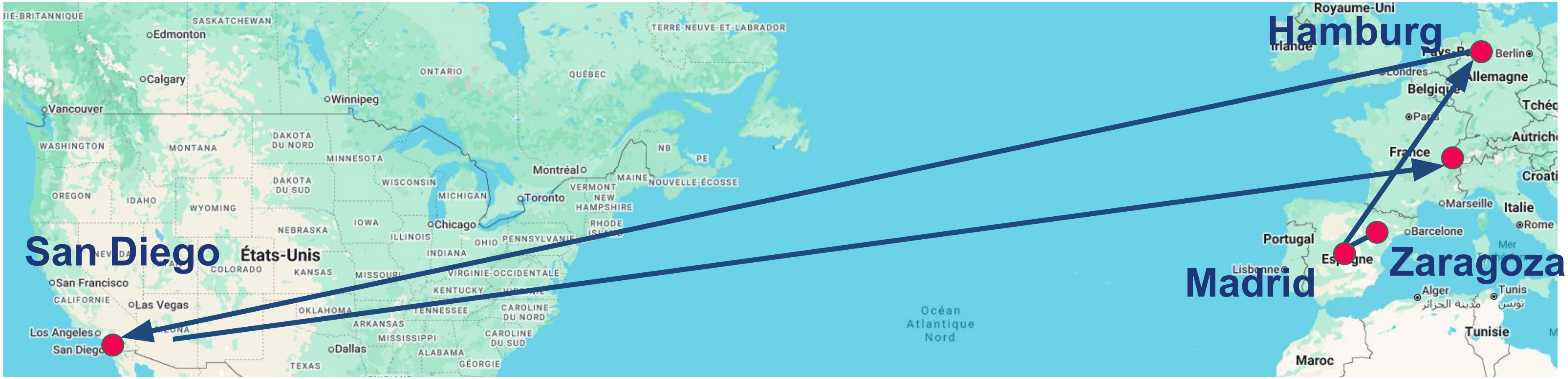
Taming forward-scattering singularities in partial waves: "Is the classical Higgs unitarity bound well-defined?"

Pablo Quílez Lasanta - *pablo.quilez.lasanta@cern.ch*

CERN MSCA fellow

Based on 2510.08784 in collaboration with M. Fuentes and B. Grinstein

Saturnalia 2025, Universidad de Zaragoza



Instituto de Física Teórica UAM-CSIC

UNIVERSIDAD AUTONOMA DE MADRID

DESY.

UNIVERSITY OF CALIFORNIA SAN DIEGO

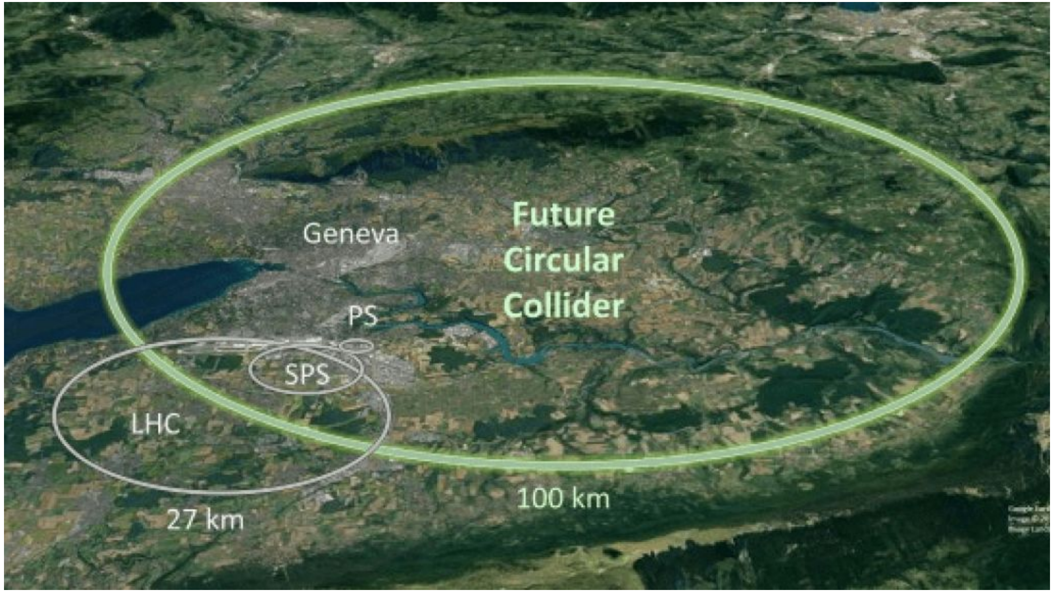
CERN

MARIE CURIE ACTIONS

Pablo Quílez Lasanta

FCC: Future Circular Collider

CERN Council: “The FCC appears technically feasible”



Timeline

- **2025**: Release of the FCC Feasibility Study report
- **2028**: Decision by CERN Member States and international partners

Tunnel

- **90.7 km** circumference
- **180 – 400 m** depths for access shafts
- **8 surface sites** (7 in France, 1 in Switzerland)

Two stages

- **FCC-ee** (precision measurements) about 15 years from the **late 2040s**
- **FCC-hh** (high energy) about 25 years from the **2070s**

Costs/benefits

- **15 billion CHF**, spread over about **12 years** for FCC-ee with four experiments
- **Positive** socio-economic benefit–cost ratio
- About **800 000** person-years of employment created

Motivation for the LHC

1975

A Phenomenological Profile of the Higgs Boson

- First attempt at systematic survey

A PHENOMENOLOGICAL PROFILE OF THE HIGGS BOSON

John ELLIS, Mary K. GAILLARD * and D.V. NANOPOULOS **
CERN, Geneva

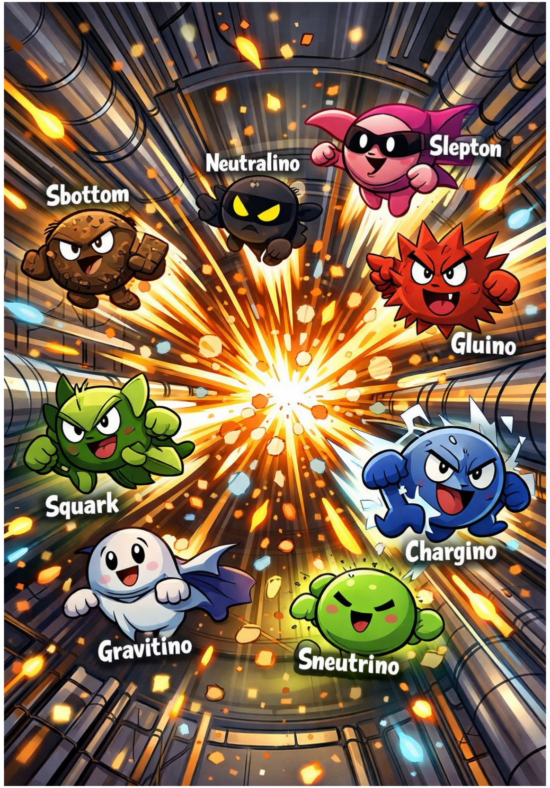
Received 7 November 1975

A discussion is given of the production, decay and observability of the scalar Higgs boson H expected in gauge theories of the weak and electromagnetic interactions such as the Weinberg-Salam model. After reviewing previous experimental limits on the mass of

We should perhaps finish with an apology and a caution. We apologize to experimentalists for having no idea what is the mass of the Higgs boson, unlike the case with charm [3,4] and for not being sure of its couplings to other particles, except that they are probably all very small. For these reasons we do not want to encourage big experimental searches for the Higgs boson, but we do feel that people performing experiments vulnerable to the Higgs boson should know how it may turn up.

Slide taken from a talk by J. R. Ellis
4th International Symposium on the History of Particle Physics

Motivation for the LHC



Perturbative unitarity bound on the Higgs

➔ Often described as a “no-lose argument”

➔ Without Higgs, longitudinal WW scattering grows with s:

$$\mathcal{T}\left(W_L W_L \to W_L W_L\right) \sim \frac{s}{v^2}$$

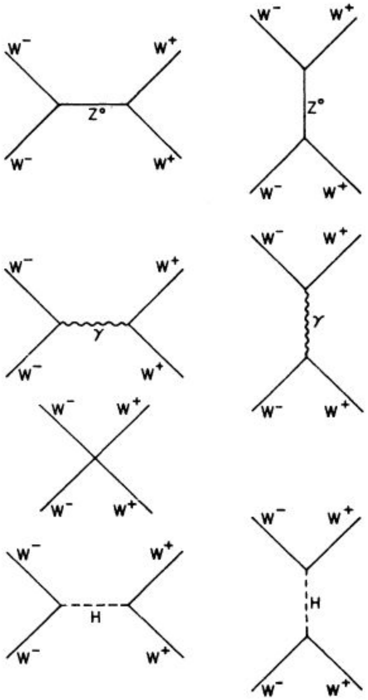


FIG. 1. Feynman graphs (in the unitary gauge) for the reaction $W^+W^- \to W^+W^-$.

Phys.Rev.Lett. 38 (1977) 883-885

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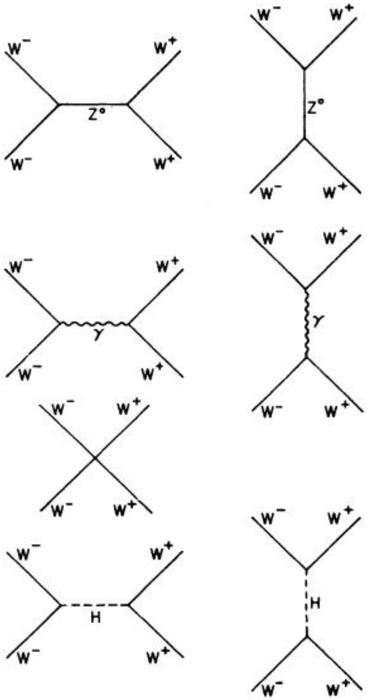
- Perturbative unitarity breaks down

$$\sqrt{s} \sim 1\text{TeV}$$

- A Higgs boson restores unitarity

$$m_h \lesssim \mathcal{O}(1\text{TeV})$$



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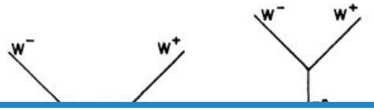


Strength of Weak Interactions at Very High Energies and the Higgs Boson Mass

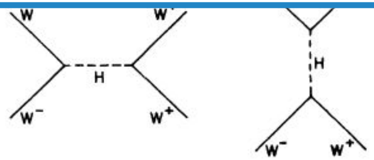
Benjamin W. Lee, C. Quigg,* and H. B. Thacker

Fermi National Accelerator Laboratory, Batavia, Illinois 60510

(Received 28 February 1977)

It is shown that if the Higgs boson mass exceeds $M_c = (8\pi\sqrt{2}/3G_F)^{1/2}$ partial-wave unitarity is not respected by the tree diagrams for two-body reactions of gauge bosons, and the weak interactions must become strong.

$$m_H^2 \leq \frac{8\sqrt{2}\pi}{3G_F} \simeq 1007\ \text{GeV}$$



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Perturbative unitarity bound on the Higgs

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- ➔ Without
- ➔ Perturba
- ➔ A Higgs boson restores unitarity

Either a Higgs boson exists, or new EW dynamics must appear around the TeV scale

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“

Lee-Quigg-Thacker WW partial wave amplitude used to bound the Higgs mass is ill defined

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2510.08784 Fuentes, Grinstein, and **PQ**

Partial wave unitarity

- Probabilities add up to 1 => S-matrix is unitary

$$S^\dagger S = 1$$

$$S = 1 + iT \quad \Longrightarrow \quad T^\dagger T = -i(T - T^\dagger)$$

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➔ Partial-wave expansion

$$a_\ell = \frac{1}{32\pi} \int_{-1}^{1} \mathrm{d}\cos\theta \; P_\ell(\cos\theta) \, \mathcal{T}(\sqrt{s}, \cos\theta) \,.$$

➔ Partial-wave perturbative unitarity bound

$$|a_\ell - i|^2 \leq 1$$

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➔ Partial-wave expansion: s- wave

$$a_0 = \frac{1}{32\pi} \int_{-1}^{1} \mathrm{d}\cos\theta \, \mathcal{T}(\sqrt{s}, \cos\theta)$$

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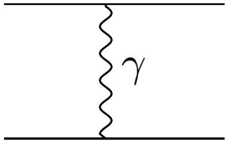
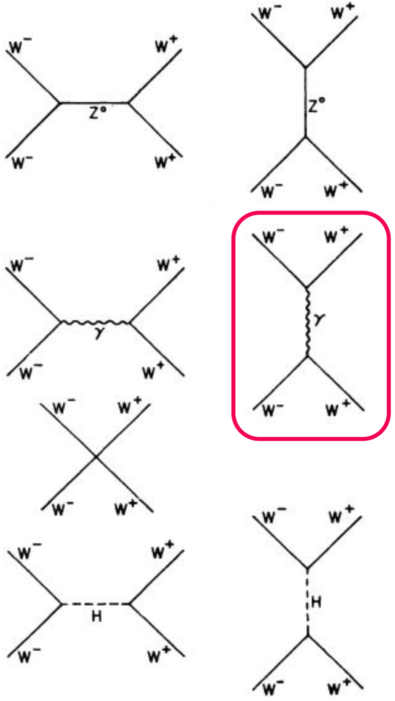
LQT Higgs mass bound

➔ The t-channel photon exchange was not included
(they focused on ∝ m^2_H and 1/p expansion)



$$\mathcal{T}^{(0)} \propto \frac{1}{t} \propto \frac{1}{1-\cos\theta} \,.$$



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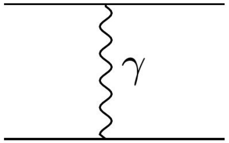
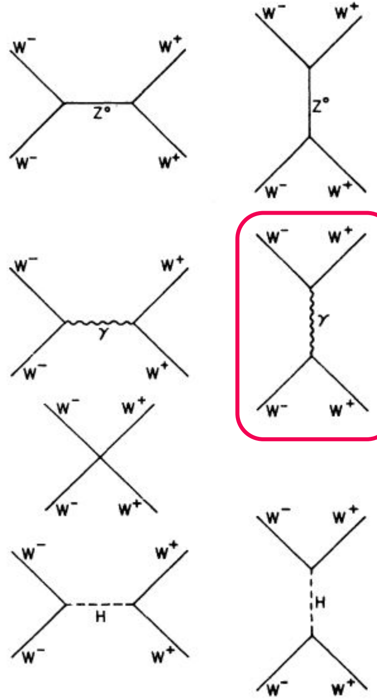


$$\mathcal{T}^{(0)} \propto \frac{1}{t} \propto \frac{1}{1-\cos\theta}\,.$$

➔ T-channel contribution to s-wave diverges!

$$a^0_{ii}\Big|_{\text{Tree}} \propto \int_{-1}^{1} \frac{e^2}{1-\cos\theta} d\cos\theta = -e^2 \lim_{x\to 1} \log(1-x) \longrightarrow \infty$$



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Taming singularities in partial waves

➔ Strong evidence for prescription

$$\mathcal{T}^{(0)} \to \mathcal{T}^{(0)} e^{iW} \qquad iW = \left(\epsilon - i\frac{z_1 z_2 \alpha_{\rm em}}{\beta}\right) \ln\left[(1-\cos\theta)/2\right]$$

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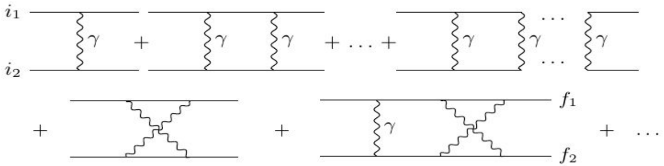
Non-relativistic exact solution

$$f_C(k;\theta) = \left[\frac{\Gamma(1-i\gamma)}{\Gamma(1+i\gamma)} e^{-i\gamma\ln[(1-\cos\theta)/2]}\right] f_C^{(0)}(k;\theta)\,.$$



First order relativistic

$$\mathcal{T}^{(1)} = \frac{z_1 z_2 e^2 \gamma^0}{m^2 + 2p^2(1-\cos\theta)}\left[1 - i\frac{z_1 z_2 \alpha}{p/E}\ln\left(\frac{2p^2(1-\cos\theta)}{m^2}\right)\right]$$

R. H. Dalitz, Proc. Roy. Soc. Lond. A 206, 509 (1951)

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Taming singularities in partial waves

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$$\frac{1}{1-\cos\theta} \longrightarrow \frac{1}{1-\cos\theta} e^{-i\alpha_{em} \log[(1-\cos\theta)/2]}$$

Non-relativistic exact solution

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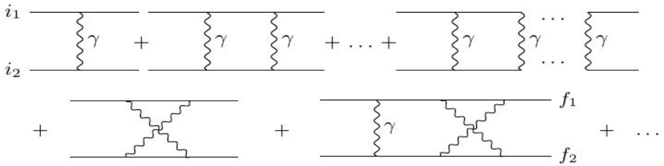

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Quasi-perturbative unitarity bound on m_H

$$a_{ii}^0 \propto \int_{-1}^{1} dx \frac{e^2}{1-x} e^{(\epsilon - i\alpha_{\rm em})\log[(1-x)/2]} = \frac{e^2}{2}\int_{-1}^{1} dx \left(\frac{1-x}{2}\right)^{\epsilon - i\alpha_{\rm em} - 1} \xrightarrow{\epsilon\to 0} \frac{ie^2}{\alpha_{\rm em}} = 4\pi i$$

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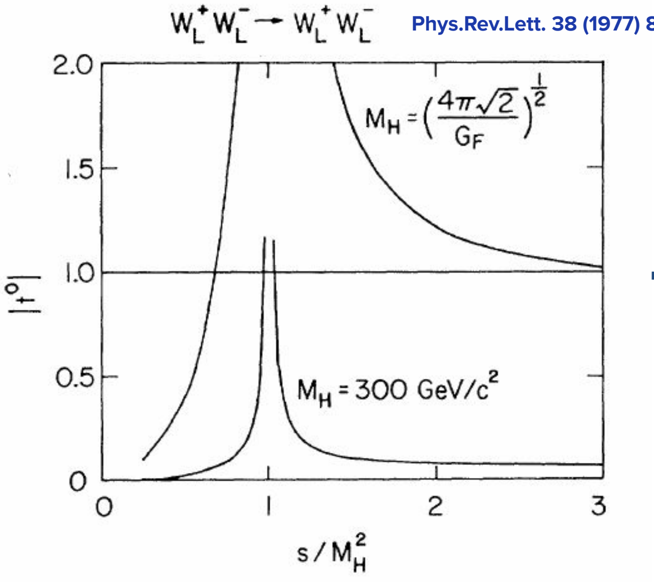

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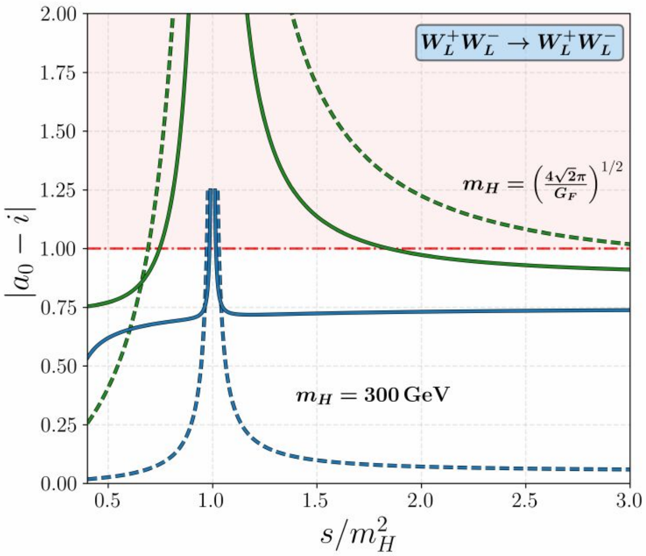

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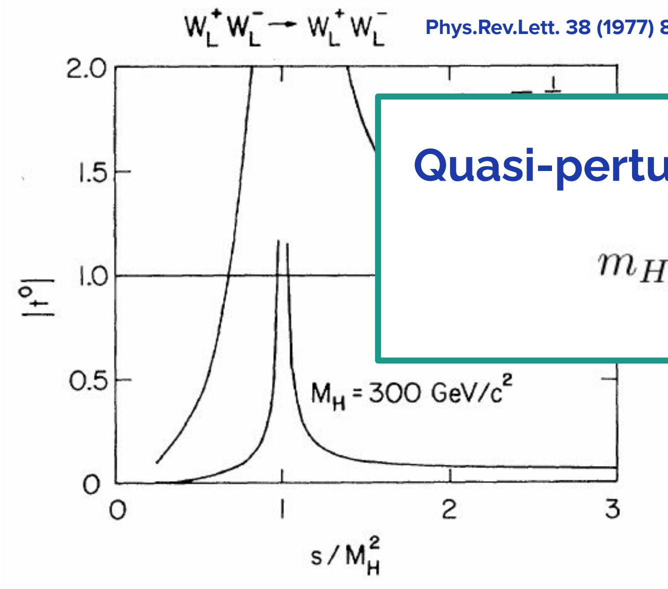
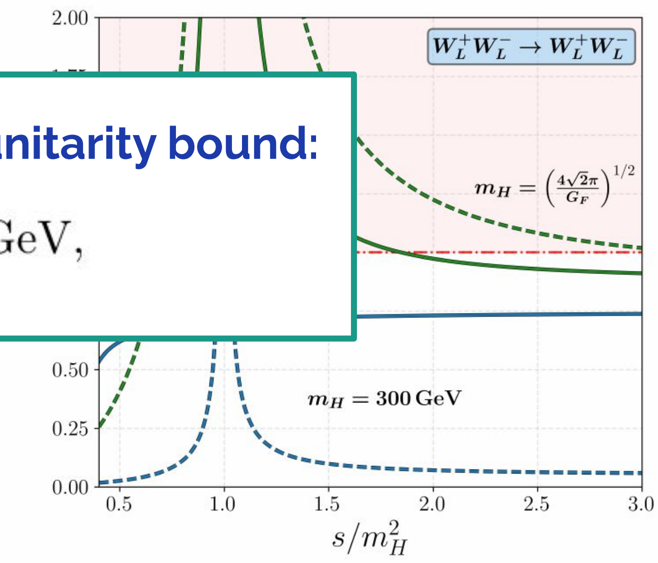


Quasi-perturbative unitarity bound:

$$m_H \lesssim 1420 \text{ GeV},$$



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What is next?

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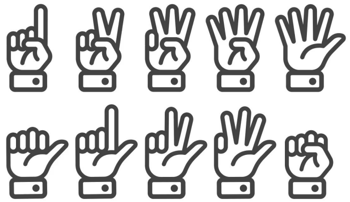
- ➔ **Prove it!** with Ryan Plestid
- ➔ **SMEFT** with B. Grinstein and M. Fuentes-Zamoro
- ➔ Other EFTs with light states: ALP EFT, dark photon, ...

Hilbert series



- ➔ Extremely useful and powerful tool to **count stuff**
- ➔ Hilbert series for covariants: ***Minimal Flavor Violation SMEFT = SMEFT***

  JHEP **2312.13349** Grinstein, Lu, Merlo, **PQ**

- ➔ **Accidental symmetries** (+ friendship relation among groups) → **Fermions**

  JHEP **2412.05359** Grinstein, Lu, Miró, **PQ**

- ➔ Operators in EFTs (solved* problem) → **Cohomology operators** ↔ **Generalized symmetries**

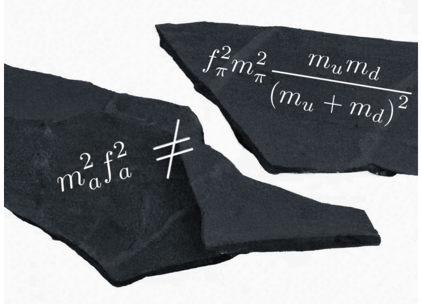
  See papers by Xiaochuan Lu

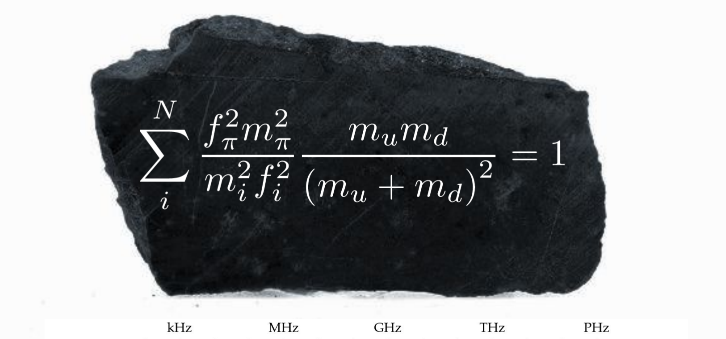
- ➔ If you need to count sth, talk to me!

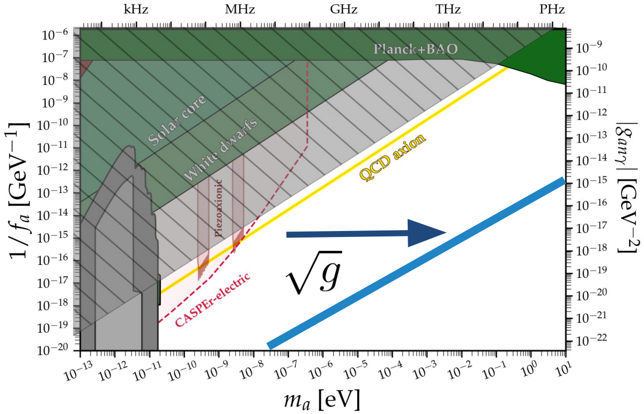
Multiple axions: A sum rule

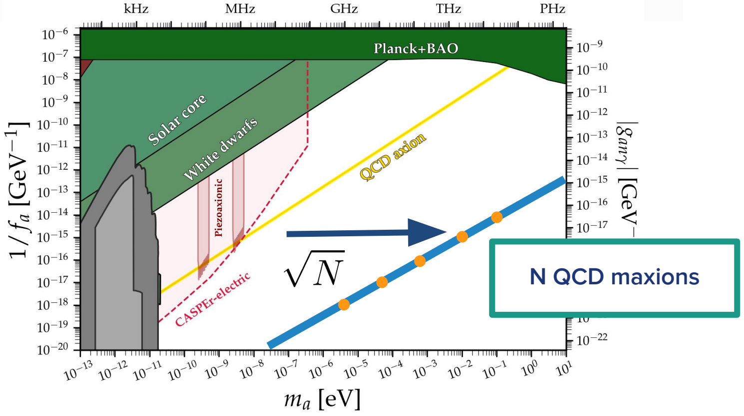
$$m_i^2 f_i^2 \equiv f_\pi^2 m_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2} \times g_i$$

$$\sum_{i=1}^{N} \frac{1}{g_i} = 1$$



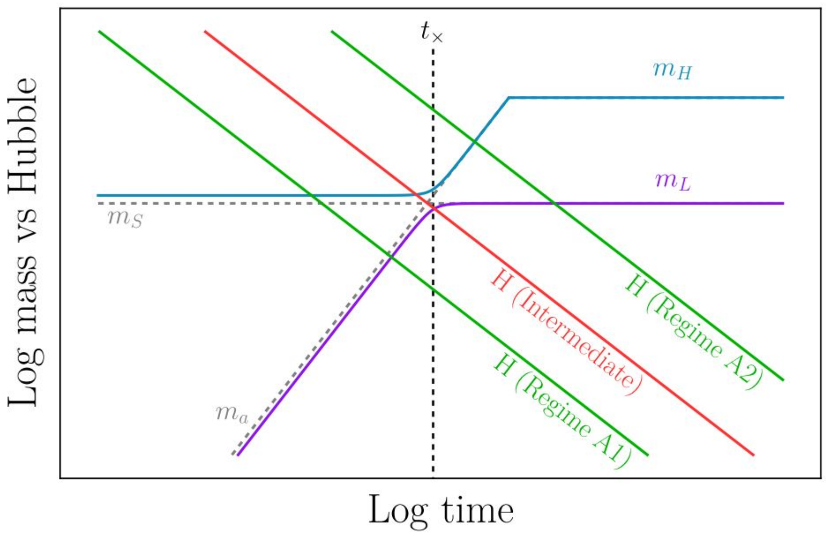


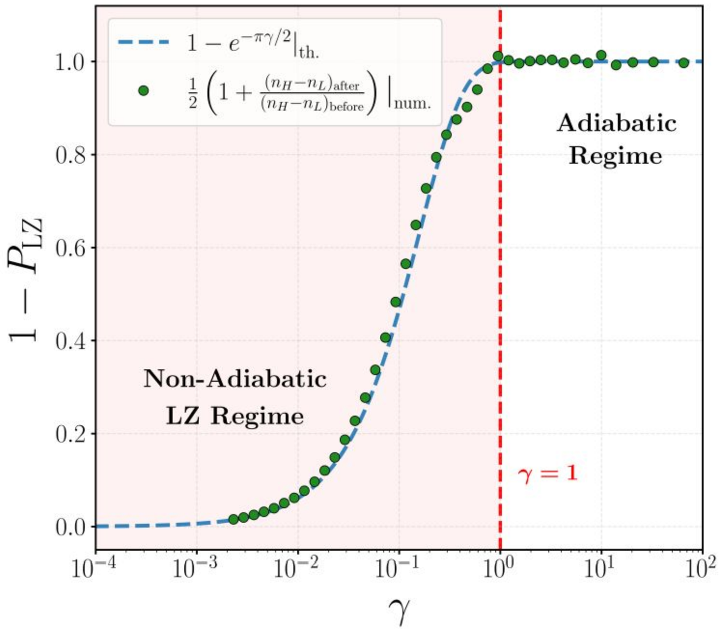




Based on **2305.15465** - Pablo Quílez

Multiple axion DM: Landau-Zener





Conclusions

- ➔ Partial-wave perturbative unitarity bounds become ill-defined when a massless particle is exchanged in the t-channel.
- ➔

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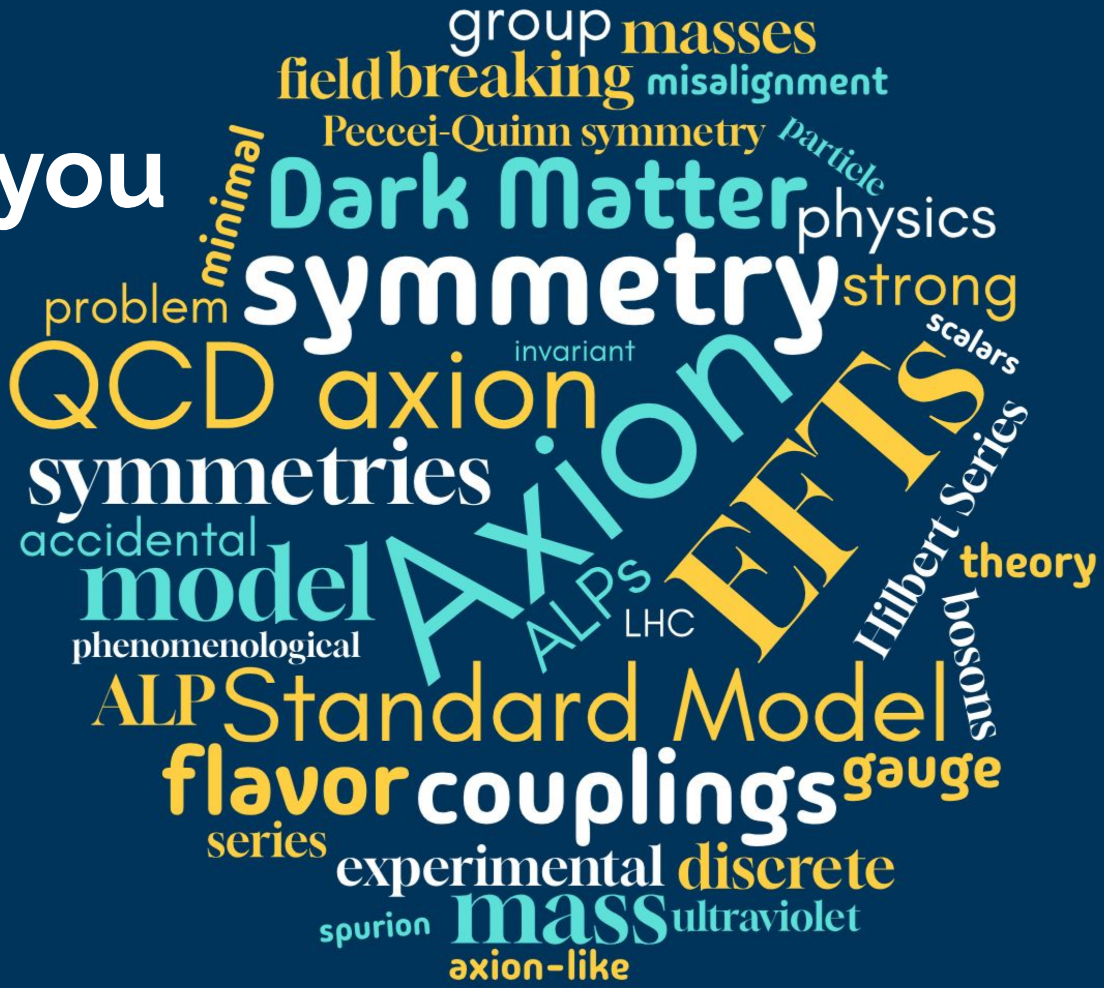
➔ **Hilbert Series** are cool

➔ **AxionS** are too

Thank you


group
masses
field
breaking
misalignment
Peccei-Quinn symmetry
particle
minimal
Dark Matter
physics
symmetry
strong
problem
scalars
invariant
QCD axion
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