Many-body wavefunctions from conformal field theory

Adrián Franco Rubio

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UNIVERSIDAD COMPLUTENSE MADRID

ICMAT INSTITUTO DE CIENCIAS MATEMÁTICAS

Time to blow my cover

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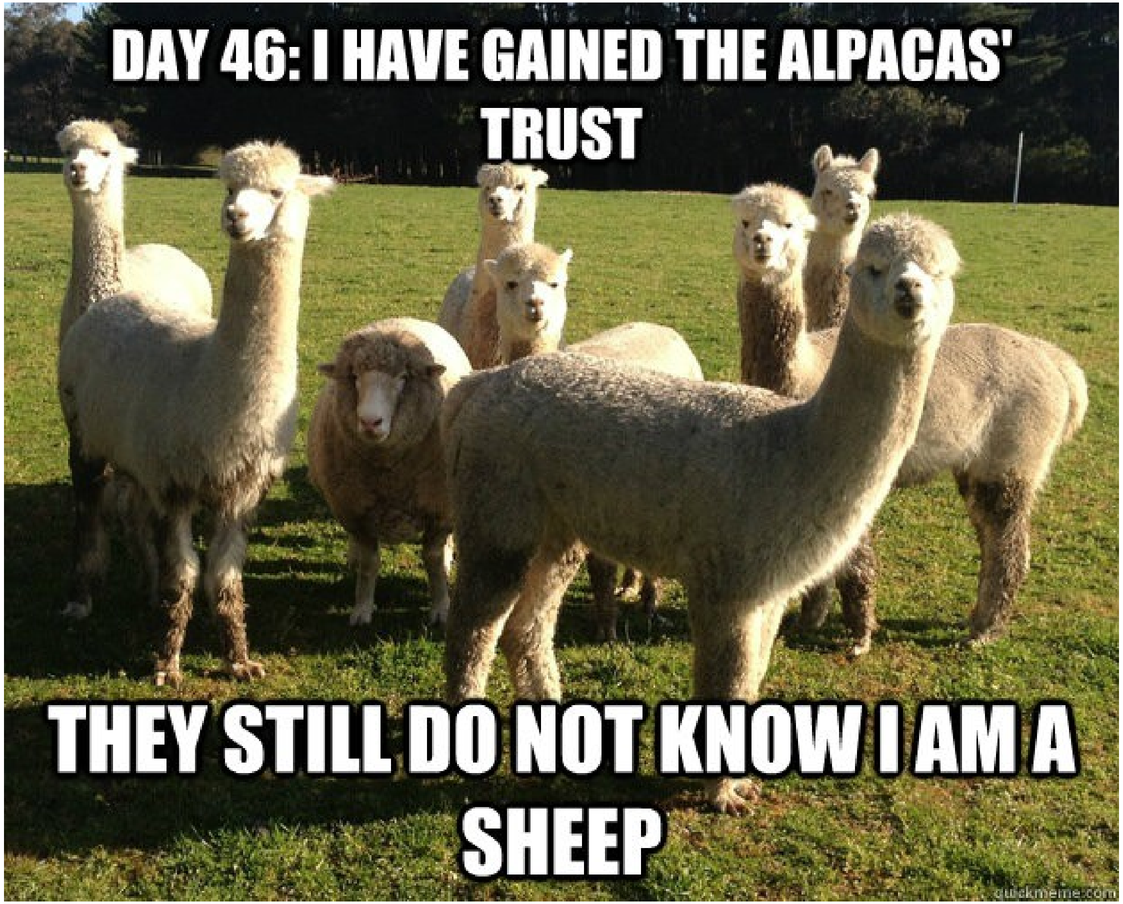
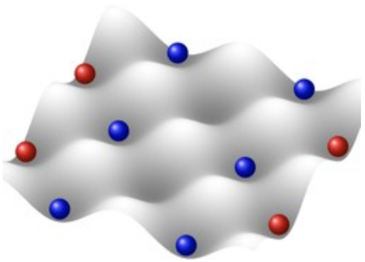


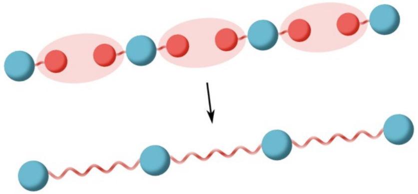
Time to blow my cover

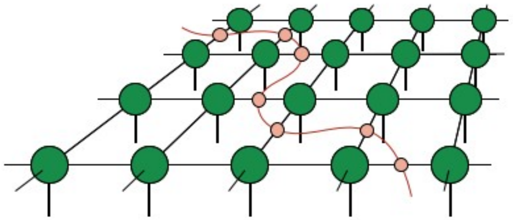
What does a quantum many-body theorist think about?

Wavefunctions on large Hilbert spaces

$$|\psi\rangle \in \mathcal{H} = (\mathbb{C}^d)^{\otimes n}$$

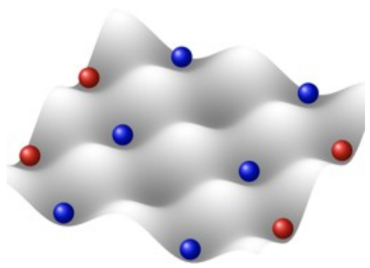






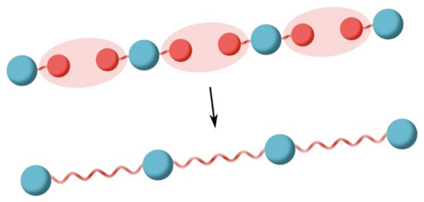
What does a quantum many-body theorist think about?

Wavefunctions on large Hilbert spaces

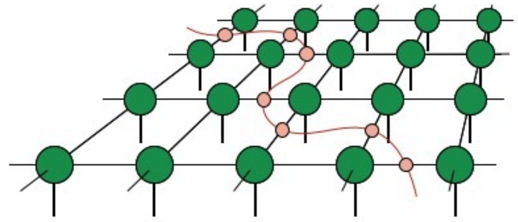


$$|\psi\rangle \in \mathcal{H} = (\mathbb{C}^d)^{\otimes n}$$

- *What is the structure of correlations and entanglement?*
- *What phase of matter does it belong to?*



- *Is it the ground state of a physical Hamiltonian?*
- *Does it have symmetries?*
- *What other states can it be turned into?*



- *Is it well-defined in the thermodynamic limit?*
- *Why did I not go into industry?*
- ...

What to talk about?

- Gauging globally symmetric quantum many-body states

arXiv:2502.20257

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- Building many-body wavefunctions from (conformal) QFT arXiv:2507.22735

Joint work with G. Sierra and J. I. Cirac

What to talk about?

- Gauging globally symmetric quantum many-body states arXiv:2502.20257
- Building many-body wavefunctions from (conformal) QFT arXiv:2507.22735

Joint work with G. Sierra and J. I. Cirac

Let’s play with some toy(model)s!

Image credit: Ralf Roletschek

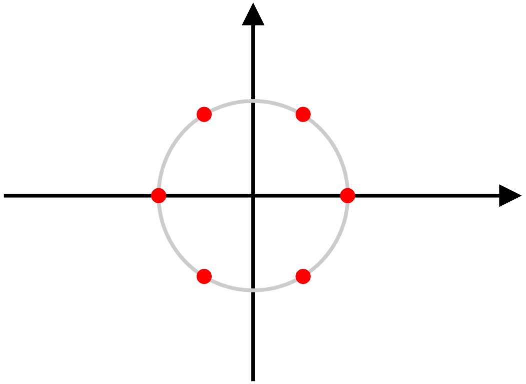
Many-body wavefunctions from a QFT

Consider a quantum state of *n* spins

$$|\psi\rangle = \sum_{s_1 \dots s_n} \psi_{s_1 \dots s_n} |s_1 \dots s_n\rangle$$

and define the wavefunction coefficient in terms of a QFT correlator

$$\psi_{s_1 \dots s_n} := \langle \phi^{s_1}(\vec{x}_1) \dots \phi^{s_n}(\vec{x}_n) \rangle$$



Many-body wavefunctions from a CFT

We'll use **Wess-Zumino-Witten models** for a few reasons

- Conformally invariant and exactly solvable (free field representations)
- Fields fall naturally into spin multiplets
- Plenty of symmetries

Many-body wavefunctions from a CFT

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OK, but *why* do this?

- Long tradition starting from (electron) ansatz wf's for the **fractional quantum Hall effect**

[Laughlin '83]
[Moore, Read '91]

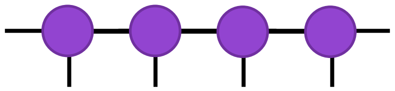
- Can be seen as an infinite dimensional generalization of **matrix product states (MPS)**

[Cirac, Sierra '09]

Matrix Product States (MPS)

[Fannes et al., 91]

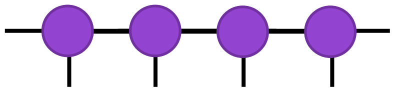
- MPS are a star of modern-day quantum many-body physics



$$\psi_{s_1 \dots s_n} := \mathrm{tr}(A_1^{s_1} \dots A_n^{s_n}) \qquad A_i^{s_i} \in \mathrm{Mat}(\mathbb{C}, D \times D)$$

Matrix Product States (MPS)

[Fannes et al., 91]



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- They provide an ansatz family for **low-entangled 1d** many-body states
- At the core of flagship numerical algorithms (DMRG) and a lot of mathematical insights!
- Some exactly solvable models have MPS ground states

$$H_{\mathrm{MG}} = \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{2} \vec{S}_i \cdot \vec{S}_{i+2} \right)$$

Majumdar-Ghosh chain (spin 1/2)

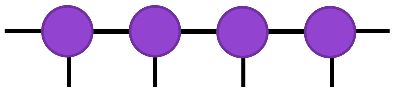
$$H_{\mathrm{AKLT}} = \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} (\vec{S}_i \cdot \vec{S}_{i+1})^2 \right)$$

AKLT chain (spin 1)

Matrix Product States (MPS)

[Fannes et al., 91]

- MPS are a star of modern-day quantum many-body physics



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- **Bond dimension**

We'll make this connection in a cleaner fashion...

$D = 1$

(product states)

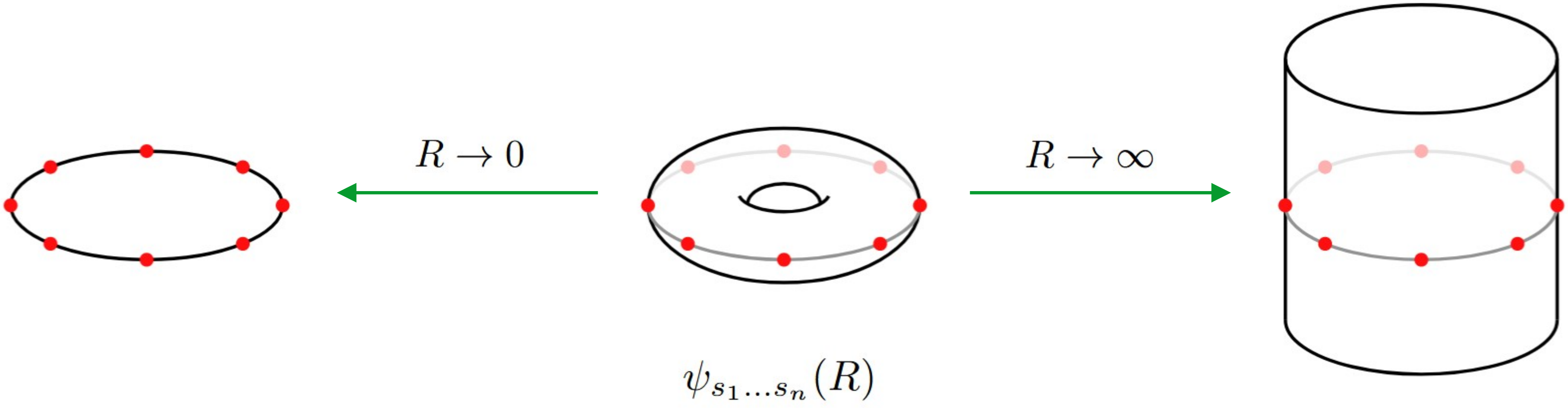
$D \to \infty$

idMPS

$$\psi_{s_1 \ldots s_n} := \langle \phi^{s_1}(\vec{x}_1) \ldots \phi^{s_n}(\vec{x}_n) \rangle$$

idMPS on a torus

[Bergholtz, Karlhede '05]
[Bernevig, Regnault '12]



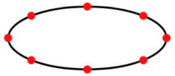
Finite bond dimension MPS

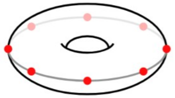
Finite correlation length
(gapped)

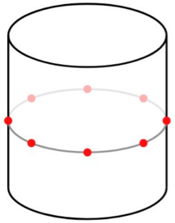
idMPS (equivalent to the plane configuration)

Infinite correlation length
(gapless/critical)

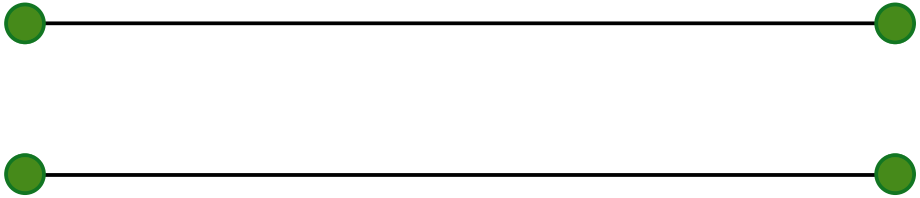
Example: $SU(2)_1$





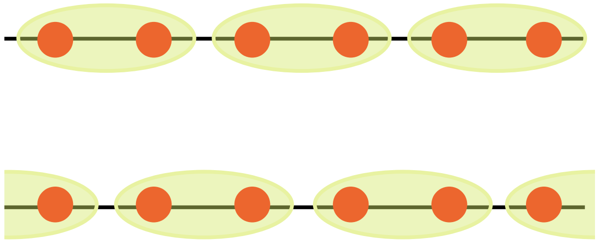


$R \to 0$ $R \to \infty$



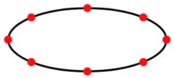
Ground states of the Majumdar-Ghosh chain (dimer coverings)

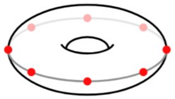
Ground states and first excited spin singlet of the Haldane-Shastry chain

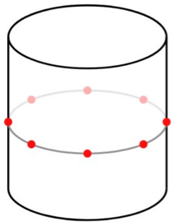


$$H_{\mathrm{HS}} = \sum_{i<j} \frac{\vec{S}_i \cdot \vec{S}_j}{\sin^2\left(\pi(i-j)/n\right)}$$

Example: $SU(2)_2$

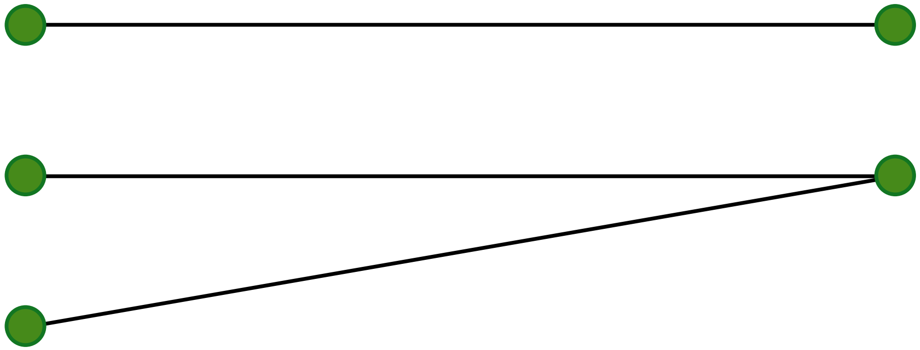






$R \to 0$ $R \to \infty$

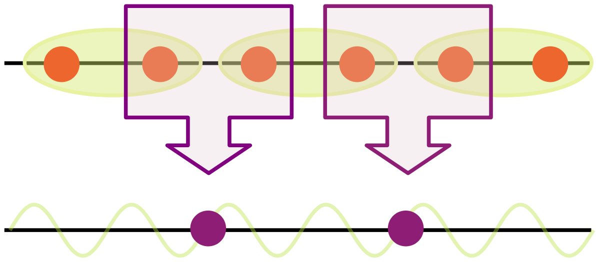
Ground states of the S=1 Majumdar-Ghosh chain



Ground states and first excited spin singlet of an unnamed spin-1 chain

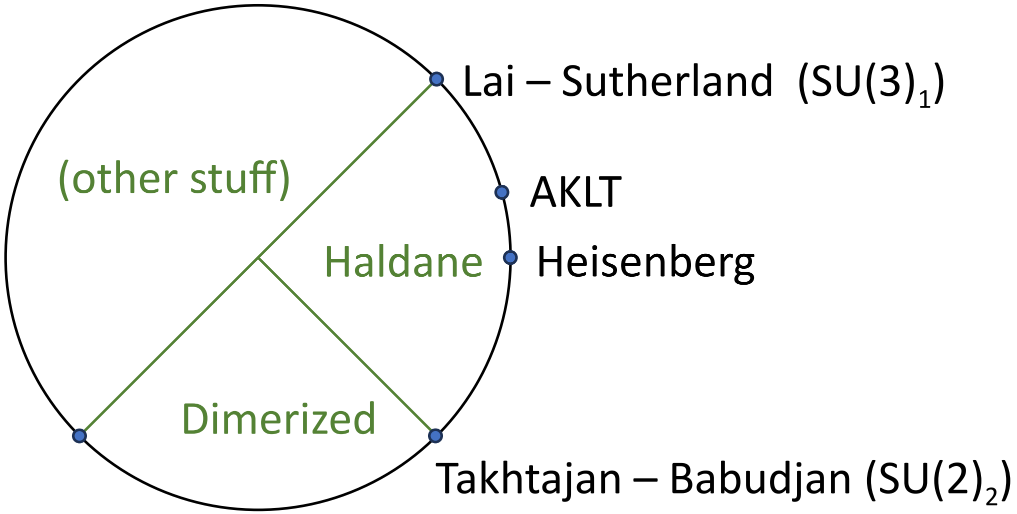
AKLT

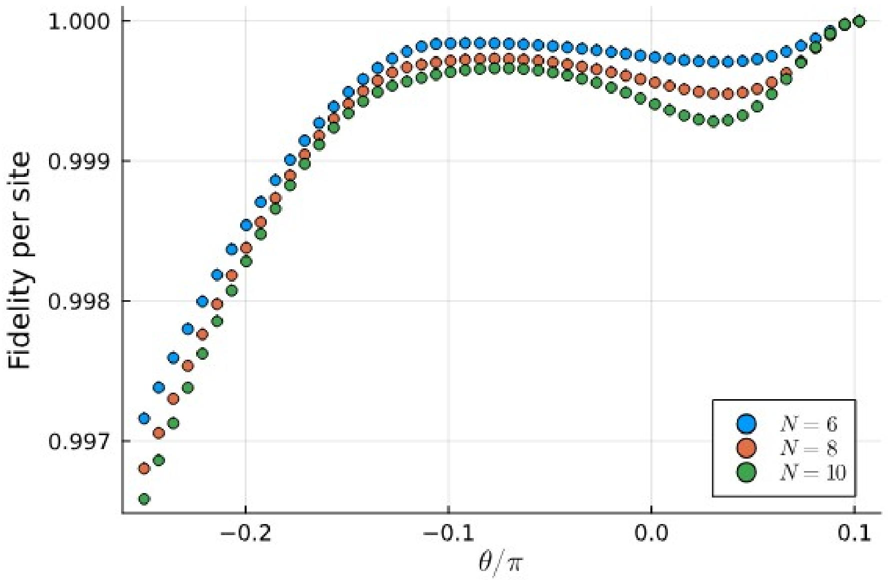
$$H_{\mathbf{AKLT}} = \sum_i \left(\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{3} (\vec{S}_i \cdot \vec{S}_{i+1})^2 \right)$$



Quadratic-biquadratic model

$$H = \sum_i \cos\theta\, (\vec{S}_i \cdot \vec{S}_{i+1}) + \sin\theta\, (\vec{S}_i \cdot \vec{S}_{i+1})^2$$





Open questions

- Other WZWs give rise to models still known from literature
  - $SO(n)_1$ – generalization of AKLT
- Intermediate states are to be understood
- Higher genus configurations can give rise to mixed states
- Interpolation of symmetry structures (group to Hopf algebra)

Bonus: Gauging quantum states

- Systematic procedure to map a globally symmetric quantum many-body state into a locally symmetric one.

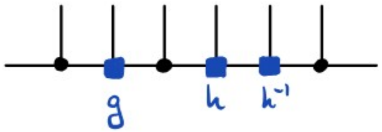
$$G|\psi\rangle = |\psi\rangle \implies \mathcal{G}_j|\psi^G\rangle = |\psi^G\rangle \quad \forall j$$

[Haegeman et al., ‘15]

- Can be interpreted as the promotion of defects (symmetry twists) to quantum degrees of freedom.

[Seifnashri, ‘23]

- Tensor networks (MPS and generalizations) provide a natural framework for its implementation.



- Symmetries can be not-on-site and noninvertible.

[arXiv:2502.20257]
[Vancraeynest-De Cuiper et al., ‘25]

Thanks!