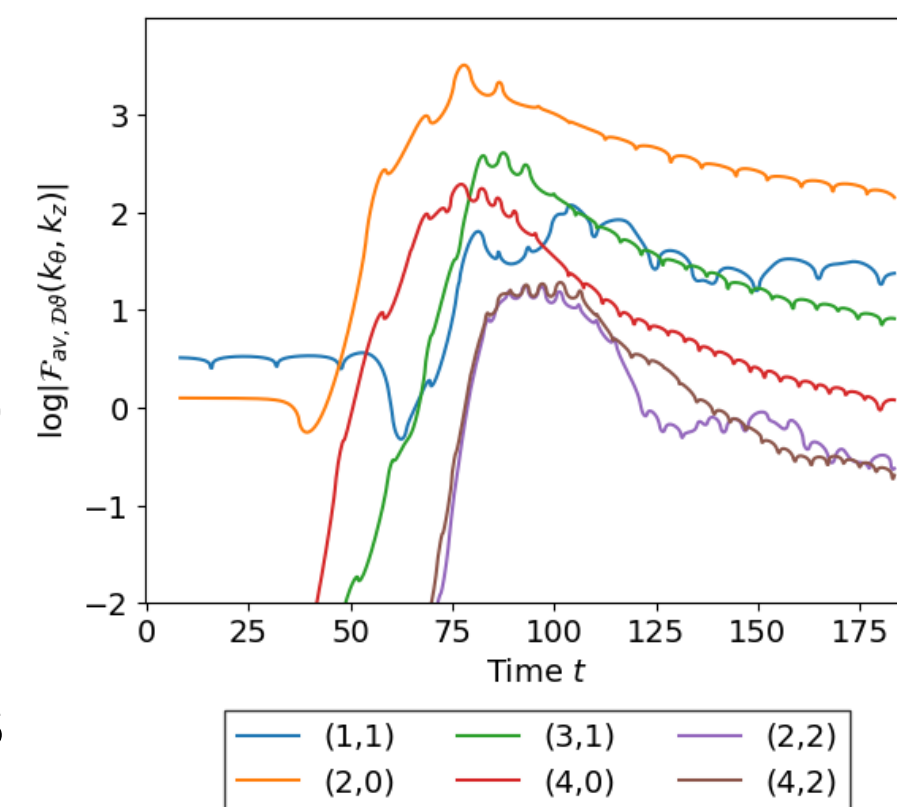
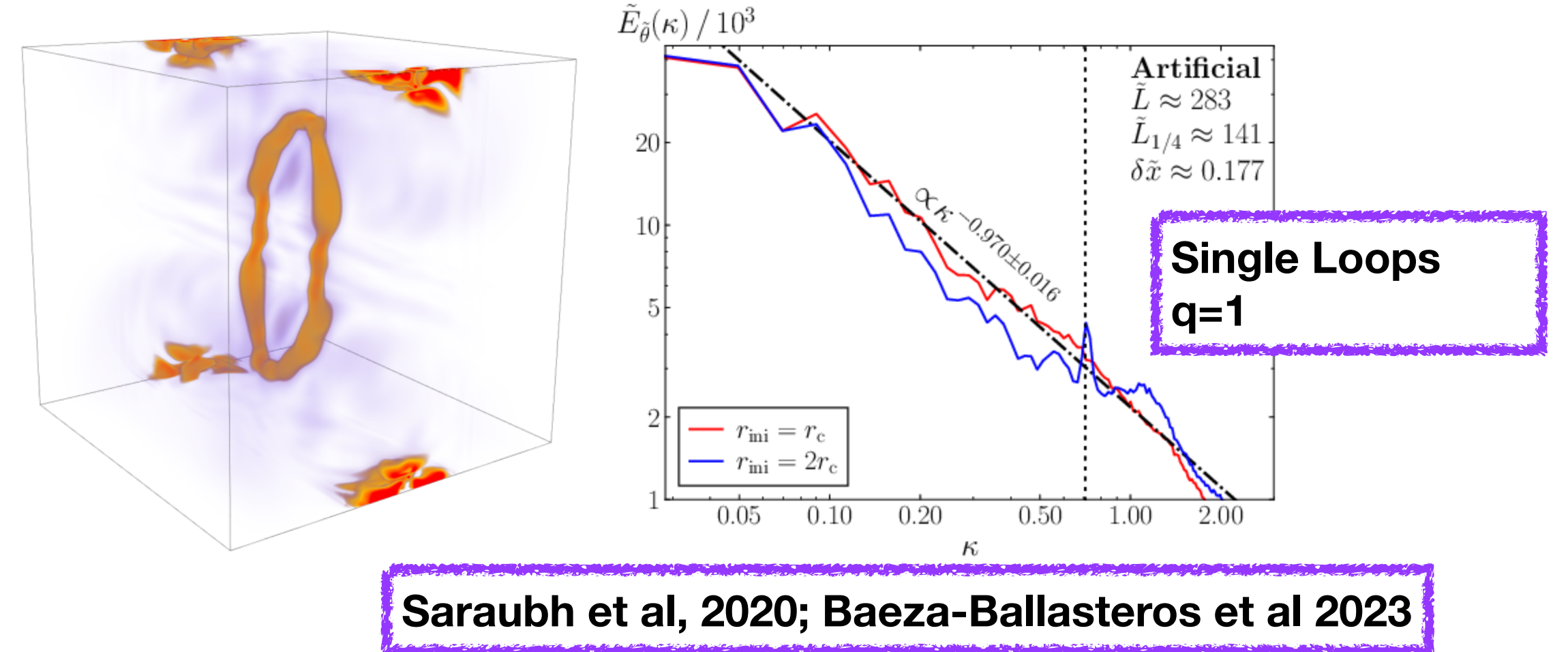
$$\mathcal{P} \propto \ln(k/k^*) \quad \text{or} \quad \mathcal{P} \propto (k/k_*)^{1-q}$$

$$q = 1 \text{ [Hariri et al, 1986]} \quad \text{or} \quad q > 1 \text{ [Davis et al, 1987]}$$

- Systematics:** self-field, fitting range, network evolution, lattice spacing, etc. Field theory networks so far:

- Smaller lattices, [Gorghetto et al, 2018] [Kim et al, 2024] extrapolations from $q < 1$ to $q \gg 1$
- Larger lattices [Saikawa et al, 2024] ($q \approx 1$ / $q \gg 1$) and AMR simulations [Benabou et al, 2025] ($q = 1$)

- Until recently **no model** for production of axions from strings for the whole network (**ensemble of strings**).



Decoherence functions

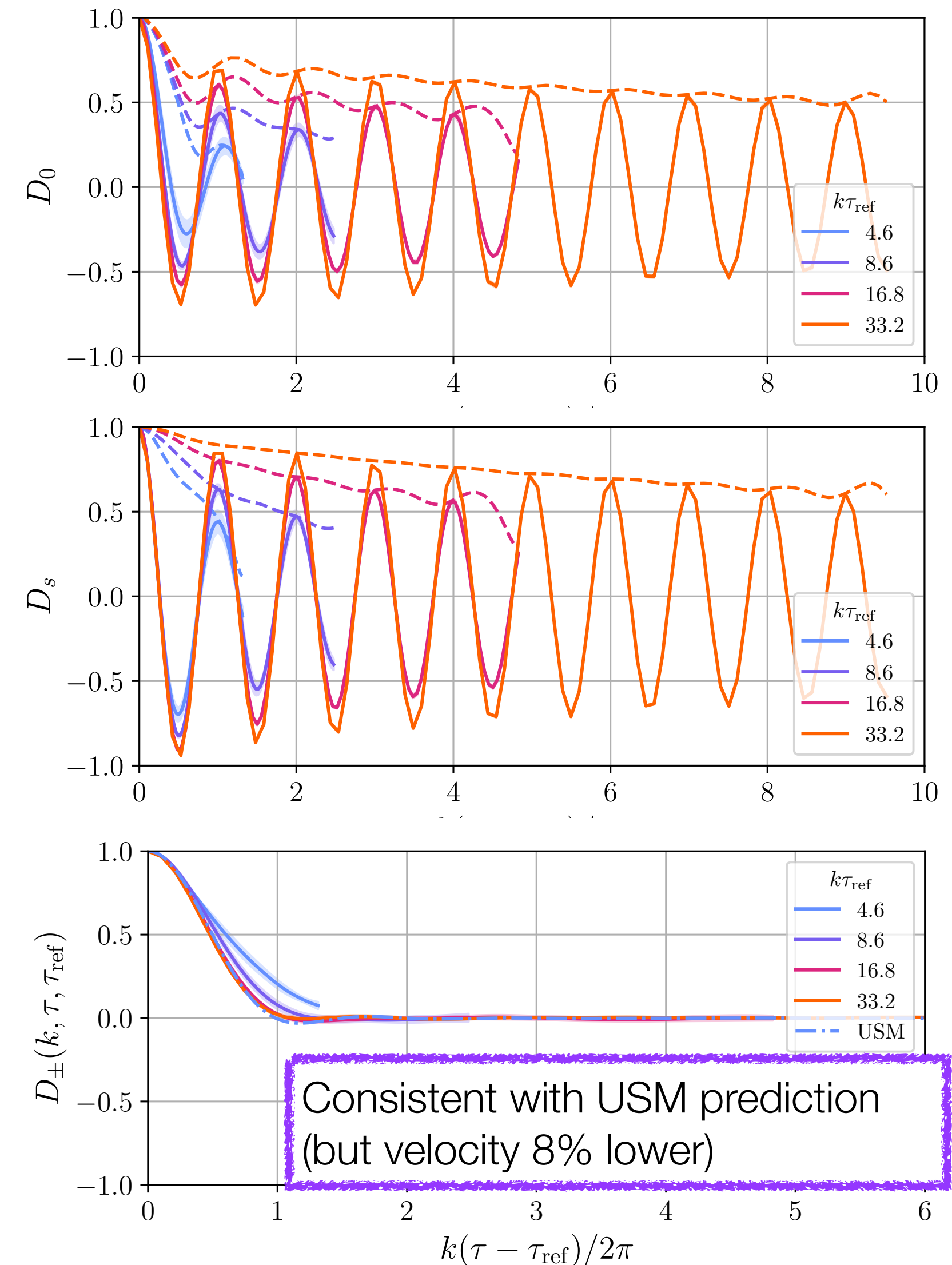
- **Unequal time Correlator:** (helps distinguish source of current)

$$U_{aa}(k, \tau_1, \tau_2) = \mathcal{V}^{-1} \langle \tilde{J}_a^*(k, \tau_1) \tilde{J}_a(k, \tau_2) \rangle$$

- **Decoherence functions:**

$$D_a(k, \tau_1, \tau_2) = \frac{U_{aa}(k, \tau_1, \tau_2)}{\sqrt{P_a(k, \tau_1)P_a(k, \tau_2)}}$$

- Axion observables (D_s, D_0) decohere similar to compressional modes in a fluid [Correia et al, 2025]
 - Self-field contribution distorts oscillating envelope of D_0 more strongly. D_s is less affected
- String observables ($D_{\pm}$) exhibit Gaussian decorrelation; similar of random sweeping in fluids [Kraichnan, 1964]



$$l_\phi = 20$$

Consistent with USM prediction
(but velocity 8% lower)

Temporal slopes of spectral index

- Fits to power law:**

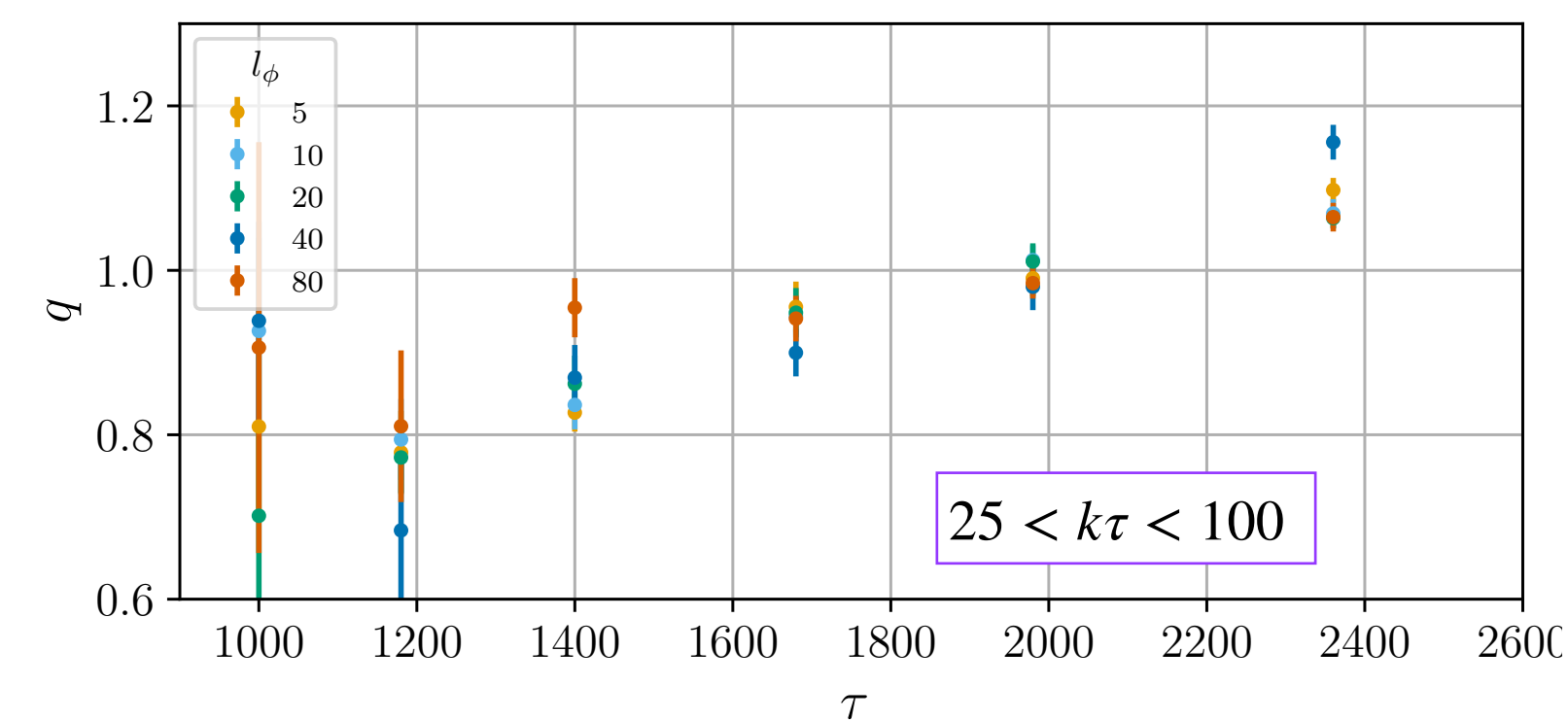
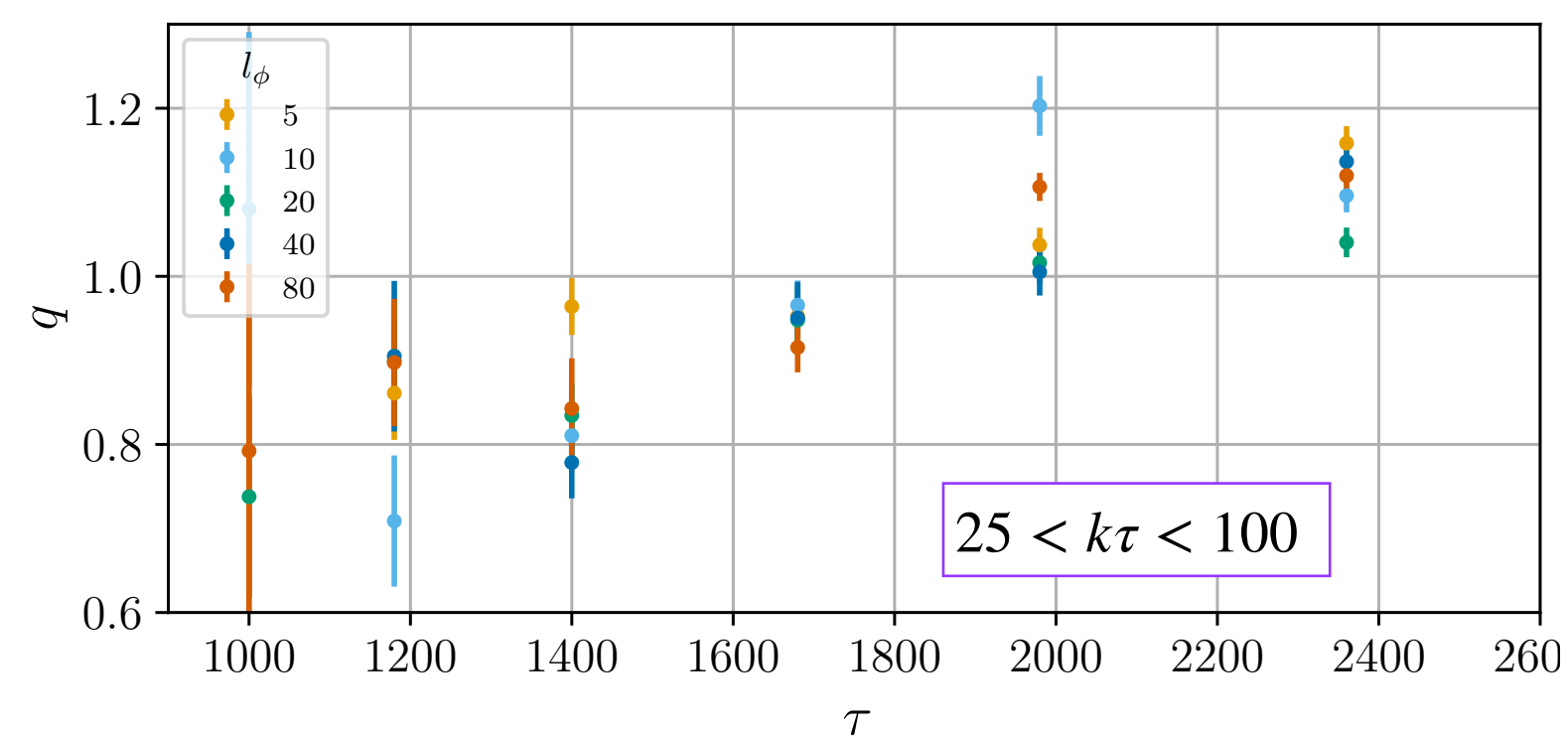
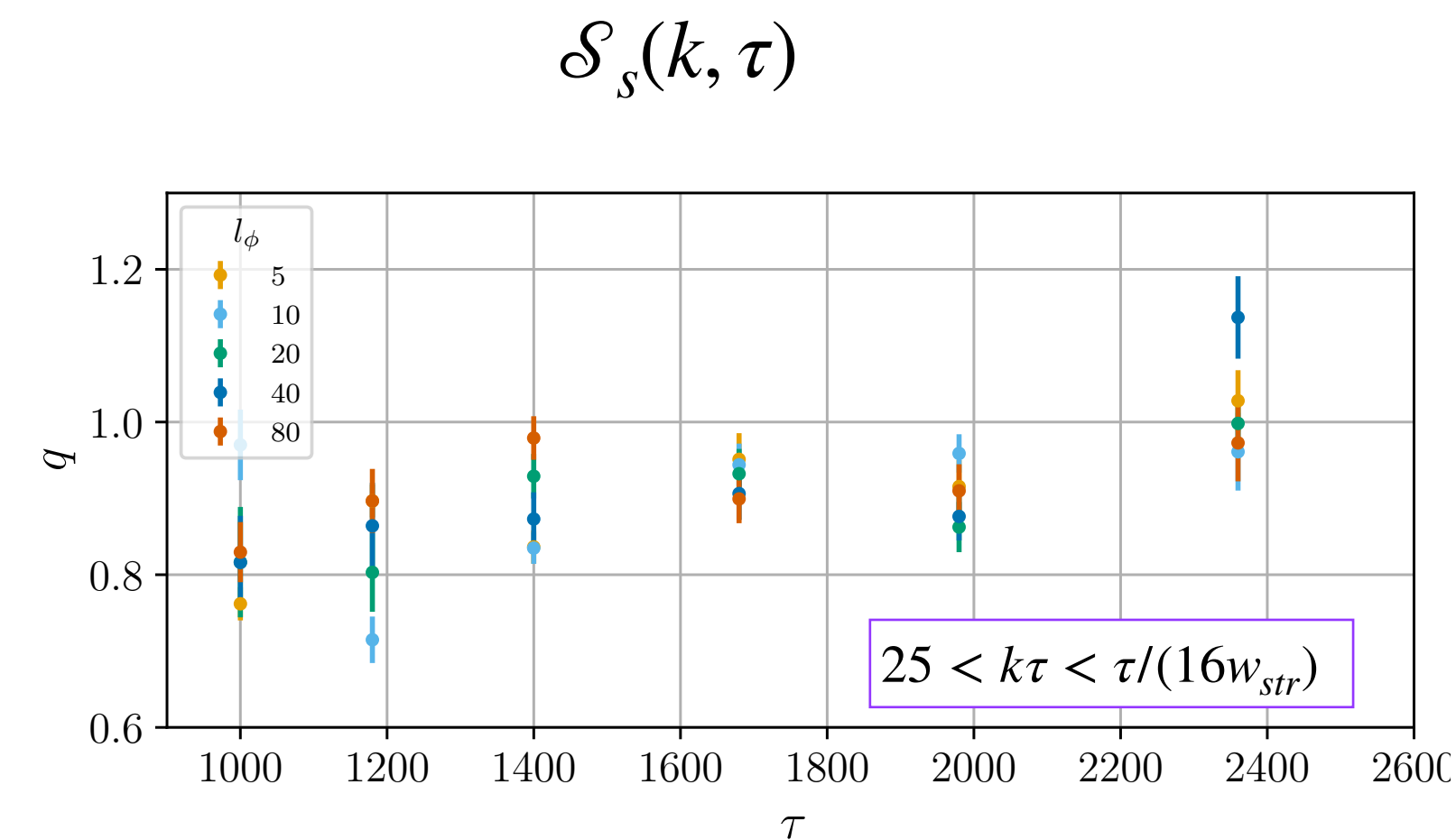
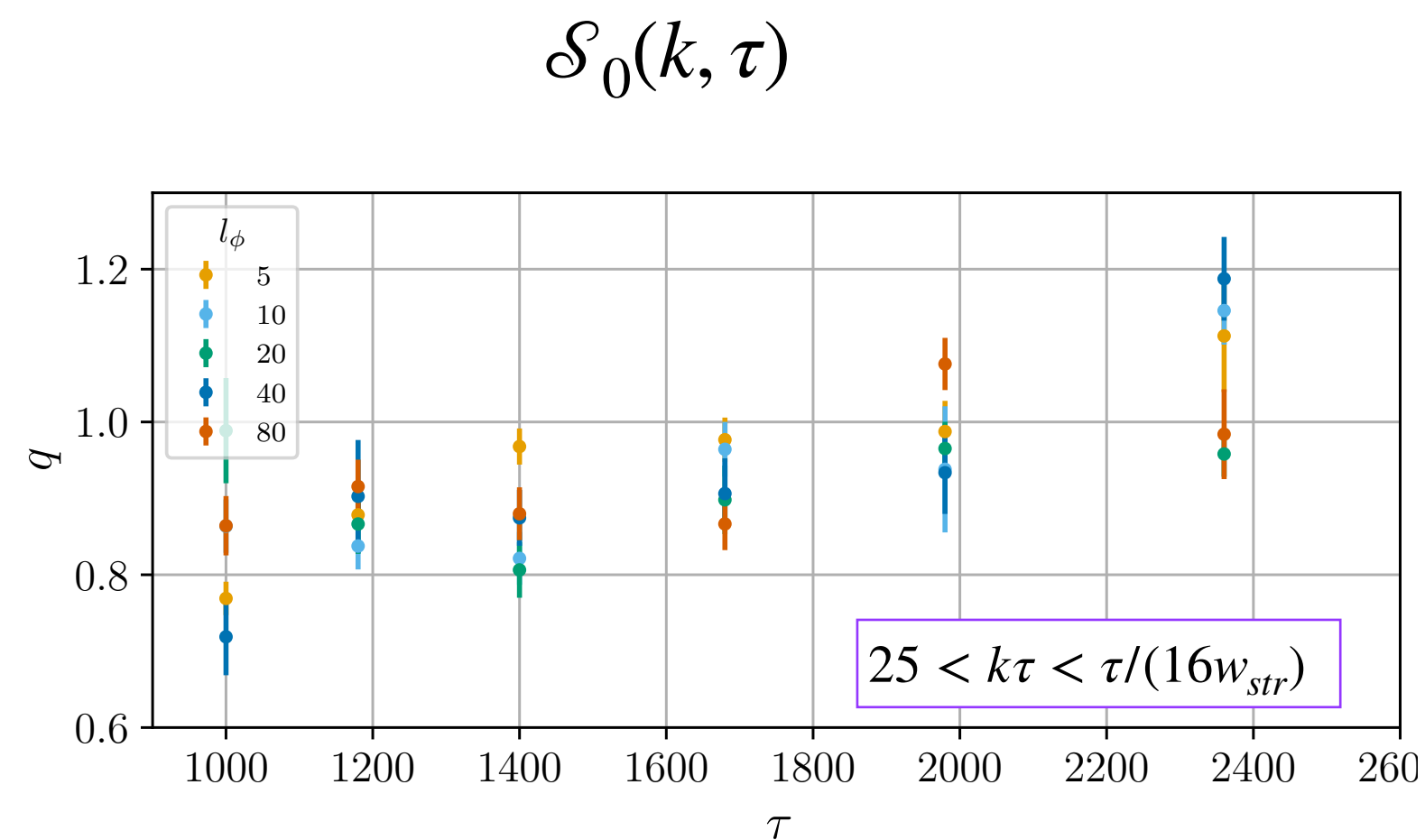
$$\mathcal{S} = (k\tau)^{1-q}$$

- Multiple systematics (see [Saikawa et al, 2024; Benabou et al, 2024]). Ex:

- Time difference of power spectra
- Lattice spacing (loss of resolution)

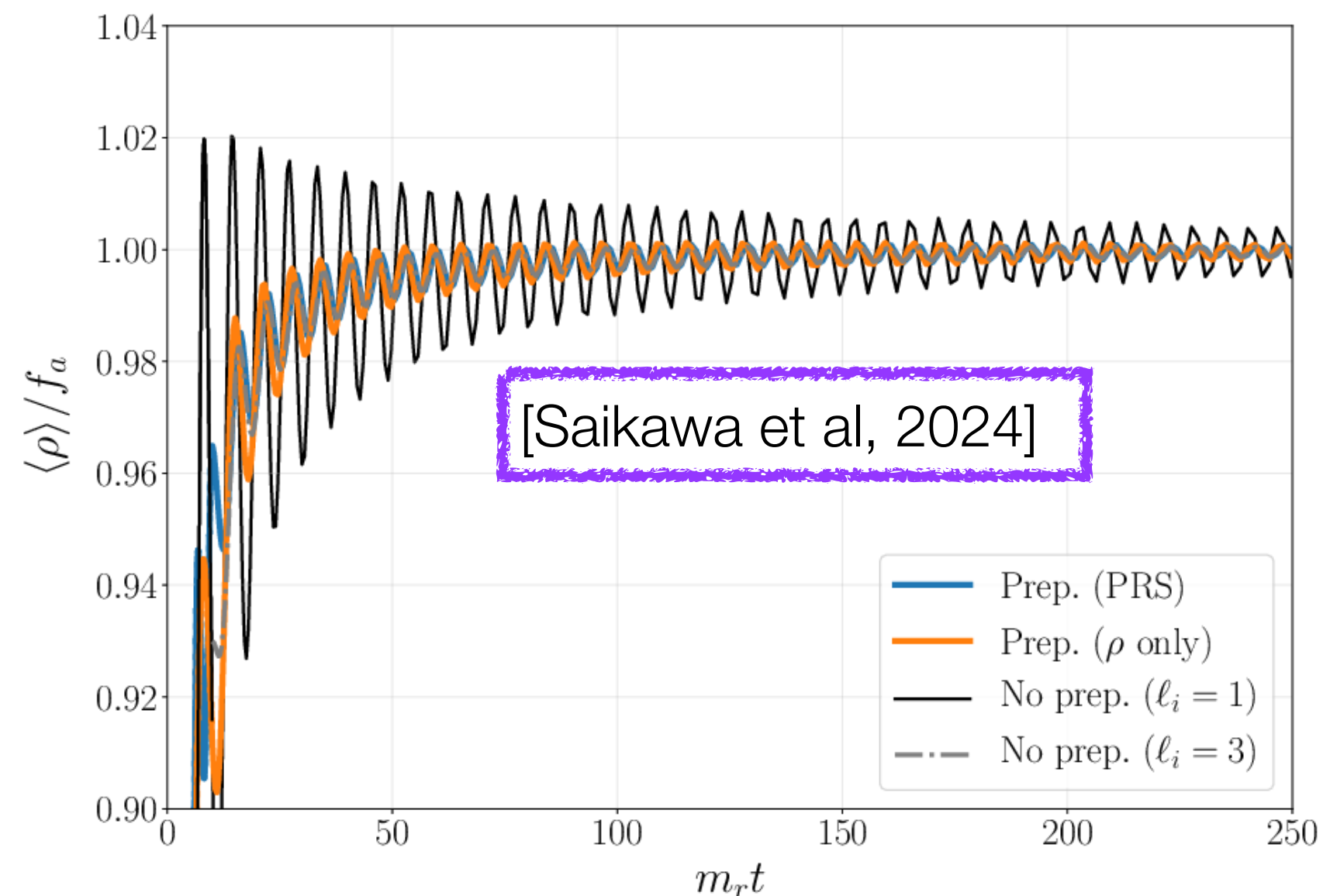
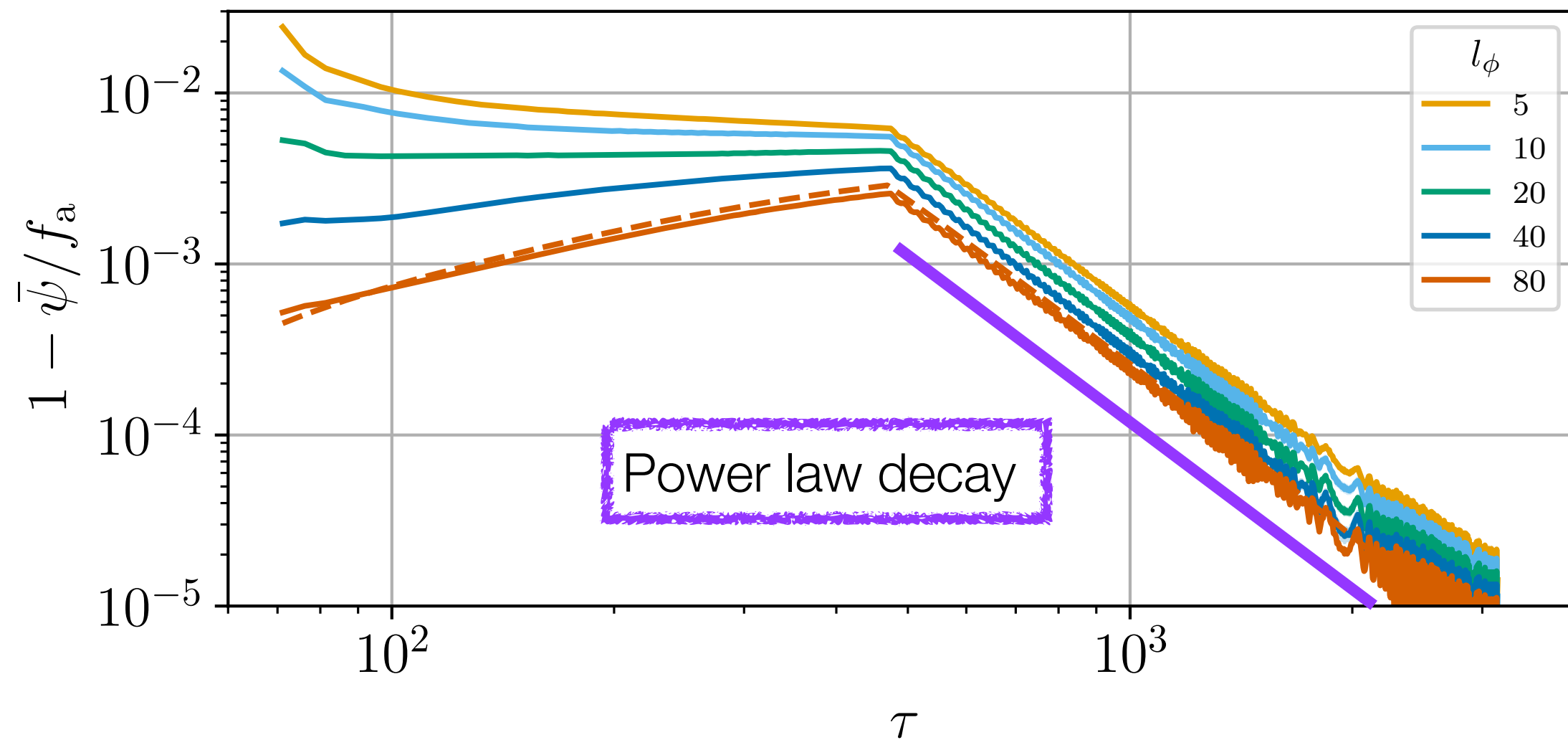
- Choice of observable (self-field)
- Fitting range (not unique)
- Evolution of $\dot{\xi}_W$ (USM prediction)
- Saxion oscillations under control! (dissipation on initial conditions)

- Number density ν_{ax} is a safer choice for extrapolations (cleaner overall)



Improved modeling and control of systematics can lead to better understanding of $\dot{q}$. Extrapolations based on $\dot{q}$ poorly understood, so far $q \approx 1$.

Saxion oscillations - dissipative initial conditions



- Following Gaussian random field Initial conditions,
 - Period of gradient flow (cooling) to remove large gradients
- $$\dot{\phi} - \nabla^2 \phi = \lambda(\phi^* \phi - \eta^2)\phi$$
- Followed by delay of physical evolution (increasing comoving string width period first)
 - Average of saxion field (radial part of ϕ) decays away as a power law in physical evolution
 - Can be understood in terms of departure from ideal string profile (see Correia et al, 2025)
- Higher density initial conditions: higher $\bar{\psi}$ overall
- Far less affected by oscillations than [Saikawa et al., 2024] ($\leftarrow$)