Lattice calculations
(Or how to do QFTs, the lazy way)

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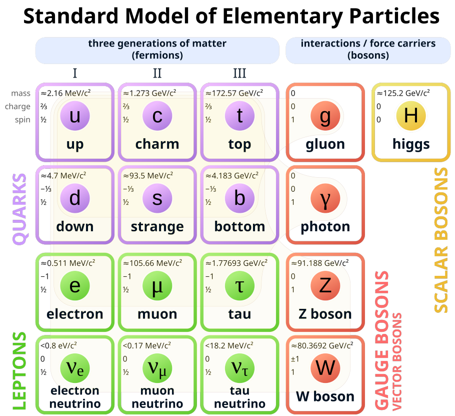
April 10^{th}, 2026

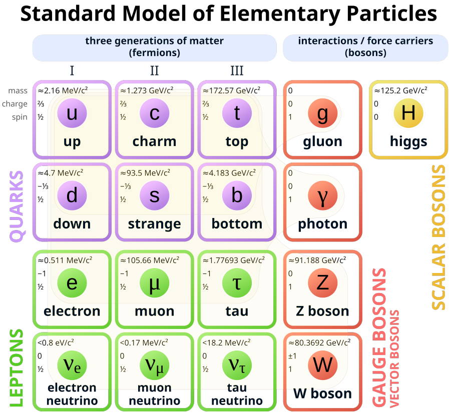
The motivation

Motivation: The Standard Model of Particle Physics



- Three interactions
  - Electromagnetism (Quantum Electro Dynamics, QED)
  - Weak (with QED makes the Electroweak theory, EW)
  - Strong (Quantum Chromo Dynamics, QCD)
- Seventeen particles
  - Six quarks (strong interaction)
  - Six leptons
  - Four bosons force carriers
  - The Higgs
- Whole particle content of known nature

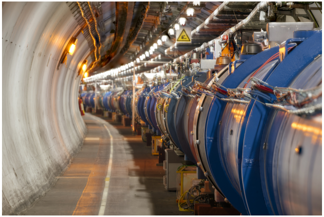
Motivation: The Standard Model of Particle Physics



- Best theory to describe nature
  - (Most of it)
- Incredible precision and accuracy
  - a_e agrees within 1 ppt (10^{-12})
- Incomplete, many unknowns
  - Where is gravity?
  - Why three families?
  - So many inputs
  - ...

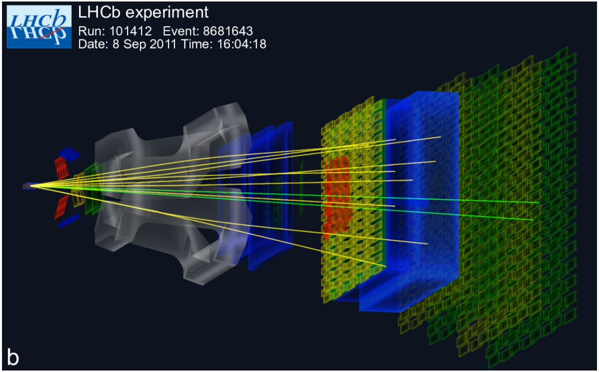
Motivation: Frontiers in physics

Energy frontier



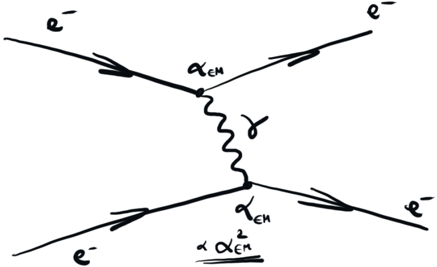
Cosmology frontier

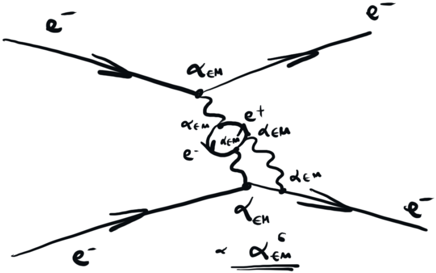
Intensity frontier



- High expectations with the LHC
- Intensity frontier becoming increasingly important

Motivation: Calculations in QFTs



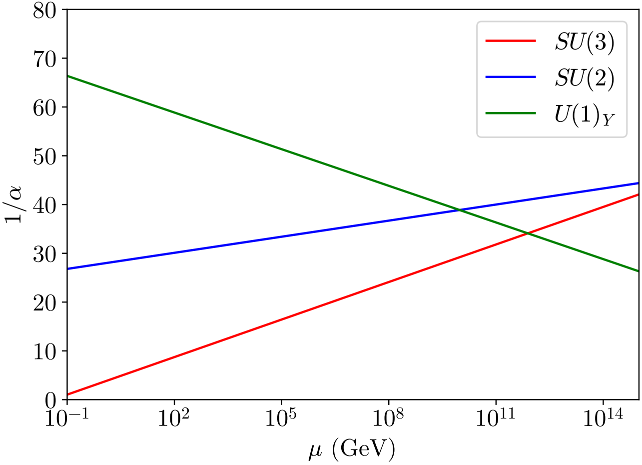


- If the coupling $\alpha << 1$, expand your observable

$$f = f_0 + \alpha f_1 + \alpha^2 f_2 + \alpha^3 f_3 + \dots$$

- Each one of the f_j can be represented as a process (Feynman diagram)

Motivation: The running of the coupling constants



- Coupling "constants" not constant

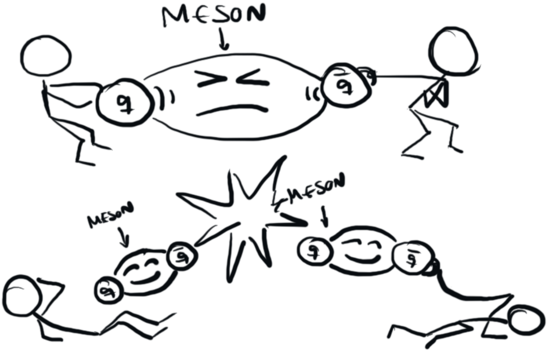
The strength of the interactions depends on the scale

Motivation: The strong interaction and QCD

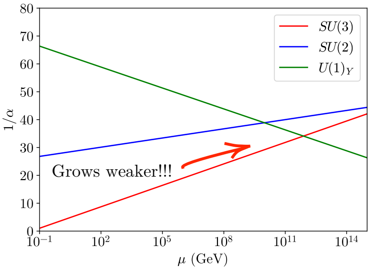
- Quarks interact to build **hadrons**
  - Baryons: three quarks
  - Mesons: quark-antiquark pair
- Two amazing properties
  - **Confinement:** can't see free quarks at low energies
  - **Asymptotic freedom:** at high energies the strong interaction is weak

Motivation: Confinement and Asymptotic freedom

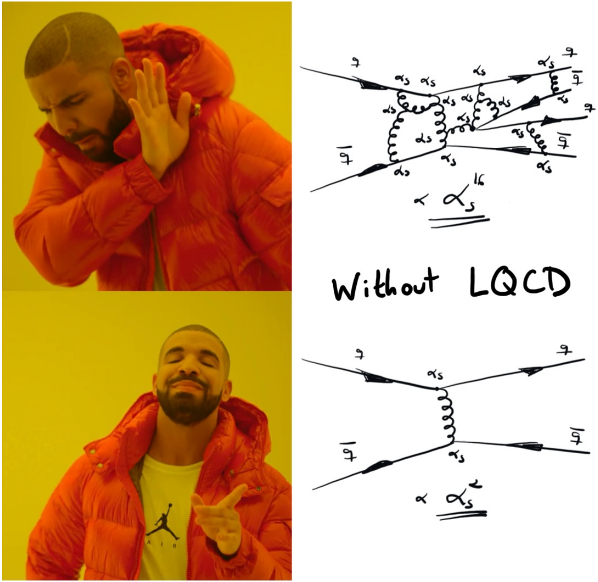
Confinement



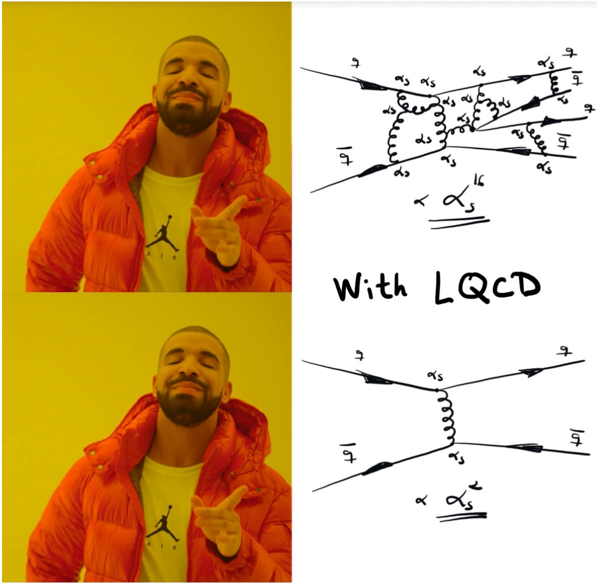
Asymptotic freedom



Motivation: Perturbation theory

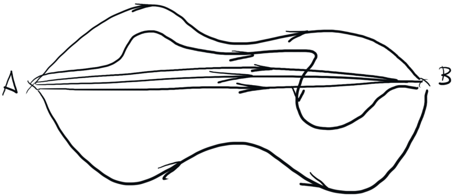


Motivation: Perturbation theory



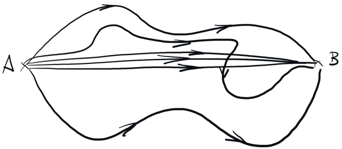
What is the lattice?

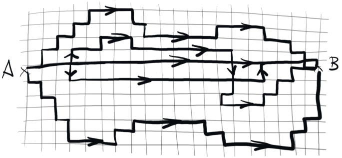
The lattice: The path integral



- A particle moving from A to B probes **all possible trajectories**
- Each trajectory has an associated phase e^{iS}
- Classical trajectories **interfere constructively** and usually prevail
- Other trajectories **interfere destructively** and do not contribute

The lattice: The path integral

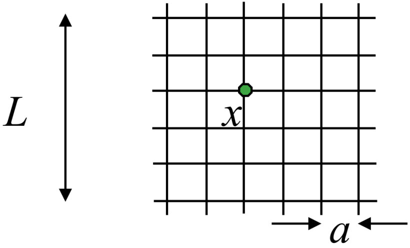


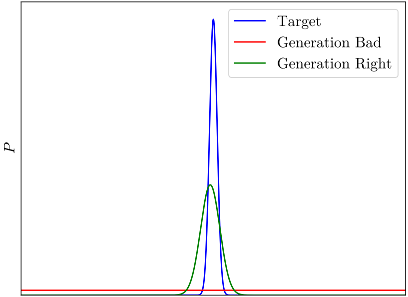


- A discrete spacetime allows to explore all possible trajectories
- The lattice spacing a is just a regulator in a QFT (cutoff $\sim \frac{1}{a}$)
- Wick rotation transform phases into positive weights $e^{iS} \to e^{-S}$

Statistical mechanics	**Lattice QFT**
$Z_\beta = \sum_\sigma e^{-\beta E(\sigma)}$	$Z = \int DU e^{-\beta S(U)}$
$\langle \sigma(x)\sigma(y)\rangle \propto e^{-\frac{\lvert x-y\rvert}{\xi}}$	$\langle O(x)O^\dagger(y)\rangle \propto e^{-\lvert x-y\rvert m}$
Each $\{\sigma\}$ is a possible configuration of the system	Each $\{U\}$ is a path in the path integral

The lattice: The path integral





- Discretize spacetime + Finite volume = Finite number of dof
  - Perform the path integral sum in a computer
  - Montecarlo + Importance sampling

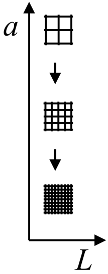
The lattice: From LQCD to QCD

- QCD discretized action (Symanzik program)

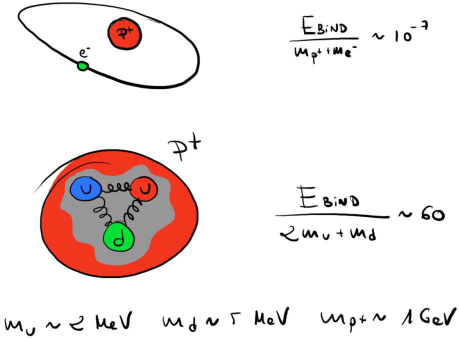
$$S_{\mathrm{QCD}} = S_0 + aS_1 + a^2 S_2 + \dots$$

- Systematically improvable
- Control over the systematic errors

- Several extrapolations taken at the same time
  - Continuum limit $a \to 0$
  - Chiral (inter)extrapolation $m_\pi^{\mathrm{LQCD}} \to m_\pi^{\mathrm{Phys}}$
    - Light quarks used to be expensive
  - Heavy quark extrapolation $m_b^{\mathrm{LQCD}} \to m_b^{\mathrm{Phys}}$
    - The b quark hits the regulator

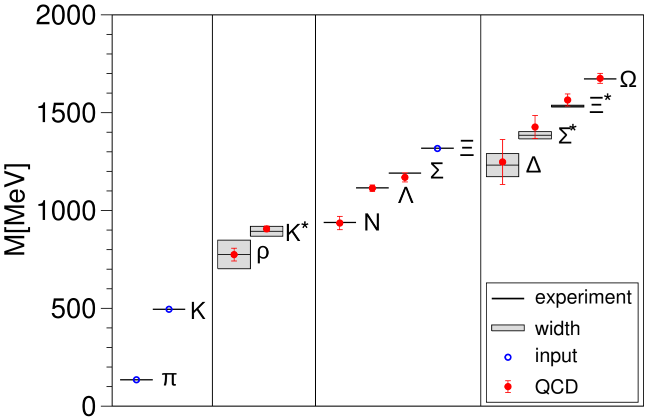


Some results



- Hadron masses come from QCD, not from constituent quarks!
  - $p^+ = uud, \quad m_u \approx 2$ MeV, $m_d \approx 5$ MeV, $m_p \approx 1$ GeV
  - Even with massless quarks, the proton is massive
  - Mass generation in hadrons is a non-perturbative effect of QCD

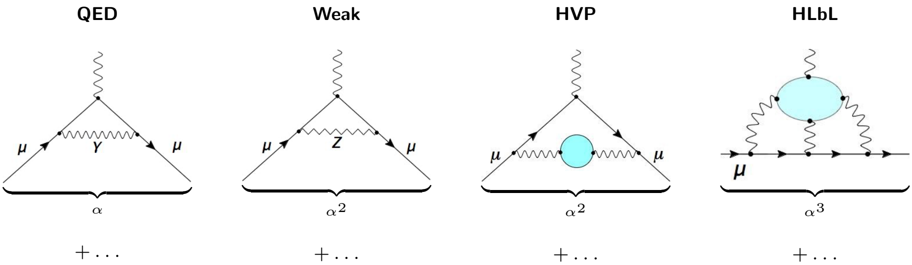
Lattice QCD results: QCD Spectrum



BMW 2008

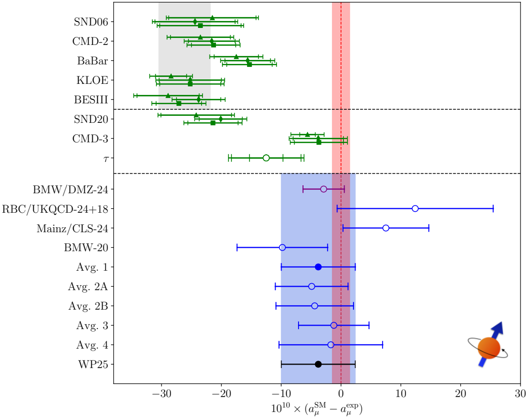
- Lattice QCD can reproduce the spectrum of known hadrons up to a few percent
- Starting to consider QED effects (difference between n^0 and p^+)

Lattice QCD results: HVP



- Muon a_μ considered a possible venue to find new physics
- QED and EW contributions computed perturbatively
- Missing ingredient: Hadronic Vacuum Polarization (HVP) effects

Lattice QCD results: HVP

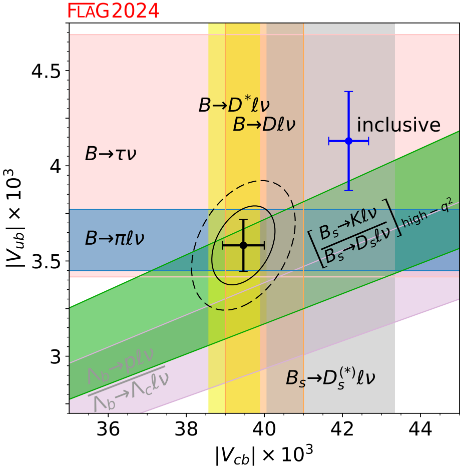


Theory Initiative 2025

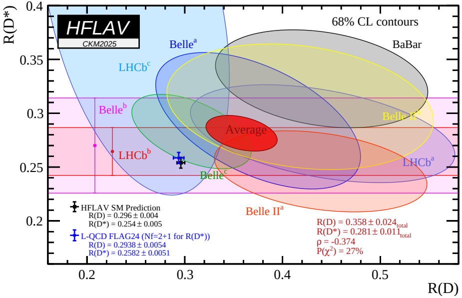
- Latest LQCD results agree with experiment
- Statement on the precision of the Standard Model

Lattice QCD results: Flavor Physics

Tensions inclusive vs exclusive CKM



Tensions in LFU ratios



Lattice can help with both

Lattice QCD results: Flavor Physics

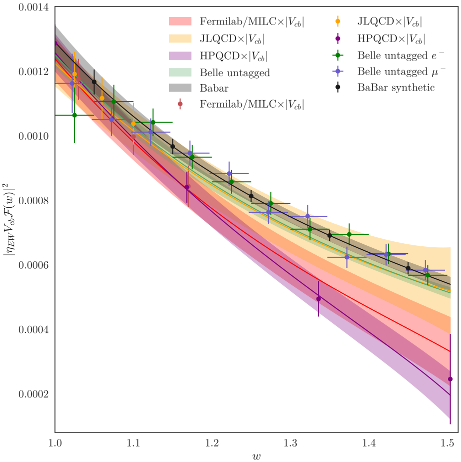
- $V_{\rm CKM}$ is an input of the SM
- Semileptonic decays allow for clean exclusive extractions

$$M_P \to M_D \ell_1 \ell_2$$

  - Efficient enough in experiments
  - Hadronic effects under control

$$\frac{d\Gamma}{dw} = K(w)\,|\mathcal{F}(w)|^2\,|V_{\rm CKM}|^2$$

- Decay amplitude $\mathcal{F}(w)$ comes from LQCD



Fermilab-MILC 2022, JLQCD 2024, HPQCD 2024, Belle 2019, BaBar 2019

Lattice QCD results: Flavor Physics

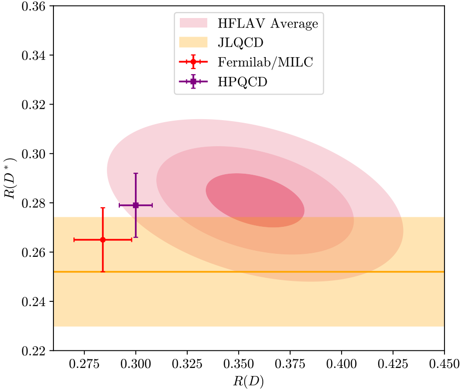
- If the form factors are known, integrate to extract Γ

$$R(X) = \frac{\Gamma(M_P \to M_D \ell_1^{\text{heavy}} \ell_2^{\text{heavy}})}{\Gamma(M_P \to M_D \ell_1^{\text{light}} \ell_2^{\text{light}})}$$

- The CKM matrix element cancels out in LFU ratios

$$R(X) = \frac{\int dw\, K_{\ell_h}(w)\, |\mathcal{F}_{\ell_h}(w)|^2\, |\cancel{V_{\text{CKM}}}|^2}{\int dw\, K_{\ell_l}(w)\, |\mathcal{F}_{\ell_l}(w)|^2\, |\cancel{V_{\text{CKM}}}|^2}$$

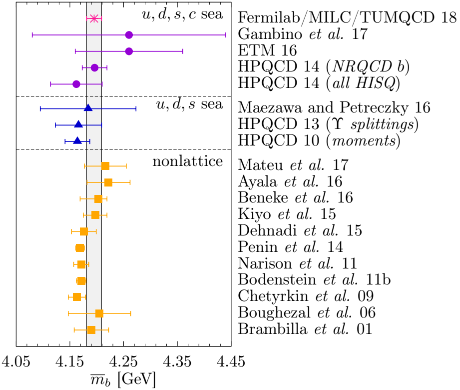
- Pure first principles determination to compare with experiment



Fermilab-MILC 2022, JLQCD 2024, HPQCD 2024, HFLAV 2025

Lattice QCD results: Quark masses

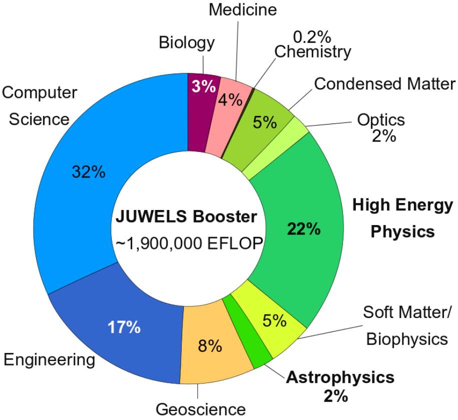
- Confinement prevents us from measuring directly m_q
- In LQCD we input bare masses, unphysical
- After renormalization (choose your scheme), m_q^R
  - QCD spectrum must match our result
- Bare mass depends on the fermion regularization
  - We know how to connect m_q in the lattice to m_q^R



Fermilab-MILC 2018

The cost

The lattice is expensive

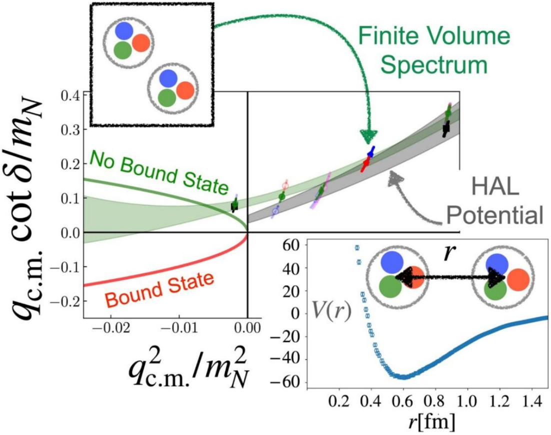


LQCD major user of HPC resources

- Usage of top HPC facilities ranges from $\approx 5\%$ to $\approx 40\%$
- Some Key Performance Parameters in HPC designs are based on LQCD
- Early access, close work with industry
  - NDAs, especialized teams, etc
- Run on any architecture
  - CPUs vectorized
  - GPUs since they were out
  - ARM low-power clusters
- Very high performant codes (e.g. QUDA)

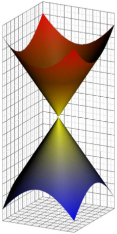
The lattice outside LQCD

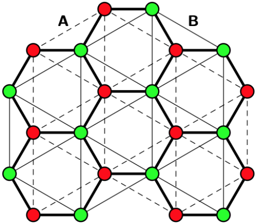
Other applications: Nuclear Physics

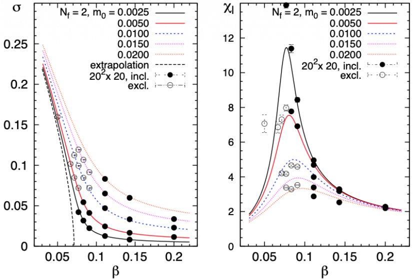


Bulava et al. 2026

- Nucleon potentials
- Spectrum of bound states (still difficult)



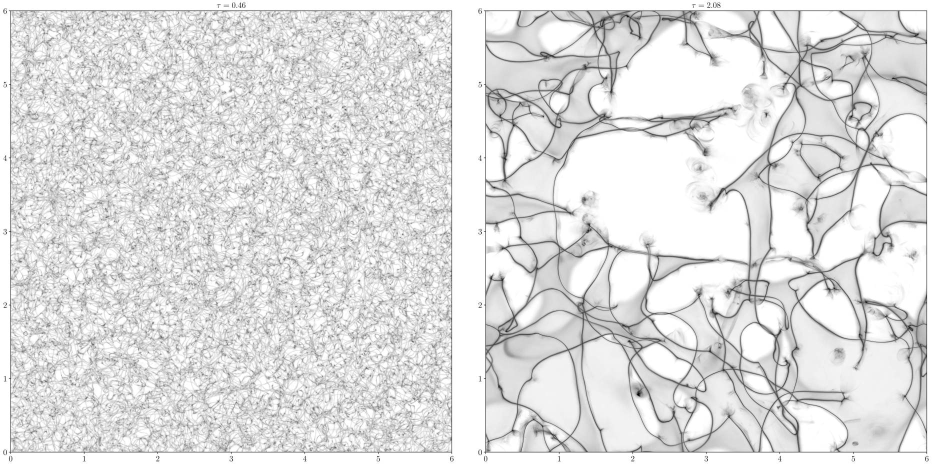




Drut, Lähde 2009

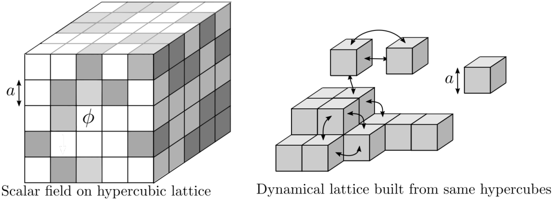
- Ab-initio simulation of QED or gauged Z_n in crystals
- Energy gaps, insulator-semimetal transitions, etc

Other applications: Axion searches



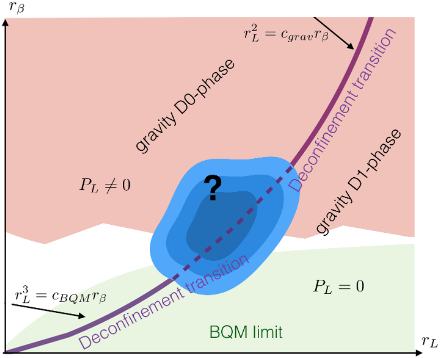
- Assuming a postinflationary breaking of the Peccei-Quinn symmetry
- Estimate of m_a, structure formation, power spectrum...

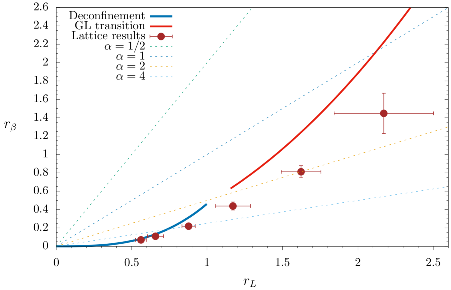
Other applications: Quantum gravity



- Assume the quantum version of Einstein theory is renormalizable, dynamical triangulations
- Loop Quantum Gravity is quite amenable for lattice simulations

Other applications: SUSY





- Simulate SUSY theories
- Spectrum of different particles

Summary

- Lattice Gauge Theories are an essential tool to explore the Standard Model
  - Best way to explore non-perturbative theories like QCD
  - Opens the door to perform high-precision particle physics

LQCD simulations are critical for the Intensity Frontier

- LQCD uses significant resources
  - Pushes the limits of HPC facilities
  - Helps to develop new simulation algorithms
- These developments find applications in other areas of physics
  - Nuclear physics
  - Condensed matter
  - Other BSM: Axions, SUSY, QG...

LQCD is one of the most important fronts of Particle Physics research

Thank you for your attention

Backup Slides

Lattice QCD results: Flavor Physics

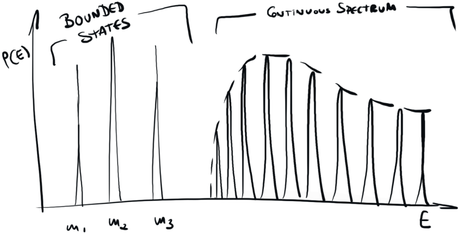
- We can compute inclusive rates in LQCD

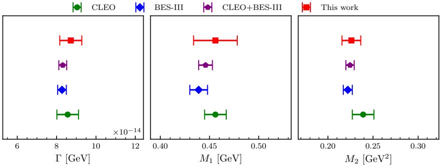
$$B \to X_c \ell \nu$$

- Extract spectral function $\rho(E)$ from correlators

$$C(t) = \int \rho(E) e^{-Et}$$

- $\rho(E)$ discrete in the lattice
  - Smeared $\rho(E)$, well defined CL
- Order of limits, important!
  - First $a \to 0$, then smearing $\to 0$





ETMC 2025